Chapter 9 Notes – Volume
Basic Geometry

Name Key

9.1 Solid Figures

Goal: Identify and name solid figures.

Solid: three-dimensional shapes

Polyhedron: a solid formed by polygons.

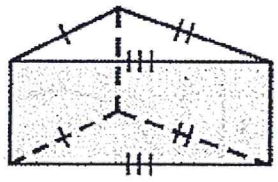
Faces: the plane surfaces of a polyhedron.

Edges: the segments joining the vertices of a polyhedron.

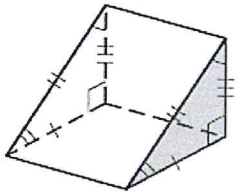
Bases: the top and/or bottom planes of a figure. A prism has 2 bases. A pyramid has 1 base.

Tell whether the solid is a polyhedron. Then name the solid and find the number of faces, vertices, and edges.

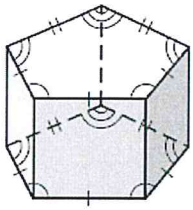
a) Polyhedron? Yes
Name: triangular prism
F: 5 V: 6 E: 9



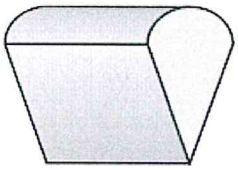
b) Polyhedron? Yes
Name: triangular prism
F: 5 V: 6 E: 9



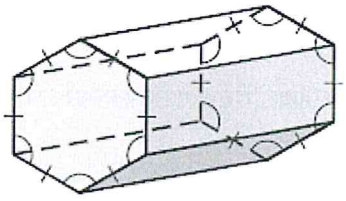
c) Polyhedron? Yes
Name: pentagonal prism
F: 7 V: 10 E: 15



d) Polyhedron? No
Name: _____
F: _____ V: _____ E: _____

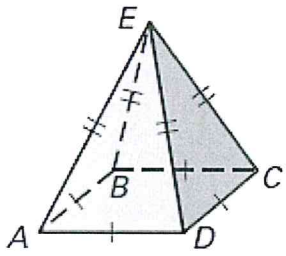


e) Polyhedron? Yes
Name: hexagonal prism
F: 8 V: 12 E: 18

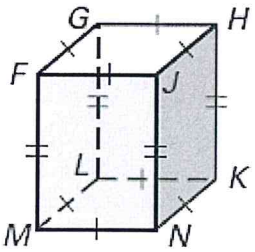


f) Polyhedron? No
Name: Cylinder
F: _____ V: _____ E: _____

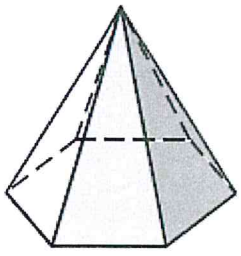
g) Polyhedron? Yes
Name: Rectangular Pyramid
F: 5 V: 5 E: 8



h) Polyhedron? Yes
Name: Rectangular Prism
F: 6 V: 8 E: 12

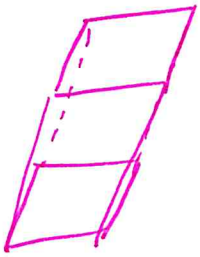


i) Polyhedron? Yes
Name: Hexagonal Pyramid
F: 7 V: 7 E: 12



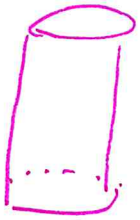
Sketch each figure.

a) a rectangular prism



b) a triangular prism

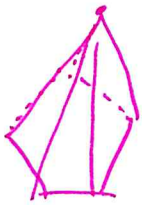
c) cylinder



d) cone

e) a rectangular pyramid

f) a pentagonal pyramid



Euler's Formula
$F + V = E + 2$

Use Euler's Formula to find the number of faces, edges, or vertices.

a) A pyramid has 9 faces and 9 vertices. How many edges does it have?

