Simplifying Radicals

Objective: You will be able to simplify radical expressions.

What is a radical? a symbol that means root. Break it apart of a number times itself. "Square root", "nth root" (index)

Example: $\sqrt{26}$

A simplified radical expression has no perfect square factors under the radical.

To simplify a radical expression, factor out the perfect square factor and then square root it!

Example: $\sqrt{24}$ $2\sqrt{6}$ $\sqrt{24}$ $2\sqrt{6}$

$\sqrt{2}$ $\sqrt{12}$

$\sqrt{2}$ $\sqrt{6}$

$\sqrt{2}$ $\sqrt{3}$

$\sqrt{4}$ $\sqrt{6}$

2 2

Simplify each radical expression.

	Possible factors	Simplified form
1. $\sqrt{32}$	$\sqrt{4}$ $\sqrt{8}$ 2 2 $\sqrt{4}$ $\sqrt{2}$ 2 2	$2\cdot 2\sqrt{2}$ $4\sqrt{2}$
2. $\sqrt{48}$	$\sqrt{4}$ $\sqrt{12}$ 2 2 $\sqrt{4}$ $\sqrt{3}$ 2 2	$4\sqrt{3}$
3. $\sqrt{150}$	$\sqrt{25}$ $\sqrt{6}$ 5 5	$5\sqrt{6}$
4. $\sqrt{147}$	$\sqrt{3}$ $\sqrt{49}$ 7 7	$7\sqrt{3}$

Multiply the radical expressions and then simplify the product.

	Product	Simplification
1. $\sqrt{5}\cdot\sqrt{8}$	$\sqrt{40}$ $\sqrt{4}\ \sqrt{10}$ $2\ 2$	$2\sqrt{10}$
2. $6\sqrt{10}\cdot\sqrt{2}$	$6\sqrt{20}$ $\sqrt{4}\ \sqrt{5}$ $2\ 2$	$6\cdot 2\sqrt{5}$ $12\sqrt{5}$
3. $2\sqrt{6}\cdot 7\sqrt{3}$	$14\sqrt{18}$ $\sqrt{9}\ \sqrt{2}$ $3\ 3$	$14\cdot 3\sqrt{2}$ $42\sqrt{2}$

To simplify the square root of a quotient, we can find the square root of the numerator, and the square root of the denominator.
(dividing) (top) (bottom)

Examples:

1. $\sqrt{\frac{100}{36}} = \frac{\sqrt{100}}{\sqrt{36}} = \frac{10}{6} = \frac{5}{3}$

2. $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$

3. $\sqrt{\frac{13}{64}}\ \frac{\sqrt{13}}{\sqrt{64}} = \frac{\sqrt{13}}{8}$

4. $\sqrt{\frac{5}{49}} = \frac{\sqrt{5}}{\sqrt{49}} = \frac{\sqrt{5}}{7}$

In Algebra, we are not allowed to have a radical in the denominator of a fraction. To resolve this problem, we will $\boxed{\text{"rationalize the denominator."}}$

Example:

1. $\frac{5}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{\sqrt{49}} = \frac{5\sqrt{7}}{7}$

2. $\frac{7}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{7\sqrt{6}}{\sqrt{36}} = \frac{7\sqrt{6}}{6}$

3. $\sqrt{\frac{144}{5}} = \frac{\sqrt{144}}{\sqrt{5}} = \frac{12}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{12\sqrt{5}}{\sqrt{25}} = \frac{12\sqrt{5}}{5}$

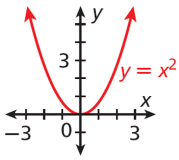
4. $\sqrt{\frac{144}{6}} = \frac{\sqrt{144}}{\sqrt{6}} = \frac{12}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{\sqrt{36}} = \frac{12\sqrt{6}}{6}$

$\boxed{2\sqrt{6}}$

8.1 Identifying Quadratic Equations

Objectives: 1. Identify quadratic functions and determine whether they have a minimum or maximum.
2. Graph a quadratic function and give its domain and range.

The function $y = x^2$ is shown in the graph. Notice that the graph is not linear. This function is a *quadratic function*. A quadratic function is any function that can be written in the standard form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The function $y = x^2$ can be written as $y = 1x^2 + 0x + 0$, where $a = 1$, $b = 0$, and $c = 0$.



You have identified linear functions by finding that a constant change in x corresponded to a constant change in y. The differences between y-values for a constant change in x-values are called *first differences*.

Constant change in x-values: +1 +1 +1 +1

x	0	1	2	3	4
$y = x^2$	0	1	4	9	16

First differences: +1 +3 +5 +7

Second differences: +2 +2 +2

Notice that the quadratic function $y = x^2$ does not have constant first differences. It has constant *second differences*. This is true for all quadratic functions.

Tell whether the function is quadratic. Explain.

x	y
−2	−9
−1	−2
0	−1
1	0
2	7

+1 +1 +1 +1

+7 +1 +1 +7

−6 +0 +6

No, because ther isn't a constant 2nd diff.

$y = 7x + 3$

No, there isn't an x^2 term.

$y - 10x^2 = 9$

$+10x^2 \quad +10x^2$

$y = 10x^2 + 9$

Yes you can write it in quadratic standard form.

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

x +1

−3 −1 +1 +3

+2 +2 +2

Yes, the 2nd diff. is constant.

$y = -x - 1$

No, there is no x^2 term.

$y + x = 2x^2$ $-x$

$-x$

$y = 2x^2 - x$

Yes, because it can be written in standard form.

Helpful Hint

In a quadratic function, only a cannot equal 0. It is okay for the values of b and c to be 0.

The graph of a quadratic function is a curve called a parabola. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.

Use a table of values to graph the quadratic function.

$y = \frac{1}{3}x^2$

$(-6)^2 \quad \frac{36}{3} = 12$

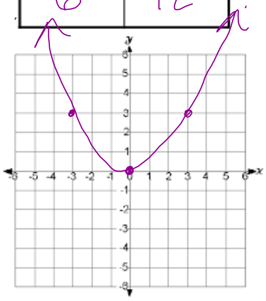
$(-3)^2 \quad \frac{9}{3} = 3$

$0^2 = \frac{0}{3} = 0$

$3^2 = \frac{9}{3} = 3$

$(6)^2 = \frac{36}{3} = 12$

x	y
-6	12
-3	3
0	0
3	3
6	12



$y = -4x^2$

$-4(-2)^2$, $-4(4)$

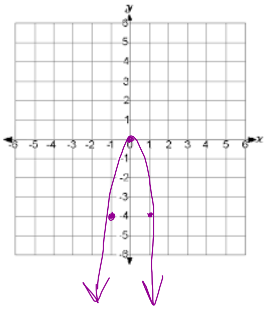
$-4(-1)^2$, $-4(1)$

$-4(0)^2$, $-4(0)$

$-4(1)^2$, $-4(1)$

$-4(2)^2$, $-4(4)$

x	y
-2	-16
-1	-4
0	0
1	-4
2	-16



$y = -3x^2 + 1$

$-3(-2)^2+1$, $-3(4)+1$, $-12+1$

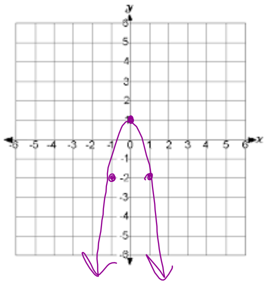
$-3(-1)^2+1$, $-3(1)+1$, $-3+1$

$-3(0)^2+1$, $0+1$

$-3(1)^2+1$, $-3(1)+1$, $-3+1$

$-3(2)^2+1$, $-3(4)+1$, $-12+1$

x	y
-2	-11
-1	-2
0	1
1	-2
2	-11



Notice that the only difference between the last two equations is the value of *a*. When a quadratic function is written in the form $y = ax^2 + bx + c$, the value of *a* determines the direction a parabola opens.

- A parabola opens upward when $a > 0$.
- A parabola opens downward when $a < 0$.

Tell whether the graph of the quadratic function opens upward or downward. Explain.

$y - \frac{1}{3}x^2 = x - 3$

$+\frac{1}{3}x^2 \quad +\frac{1}{3}x^2$

$y = \frac{1}{3}x^2 + x - 3$

upward because $a > 0$

$y = 5x - 3x^2$

$y = -3x^2 + 5x$

downward $a < 0$

$f(x) = -4x^2 - x + 1$

downward because $a < 0$

$y - 5x^2 = 2x - 6$

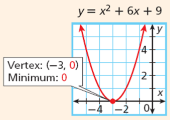
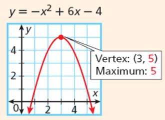
$+5x^2 \quad +5x^2$