$9 + 9 = E + 2$

$E = 16$

b) A prism has 8 faces and 18 edges. How many vertices does it have?

$8 + V = 18 + 2$

$V = 12$

c) A polyhedron has 15 edges and 7 vertices. How many faces does it have?

$F + 7 = 15 + 2$

$F = 10$

9.2 Surface Area of Prisms and Cylinders

Goal: Find the surface areas of prisms and cylinders.

Prism: a polyhedron with two congruent bases that lie in parallel planes

Surface area: on a polyhedron, the sum of the area of its faces

Cylinder: a solid with two congruent circular bases that lie in parallel planes

*Note: Surface area is always labeled units^2

Surface Area of a Prism	Surface Area of a Cylinder
$SA = 2B + Ph$ (h: height; B: area of Base; P: Perimeter of Base)	$SA = 2\pi r^2 + 2\pi rh$ (h: height; πr^2: Area of Base; $2\pi r$: Perimeter of Base)

Find the surface area of each figure

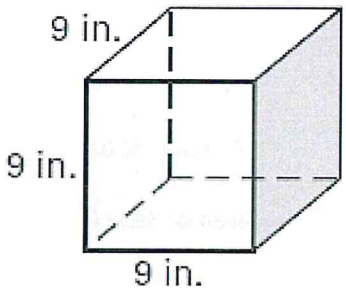
a) Shape of base: square

Area of Base (B): 81 $9(9)$

Perimeter of Base (P): 36 $9(4)$

Height of shape (h): 9

Surface Area: 486 in^2 $2(81) + 36(9)$



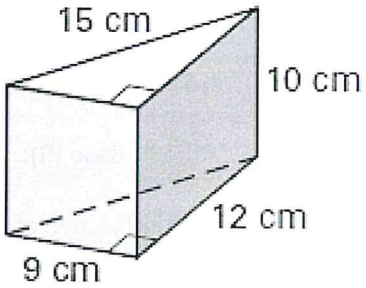
b) Shape of base: triangle

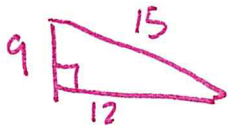
Area of Base (B): 54 $\frac{1}{2}(12)(9)$