$y = 5x^2 + 2x - 6$

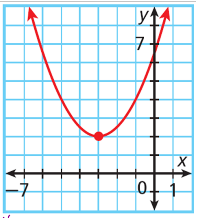
upward because $a > 0$

The highest or lowest point on a parabola is the <u>Vertex</u>. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

Minimum and Maximum Values

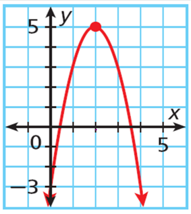
WORDS	If $a > 0$, the parabola opens upward, and the y-value of the vertex is the **minimum** value of the function.	If $a < 0$, the parabola opens downward, and the y-value of the vertex is the **maximum** value of the function.
GRAPHS	$y = x^2 + 6x + 9$ 	$y = -x^2 + 6x - 4$ 

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A. 

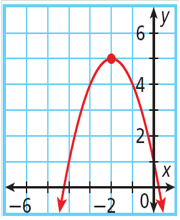
vertex $(-3, 2)$

min: 2

B. 

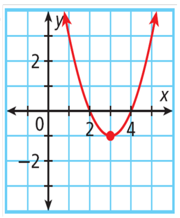
vertex $(2, 5)$

max: 5

a. 

vertex $(-2, 5)$

max: 5

b. 

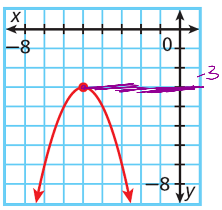
vertex $(3, -1)$

min: −1

Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph.

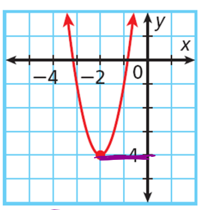
For the graph of $y = x^2 - 4x + 5$, the range begins at the minimum value of the function, where $y = 1$. All the y-values of the function are greater than or equal to 1. So the range is $y \geq 1$.

Find the domain and range.



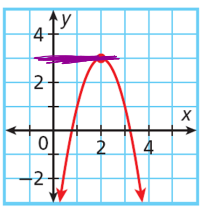
D: all reall numb
ℝ ℝ

Range: $y \leq -3$



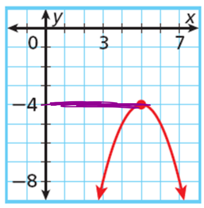
D: ℝ

R: $y \geq -4$



D: ℝ

R: $y \leq 3$



D: ℝ

R: $y \leq -4$

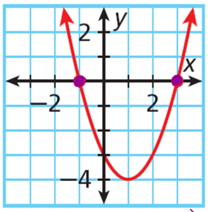
8.2 Characteristics of Quadratic Functions

Objectives: 1. Find the zeros of a quadratic function from its graph.
2. Find the axis of symmetry and the vertex of a parabola.

Find the zeros of the quadratic function from its graph. Check your answer.

(cross x-axis)

$y = x^2 - 2x - 3$



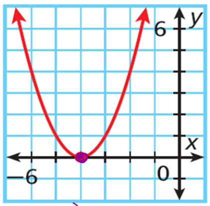
$(-1,0)$ $(3,0)$

$y=(-1)-2(-1)-3$
$y=1+2-3$
$y=3-3$
$y=0$

two

$y=(3)^2-2(3)-3$
$y=9-6-3$
$y=3-3$
$y=0$

$y = x^2 + 8x + 16$

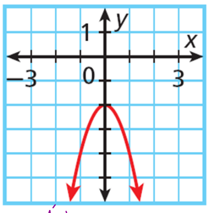


$(-4,0)$

$y=(-4)^2+8(-4)+16$
$y=16-32+16$
$y=-16+16$
$y=0$

one

$y = -2x^2 - 2$



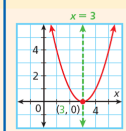
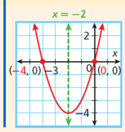
None

No y value of zero.

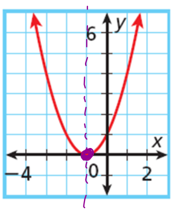
A vertical line that divides a parabola into two symmetrical halves is the **axis of symmetry**. The axis of symmetry always passes through the vertex of the parabola. You can use the zeros to find the axis of symmetry.