Perimeter of Base (P): 36 $9 + 12 + 15$

Height of shape (h): 10

Surface Area: 468 cm^2 $2(54) + 36(10)$





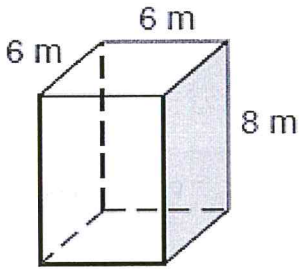
c) Shape of base: Square

Area of Base (B): 36 $6(6)$

Perimeter of Base (P): 24 $(6)(4)$

Height of shape (h): 8

Surface Area: 264 m^2 $2(36) + 24(8)$



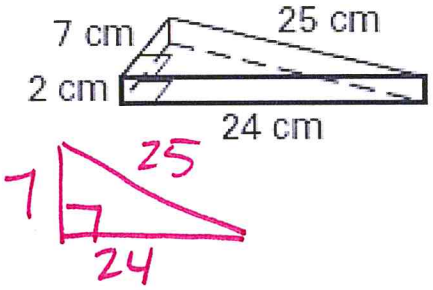
d) Shape of base: triangle

Area of Base (B): 84 $\frac{1}{2}(7)(24)$

Perimeter of Base (P): 56 $7+24+25$

Height of shape (h): 2

Surface Area: 280 cm^2 $2(84) + 56(2)$



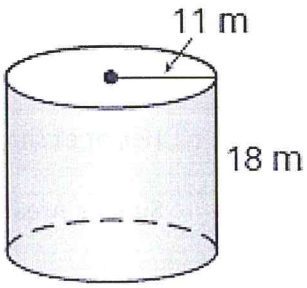
e) Shape of base: Circle

Area of Base (B): 380 $\pi 11^2$

Circumference of Base (P): 69 $2\pi(11)$

Height of shape (h): 18

Surface Area: 2002 m^2 $2(380) + 69(18)$



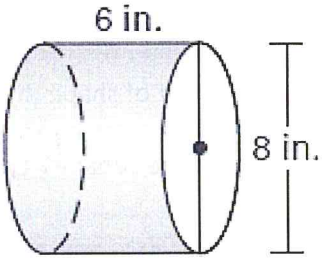
f) Shape of base: Circle

Area of Base (B): 50 $\pi 4^2$

Circumference of Base (P): 25 $2\pi 4$

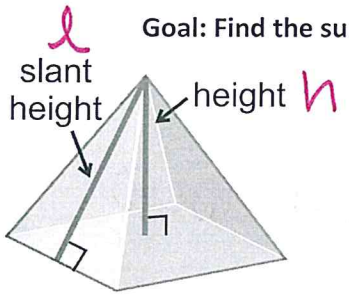
Height of shape (h): 6

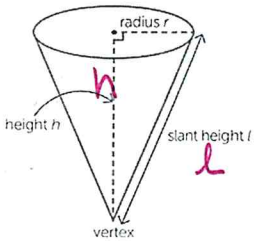
Surface Area: 250 in^2 $2(50) + 25(6)$



9.3 Surface Area of Pyramids and Cones – Day 1

Goal: Find the surface area of pyramids and cones.





Surface Area of a Pyramid	Surface Area of a Cone
$SA = B + \frac{1}{2}Pl$	$SA = \pi r^2 + \pi rl$

Find the surface area of each figure.

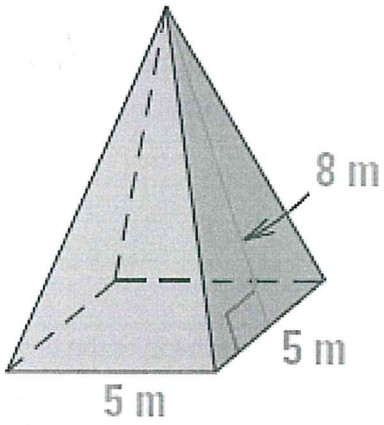
a) Shape of the base: Square

Area of the base: 25 5(5)

Perimeter of the base: 20 5(4)

Slant height: 8

Surface Area: 105 m^2 25 + 1/2(20)(8)



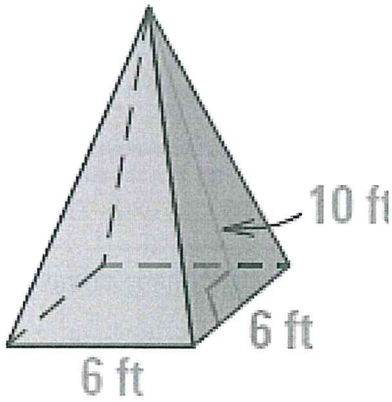
b) Shape of the base: Square

Area of the base: 36 6(6)

Perimeter of the base: 24 6(4)

Slant height: 10

Surface Area: 156 ft^2 36 + 1/2(24)(10)



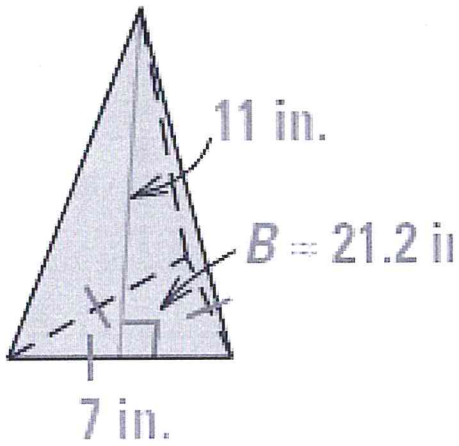
c) Shape of the base: triangle

Area of the base: 21.2

Perimeter of the base: 21 7x3

Slant height: 11

Surface Area: 136.7 in² 21.2 + 1/2(11)(21)