$x = \boxed{}$

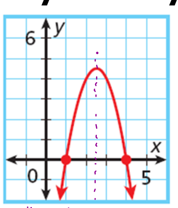
Finding the Axis of Symmetry by Using Zeros

WORDS	NUMBERS	GRAPH
One Zero		
If a function has one zero, use the *x*-coordinate of the vertex to find the axis of symmetry.	Vertex: (3, 0) Axis of symmetry: $x = 3$	
Two Zeros		
If a function has two zeros, use the average of the two zeros to find the axis of symmetry.	$\frac{-4+0}{2} = \frac{-4}{2} = -2$ Axis of symmetry: $x = -2$	

Find the axis of symmetry of each parabola.

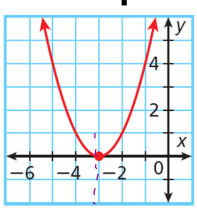


$x=-1$

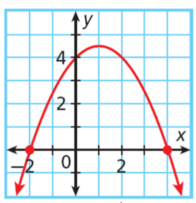


$\frac{1+4}{2} = \frac{5}{2}$

$x=\frac{5}{2}$



$x=-3$



$x = \frac{-2+4}{2} = \frac{2}{2} = 1$

$x=1$

If a function has no zeros or they are difficult to identify from a graph, you can use a formula to find the axis of symmetry. The formula works for all quadratic functions.

Finding the Axis of Symmetry by Using the Formula

FORMULA	EXAMPLE
For a quadratic function $y = ax^2 + bx + c$, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.	$y = 2x^2 + 4x + 5$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(2)} = -1$ The axis of symmetry is $x = -1$.

Find the axis of symmetry.

$y = -3x^2 + 10x + 9$

$a = -3 \quad b = 10 \quad c = 9$

$x = \frac{-b}{2a} = \frac{-(10)}{2(-3)} = \frac{-10}{-6}$

$x = \frac{5}{3}$

$y = 2x^2 + x + 3$

$a = 2 \quad b = 1 \quad c = 3$

$x = \frac{-b}{2a} = \frac{-1}{2(2)} = \boxed{\frac{-1}{4} = x}$

$y = 0.25x^2 + 2x + 3$

$a = .25 \quad b = 2 \quad c = 3$

$x = \frac{-b}{2a} = \frac{-2}{2(.25)} = \frac{-2}{.5}$

$\frac{-2}{1/2} = -2(2) = -4$

Finding the Vertex of a Parabola	
Step 1	To find the x-coordinate of the vertex, find the axis of symmetry by using zeros or the formula.
Step 2	To find the corresponding y-coordinate, substitute the x-coordinate of the vertex into the function.
Step 3	Write the vertex as an ordered pair.

Find the vertex.

1. $y = 0.25x^2 + 2x + 3$

$a = .25 \quad b = 2 \quad c = 3$

$x = \frac{-2}{2(.25)} = \frac{-2}{.5} = \frac{-2}{1/2} = -2(2) = -4$

$(-4, -1)$

$y = .25(-4)^2 + 2(-4) + 3$

$y = \frac{1}{4}(16) - 8 + 3$

$y = 4 - 8 + 3$

$y = -4 + 3 = -1$

2. $y = -3x^2 + 6x - 7$

$a = -3 \quad b = 6 \quad c = -7$

$x = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1$

$\boxed{(1, -4)}$

$y = -3(1)^2 + 6(1) - 7$

$y = -3(1) + 6(1) - 7$

$y = -3 + 6 - 7$

$y = 3 - 7$

$y = -4$

3. $y = x^2 - 4x - 10$

$a = 1 \quad b = -4 \quad c = -10$

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$\boxed{(2, -14)}$

$y = (2)^2 - 4(2) - 10$

$4 - 8 - 10$

$-4 - 10$

$y = -14$

4. $y = 3x^2 + 12x + 8$

$a = 3 \quad b = 12 \quad c = 8$

$x = \frac{-b}{2a} = \frac{-12}{2(3)} = \frac{-12}{6} = -2$

$\boxed{(-2, -4)}$

$y = 3(-2)^2 + 12(-2) + 8$

$3(4) + 12(-2) + 8$

$y = 12 - 24 + 8$

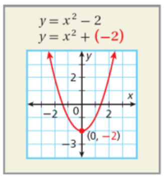
$-12 + 8$

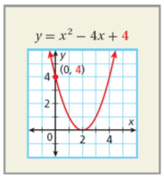
$y = -4$

8.3 Graphing Quadratic Functions

Objective: Graph a quadratic function in the form $y = ax^2 + bx + c$.