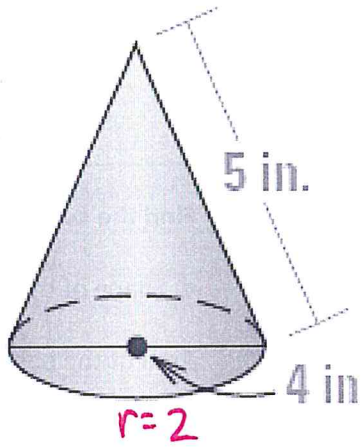


d) Shape of the base: circle

Area of the base: 12.6 π2²

Slant height: 5

Surface Area: 44 in² 12.6 + π(2)(5)

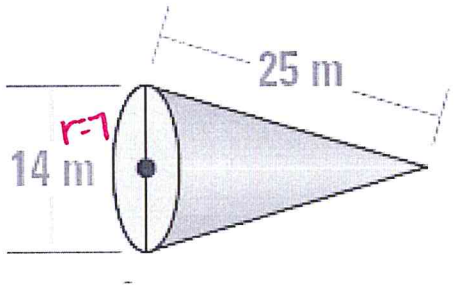


e) Shape of the base: circle

Area of the base: 154 π7²

Slant height: 25

Surface Area: 704 m² 154 + π(7)(25)

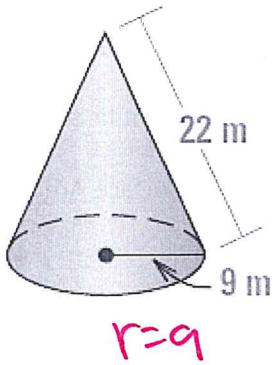


f) Shape of the base: circle

Area of the base: 254.5 π9²

Slant height: 22

Surface Area: 876.5 m² 254.5 + π(9)(22)



9.3 Surface Area of Pyramids and Cones – Day 2

Goal: Find the surface area of pyramids and cones.

***If the slant height of a cone or pyramid is not given, you just first use the** Pythagorean Theorem **to find it.**

$a^2 + b^2 = c^2$

Surface Area of a Pyramid	Surface Area of a Cone
$SA = B + \frac{1}{2}Pl$ (B: Area of Base; P: Perimeter of Base; l: slant height)	$SA = \pi r^2 + \pi rl$ (πr^2: B; r: radius; l: slant height)

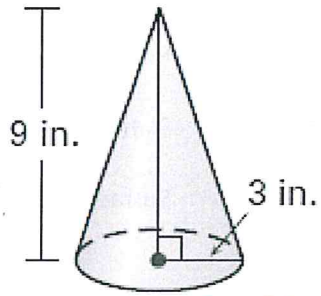
Find the surface area of each figure.

a) Shape of the base: circle

Area of the base: 28.3 $\pi 3^2$

Slant height: 9.5

Surface Area: 117.8 in^2 $28.3 + \pi(3)(9.5)$



$9^2 + 3^2 = l^2$

$81 + 9 = l^2$

$\sqrt{90} = \sqrt{l^2}$

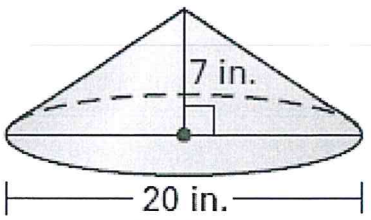
$l = 9.5$

b) Shape of the base: circle

Area of the base: 314.2 $\pi 10^2$

Slant height: 12.2

Surface Area: 1097.5 in^2 $314.2 + \pi(10)(12.2)$



r = 10

$7^2 + 10^2 = l^2$

$49 + 100 = l^2$

$\sqrt{149} = \sqrt{l^2}$

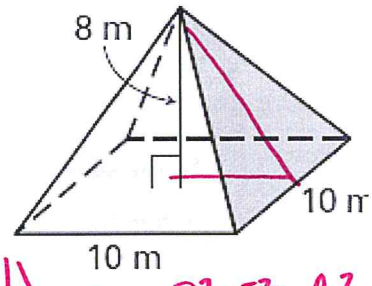
c) Shape of the base: Square

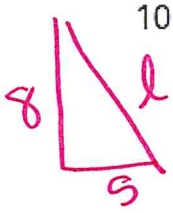
Area of the base: 100 (10)(10)

Perimeter of the base: 40 10(4)

Slant height: 9.4

Surface Area: 288 m^2 100 + 1/2(40)(9.4)





$8^2+5^2=l^2$
$64+25=l^2$
$\sqrt{89}=\sqrt{l^2}$
$l=9.4$

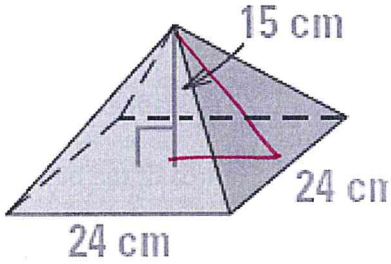
d) Shape of the base: Square

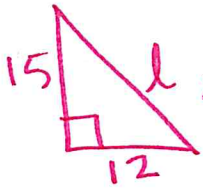
Area of the base: 576 24(24)

Perimeter of the base: 96 24(4)

Slant height: 19.2

Surface Area: 1497.6 cm^2 576 + 1/2(96)(19.2)





$15^2+12^2=l^2$
$225+144=l^2$
$\sqrt{369}=\sqrt{l^2}$
$l=19.2$

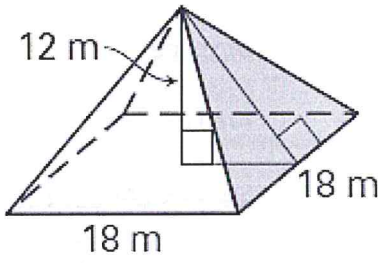
e) Shape of the base: Square

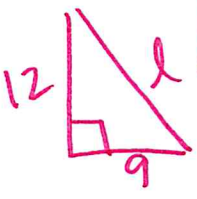
Area of the base: 324 18(18)

Perimeter of the base: 72 18(4)

Slant height: 15

Surface Area: 864 m^2 324 + 1/2(72)(15)





$12^2+9^2=l^2$
$144+81=l^2$
$\sqrt{225}=\sqrt{l^2}$
$l=15$

9.4 Volume of Prisms and Cylinders

Goal: Find the volumes of prisms and cylinders.

Volume: the number of cubic units contained in a solids interior.

*Note: Volume is always labeled $units^3$

Volume of a Prism	Volume of a Cylinder
$V = Bh$ ← height; B: Area of Base	$V = \pi r^2 h$ ← height; πr^2: Area of Base

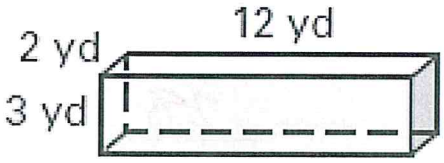
Find the volume of each solid.

a) Shape of the base: rectangle

Area of the base: 24 2(12)

Height: 3

Volume: 72 yd^3 24(3)

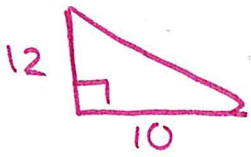


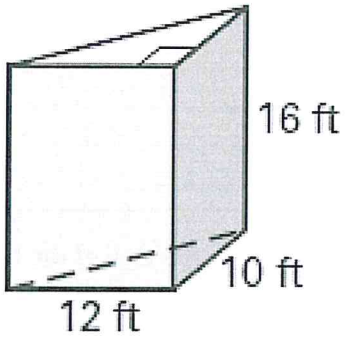
b) Shape of the base: triangle

Area of the base: 60 1/2(12)(10)