Recall that a y-intercept is the y-coordinate of the point where a graph intersects the y-axis. The x-coordinate of this point is always 0. For a quadratic function written in the form $y = ax^2 + bx + c$, when $x = 0$, $y = c$. So the y-intercept of a quadratic function is c.





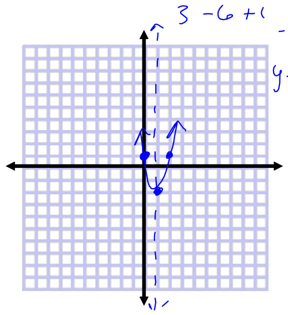
Graph. $a=3 \quad b=-6 \quad c=1$

1. $y = 3x^2 - 6x + 1$

$x = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$

$(1,-2)$

$3(1)^2 - 6(1) + 1$
$3(1) - 6(1) + 1$
$3 - 6 + 1$
$-3 + 1$
$y = -2$



Step 1: Find axis of symmetry

Step 2: Find the vertex

Step 3: Find the y-intercept

Step 4: Reflect the y-intercept over the axis of symmetry.

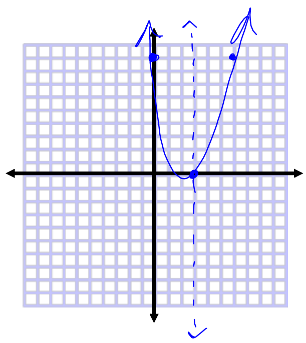
2. $y = x^2 - 6x + 9$

$a=1 \quad b=-6 \quad c=9$

$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$(3,0)$ $\quad y = 3^2 - 6(3) + 9$
$9 - 18 + 9$
$-9 + 9$
0

y-int: 9



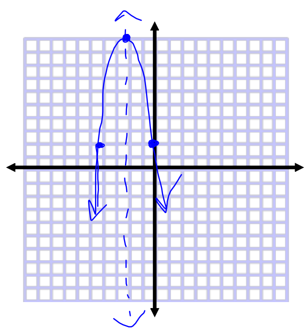
3. $y = -2x^2 - 8x + 2$

$a=-2 \quad b=-8 \quad c=2$

$x = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2$

$(-2,10)$ $\quad y = -2(-2)^2 - 8(-2) + 2$
$y = -2(4) - 8(-2) + 2$
$= -8 + 16 + 2$
$= 8 + 2$
$= 10$

y int: 2



4. $y = x^2 - 2x - 3$

$a=1 \quad b=-2 \quad c=-3$

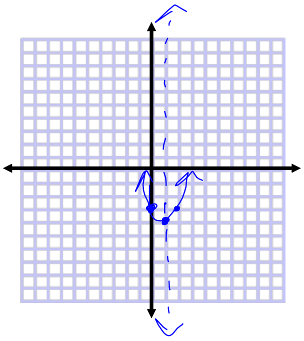
$\frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$y=(1)^2-2(1)-3$

$1-2-3$

-4

$(1,-4)$ y-int: -3



5. $y = 2x^2 + 2$

$a=2 \quad b=0 \quad c=2$

$x=\frac{-b}{2a}=\frac{0}{2(2)}=0$

$2(0)^2+2$

2

$(0,2)$

$(1,4)$ *reflect this*

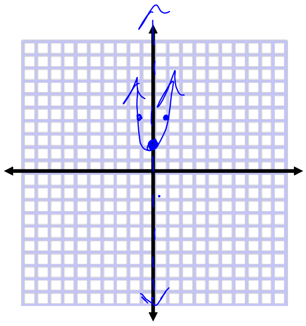
$2(1)^2+2$

$2(1)+2$

$2+2$

4

Pick another value close to axis of symmetry Then reflect that point.



6. $y = 0.25x^2 + 2x + 3$

$a=\frac{1}{4} \quad b=2 \quad c=3$

$x=\frac{-b}{2a}=\frac{-2}{2(1/4)}=\frac{-2}{1/2}=-2(2)=-4$

$(-4,-1)$

$\frac{1}{4}(-4)^2+2(-4)+3$

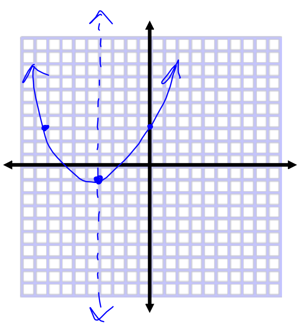
$\frac{1}{4}(16)-8+3$

$4-8+3$

$-4+3$

-1

y-int $=3$



7. $y = 4x^2 + 3$

$a=4 \quad b=0 \quad c=3$

$x=\frac{-b}{2a}=\frac{0}{2(4)}=0$

$4(0)^2+3$

3

$(0,3)$

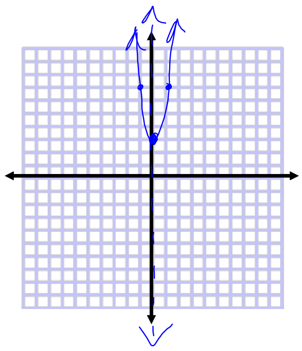
$(1,7)$

$4(1)^2+3$

$4(1)+3$

$4+3$

7



8.4 Transforming Quadratic Functions

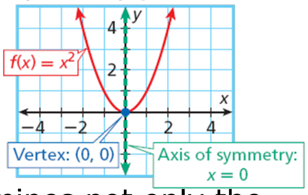
Objective: Graph and transform quadratic functions.

Remember from Chapter 4, that the graphs of all linear functions are transformations of the linear parent function $y = x$.

The quadratic parent function is $f(x) = x^2$. The graph of all other quadratic functions are transformations of the graph of $f(x) = x^2$.

For the parent function $f(x) = x^2$:

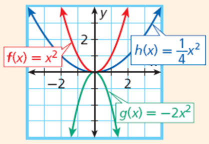
- The axis of symmetry is $x = 0$, or the y-axis.
- The vertex is $(0, 0)$
- The function has only one zero, 0



The value of a in a quadratic function determines not only the direction a parabola opens, but also the width of the parabola.

Width of a Parabola

WORDS	EXAMPLES
The graph of $f(x) = ax^2$ is **narrower** than the graph of $f(x) = x^2$ if $\|a\| > 1$ and **wider** if $\|a\| < 1$.	Compare the graphs of $g(x)$ and $h(x)$ with the graph of $f(x)$. $\|-2\| \ ? \ 1$; $\left\|\frac{1}{4}\right\| \ ? \ 1$; $2 > 1$ **narrower** ; $\frac{1}{4} < 1$ **wider**



Order the functions from narrowest graph to widest.

1. $f(x) = 3x^2$, $g(x) = 0.5x^2$

f(x), g(x)
narrow wider

2. $f(x) = -x^2$, $g(x) = \frac{2}{3}x^2$

f(x), g(x)
narrow wider

3. $f(x) = x^2$, $g(x) = \frac{1}{2}x^2$, $h(x) = -2x^2$

h(x), f(x), g(x)

4. $f(x) = -4x^2$, $g(x) = 6x^2$, $h(x) = 0.2x^2$

g(x), f(x), h(x)

The value of c makes these graphs look different. The value of c in a quadratic function determines not only the value of the y-intercept but also a vertical translation of the graph of $f(x) = ax^2$ up or down the y-axis.

Vertical Translations of a Parabola

The graph of the function $f(x) = x^2 + c$ is the graph of $f(x) = x^2$ translated vertically.

- If $c > 0$, the graph of $f(x) = x^2$ is translated c units **up**.
- If $c < 0$, the graph of $f(x) = x^2$ is translated c units **down**.

Compare the graph of the function with the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 3$

   $c = 3$
   up 3 units

2. $g(x) = x^2 - 3$

   $c = -3$
   down 3units

3. $g(x) = x^2 - 4$

   $c = -4$
   down 4 units

4. $g(x) = 3x^2$

   $f(x) = x^2$ start.
   $a = 1$
   transformation
   $a = 3$
   narrower

5. $g(x) = -\frac{1}{2}x^2$

   wider
   opens
   down.

6. $g(x) = \frac{3}{2}x^2$

   $a = \frac{3}{2} = 1.5$
   narrower

7. $g(x) = 3x^2 + 9$

   narrower
   moved up 9 units

8. $g(x) = \frac{1}{2}x^2 + 2$

   wider
   and translation
   up 2 units

9. $g(x) = -x^2 - 4$

   opens down
   and translation
   down 4 units

8.6: Solving Quadratic Equations by Factoring

Zero Product Property

For all real numbers a and b,

WORDS	NUMBERS	ALGEBRA
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

Use the Zero Product Property to solve the equation. Check your answer.