Height: 16

Volume: 960 ft^3 60(16)



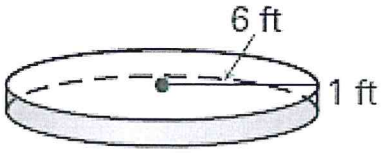


c) Shape of the base: circle

Area of the base: 113 $\pi 6^2$

Height: 1

Volume: 113 ft^3 (113)(1)

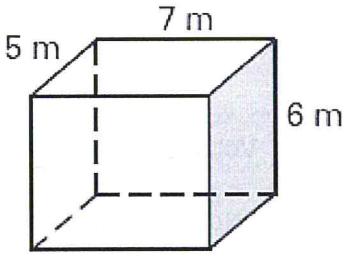


d) Shape of the base: Rectangle

Area of the base: 35 5(7)

Height: 6

Volume: 210 m^3 35(6)

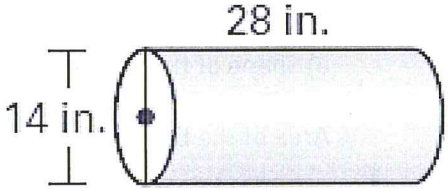


e) Shape of the base: circle

Area of the base: 154 $\pi 7^2$

Height: 28

Volume: 4312 in^3 154(28)

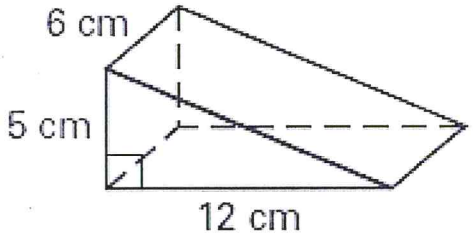


f) Shape of the base: triangle

Area of the base: 30 1/2(12)(5)

Height: 6

Volume: 180 cm^3 30(6)



9.5 Volume of Pyramids and Cones

Goal: Find the volumes of pyramids and cones.

Volume of a Pyramid	Volume of a Cone
$V = \frac{1}{3}Bh$ (Area of Base; height)	$V = \frac{1}{3}\pi r^2 h$ ($B = \pi r^2$)

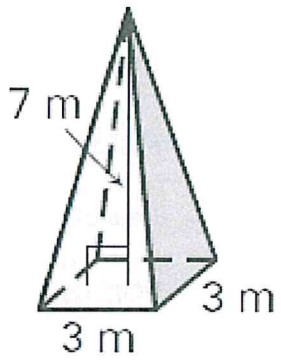
Find the volume of each figure.

a) Shape of the base: square

Area of the base: 9 $3(3)$

Height: 7

Volume: 21 m^3 $\frac{1}{3}(9)(7)$

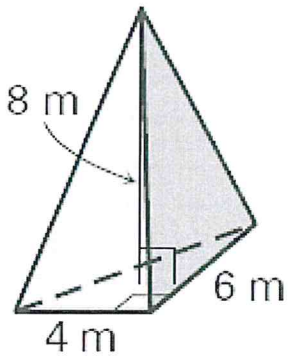


b) Shape of the base: triangle

Area of the base: 12 $\frac{1}{2}(4)(6)$

Height: 8

Volume: 32 m^3 $\frac{1}{3}(12)(8)$

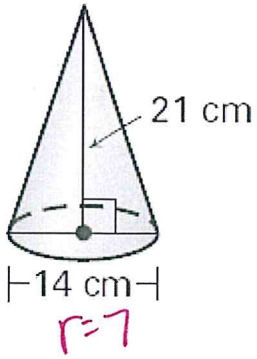


c) Shape of the base: circle

Area of the base: 154 $\pi 7^2$

Height: 21

Volume: 1078 cm^3 $\frac{1}{3}(154)(21)$

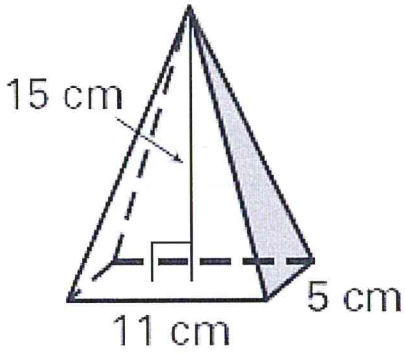


d) Shape of the base: rectangle

Area of the base: 55 11(5)