$(x - 7)(x + 2) = 0$

$x-7=0$, $+7\ \ +7$, $x=7$ $\qquad$ $x+2=0$, $-2\ \ -2$, $x=-2$

$(7-7)(7+2)=0$, $0(9)=0$, $0=0$ ✓

$(-2-7)(-2+2)=0$, $(-9)(0)=0$, $0=0$ ✓

$(x - 2)(x) = 0$

$x-2=0$, $+2\ \ +2$, $x=2$ $\qquad$ $x=0$

$(2-2)(2)=0$, $0(2)=0$, $0=0$ ✓

$(0-2)(0)=0$, $-2(0)=0$, $0=0$ ✓

$(x)(x + 4) = 0$

$x=0$ $\qquad$ $x+4=0$, $x=-4$

$0(0+4)$, $0(4)=0$ ✓

$-4(-4+4)=0$, $-4(0)=0$, $0=0$ ✓

$(x + 4)(x - 3) = 0$

$x+4=0$, $x=-4$ $\qquad$ $x-3=0$, $+3\ \ +3$, $x=3$

$(-4+4)(-4-3)$, $0(-7)=0$, $0=0$

$(3+4)(3-3)=0$, $7(0)=0$, $0=0$

If a quadratic equation is written in standard form, $ax^2 + bx + c = 0$, then to solve the equation, you may need to factor before using the Zero Product Property.

Solve the quadratic equation by factoring. Check your answer.

$x^2 - 6x + 8 = 0$

1, 8
2, 4

$(x-2)(x-4)=0$

$x-2=0$, $+2\ \ +2$, $x=2$ $\qquad$ $x-4=0$, $+4\ \ +4$, $x=4$

$2^2-6(2)+8=0$, $4-12+8=0$, $-8+8=0$, $0=0$ ✓

$4^2-6(4)+8=0$, $16-24+8=0$, $-8+8=0$, $0=0$ ✓

$x^2 + 4x = 21$

$-21\ \ -21$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0$, $x=-7$ $\qquad$ $x-3=0$, $x=3$

$(-7)^2+4(-7)=21$, $49-28=21$, $21=21$ ✓

$3^2+4(3)+21$, $9+12=21$, $21=21$ ✓

$x^2 - 12x + 36 = 0$

$(x-6)(x-6)=0$

$x-6=0$, $x=6$ $\qquad$ $x-6=0$, $x=6$

$6^2-12(6)+36=0$, $36-72+36=0$, $-36+36=0$, $0=0$ ✓

$-2x^2 = 20x + 50$

$0=2x^2+20x+50$

$0=2(x^2+10x+25)$

$0=2(x+5)(x+5)$

$0\neq 2$ $\qquad$ $x+5=0$, $x=-5$ $\qquad$ $x+5=0$, $x=-5$

$-2(-5)^2=20(-5)$ $+50$, $-2(25)=-100+50$, $-50=-50$ ✓

Solve the quadratic equation by factoring. Check your answer.

$x^2 - 6x + 9 = 0$

$(x-3)(x-3)=0$

$x-3=0 \quad x-3=0$

$\boxed{x=3 \quad x=3}$

$3^2-6(3)+9=0$

$9-18+9=0$

$-9+9=0$

$0=0$ ✓

$x^2 + 4x = 5$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x+5=0 \quad x-1=0$

$\boxed{x=-5 \quad x=1}$

$(-5)^2+4(-5)=5$

$25-20=5$

$5=5$ ✓

$1^2+4(1)=5$

$1+4=5$

$5=5$ ✓

$30x = -9x^2 - 25$

$+9x^2 +25 \quad +9x^2 \quad +25$

$9x^2+30x+25=0$

$(3x+5)(3x+5)=0$

$3x+5=0$

$-5 \quad -5$

$\frac{3x}{3}=\frac{-5}{3}$

$3x+5=0$

$-5 \quad -5$

$\frac{3x}{3}=\frac{-5}{3}$

$\boxed{x=-5/3}$

$x^2 + 2x = 8$

$-8 \quad -8$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0 \quad x-2=0$

$-4 \quad -4 \quad +2 \quad +2$

$\boxed{x=-4 \quad x=2}$

1, 8

4, 2

The height of a toy rocket in meters can be approximated by $h= -6t^2+42t$, where h is the height in meters and t is time in seconds. Find the time it takes the rocket to reach the ground after it has been launched.