Height: 15

Volume: 275 cm^3 1/3(55)(15)

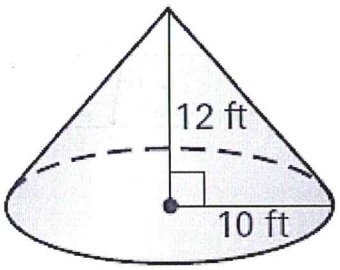


e) Shape of the base: circle

Area of the base: 314 $\pi 10^2$

Height: 12

Volume: 1256 ft^3 1/3(314)(12)

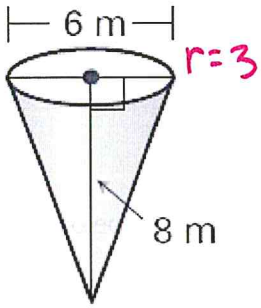


f) Shape of the base: circle

Area of the base: 28.3 $\pi 3^2$

Height: 8

Volume: 75.47 m^3 1/3(28.3)(8)



9.5 Volume of Pyramids and Cones – Day 2

Goal: Find the volumes of pyramids and cones.

***If the height of a cone or pyramid is not given, you just first use the Pythagorean Theorem to find it.**

Volume of a Pyramid	Volume of a Cone
$V = \frac{1}{3}Bh$ (Area of Base; height)	$V = \frac{1}{3}\pi r^2 h$ (πr^2 = B)

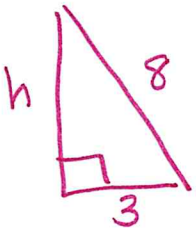
Find the volume of each figure.

a) Shape of the base: rectangle

Area of the base: 36 $6(6)$

Height: 7.4

Volume: 88.8 m^3 $\frac{1}{3}(36)(7.4)$

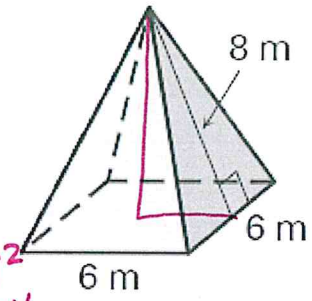


$3^2 + h^2 = 8^2$

$9 + h^2 = 64$

$\sqrt{h^2} = \sqrt{55}$

$h = 7.4$

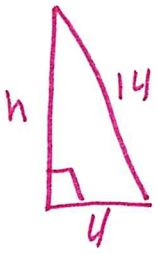


b) Shape of the base: circle

Area of the base: 50.3 $\pi 4^2$

Height: 13.4

Volume: 224.1 cm^3 $\frac{1}{3}(50.3)(13.4)$

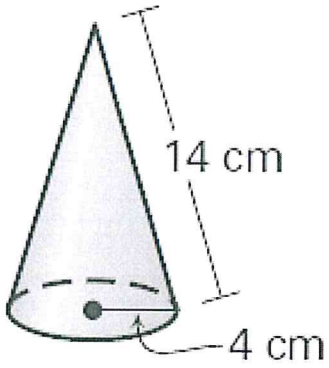


$4^2 + h^2 = 14^2$

$16 + h^2 = 196$

$\sqrt{h^2} = \sqrt{180}$

$h = 13.4$

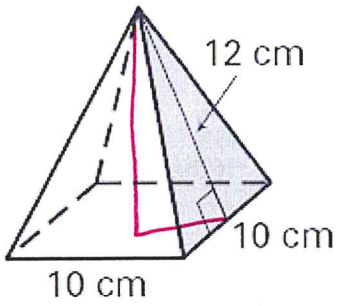


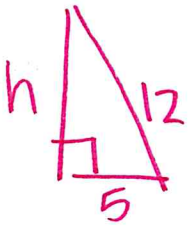
c) Shape of the base: Square

Area of the base: 100 10×10

Height: 10.9

Volume: 363.3 cm^3 $1/3(100)(10.9)$





$h^2 + 5^2 = 12^2$

$h^2 + 25 = 144$

$\sqrt{h^2} = \sqrt{119}$

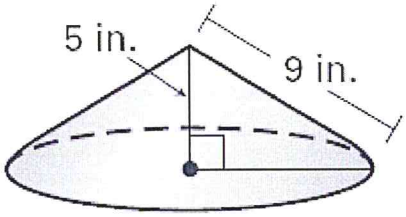
$h = 10.9$

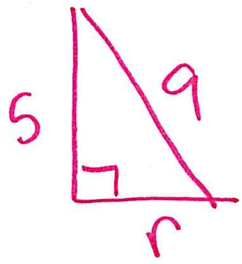
d) Shape of the base: circle

Area of the base: 176.7 $\pi(7.5)^2$

Height: 5

Volume: 294.5 in^3 $1/3(176.7)(5)$





$5^2 + r^2 = 9^2$

$25 + r^2 = 81$

$\sqrt{r^2} = \sqrt{56}$

$r = 7.5$

9.6 Surface Area and Volume of Spheres

Goal: Find the surface area and volume of spheres.

r = radius

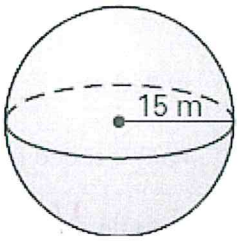
Surface Area of a Sphere	Volume of a Sphere	Volume of a Hemisphere
$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$	$V = \frac{2}{3}\pi r^3$
units^2	units^3	units^3

Find the surface area of each sphere.

a) Radius: 15

Radius squared (r^2): 225

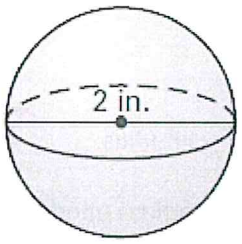
Surface Area: 2827.4 m^2 $4\pi(225)$



b) Radius: 1

Radius squared (r^2): 1

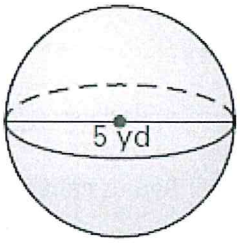
Surface Area: 12.56 in^2 $4\pi(1)$



c) Radius: 2.5

Radius squared (r^2): 6.25

Surface Area: 78.5 yd^2 $4\pi(6.25)$

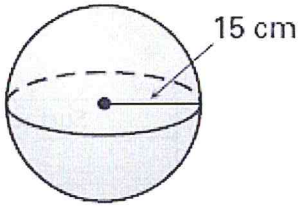


Find the volume of each sphere or hemisphere.

a) Radius: 15

Radius cubed (r^3): 3375

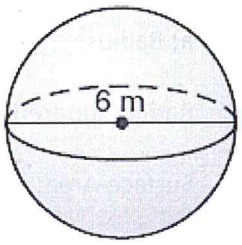
Volume: 14137 cm^3 $4/3\pi(3375)$



b) Radius: 3

Radius cubed (r^3): 27

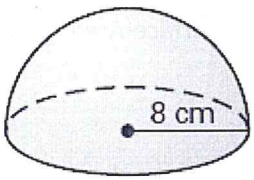
Volume: 113 m^3 $4/3\pi(27)$



c) Radius: 8

Radius cubed (r^3): 512

Volume: 1072 cm^3 $2/3\pi(512)$



d) Radius: 5.5

Radius cubed (r^3): 166.375

Volume: 348.5 yd^3 $2/3\pi(166.375)$