$h=-6t^2+42t$

$0=-6t^2+42t$

$0=\boxed{-6t}\boxed{(t-7)}$

$\frac{0}{-6}=\frac{-6t}{-6}$

$0=t$

$0=t-7$

$+7 \quad +7$

$\boxed{7_{sec}=t}$

8.7 Solving Quadratic Equations by using Square Roots

Objective: Solve quadratic equations using square roots.

Some quadratic equations cannot be easily solved by factoring. Square roots can be used to solve some of these quadratic equations. Recall that every positive real number has two square roots, one positive and one negative.

$3(3) = 3^2 = 9 \rightarrow \sqrt{9} = 3 \leftarrow$ **Positive Square root of 9**

$(-3)(-3) = (-3)^2 = 9 \rightarrow -\sqrt{9} = -3 \leftarrow$ **Negative Square root of 9**

$\sqrt{9} = \pm 3$

When you take the square root of a positive number and the sign of the square root is not indicated, you must find both the positive and negative square root. This is indicated by $\pm\sqrt{}$

$\pm\sqrt{9} = \pm 3 \leftarrow$ **Positive and negative Square roots of 9**

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation in the form $x^2 = a$, take the square root of both sides.	$x^2 = 15$ $\sqrt{x^2} = \pm\sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a positive real number, then $x = \pm\sqrt{a}$.

Solve using square roots. Check your answer.

$\sqrt{x^2} = \sqrt{169}$

$x = \pm 13$

Check

$13 \cdot 13 = 169$

$-13 \cdot -13 = 169$

$\sqrt{x^2} = \sqrt{-49}$

$x =$

No real Solution

Check

$\sqrt{x^2} = \sqrt{20}$

$\sqrt{20}$

$\sqrt{10}\ \sqrt{2}$

$\sqrt{5}\ \sqrt{2}$

$2\sqrt{5}$

$x = \pm 2\sqrt{5}$

Check

$(2\sqrt{5})(2\sqrt{5}) = 4\sqrt{25} = 4(5) = 20$

$(-2\sqrt{5})(-2\sqrt{5}) = 4\sqrt{25} = 4(5) = 20$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

Check

$0 \cdot 0 = 0$

$\sqrt{x^2} = \sqrt{-16}$

$x =$

No real Solution

Check

$\sqrt{16}$

$\sqrt{x^2} = \sqrt{18}$

$\sqrt{18}$

$\sqrt{9}\ \sqrt{2}$

$\sqrt{3}\sqrt{3}$

$x = \pm 3\sqrt{2}$

Check

$(3\sqrt{2})(3\sqrt{2}) = 9\sqrt{4} = 9(2) = 18$

$(-3\sqrt{2})(-3\sqrt{2}) = 9\sqrt{4} = 9(2) = 18$

Solve using square roots.

$x^2 + 7 = 7$

$-7 \quad -7$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

Check

$0^2 + 7 = 7$

$7 = 7$ ★

$16x^2 - 49 = 0$

$+49 \quad +49$

$\frac{16x^2}{16} = \frac{49}{16}$

$\sqrt{x^2} = \sqrt{\frac{49}{16}}$ $\quad \frac{\sqrt{49} = 7}{\sqrt{16} = 4}$

$x = \pm 7/4$

Check

$16\left(\frac{7}{4}\right)^2$

$16\left(\frac{49}{16}\right) - 49 = 0$

$0 = 0$ ★

$100x^2 + 49 = 0$

$-49 \quad -49$

$\frac{100x^2}{100} = \frac{-49}{100}$

$\sqrt{x^2} = \sqrt{\frac{-49}{100}}$

$x =$

Check

No real Solutions

$\sqrt{(x - 5)^2} = \sqrt{16}$

$x - 5 = \pm 4$

$x - 5 = 4 \quad x - 5 = -4$

$+5 \quad +5 \qquad +5 \quad +5$

$\boxed{x = 9 \qquad x = 1}$

$-3x^2 + 120 = 0$

$-120 \quad -120$

$\frac{-3x^2}{-3} = \frac{-120}{-3}$

$\sqrt{x^2} = \sqrt{40}$

$x = \pm 2\sqrt{10}$

$\sqrt{40}$

$\sqrt{4} \quad \sqrt{10}$

$2\sqrt{2}$

$x^2 + 45 = 0$

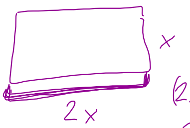
$-45 \quad -45$

$\sqrt{x^2} = \sqrt{-45}$

$x =$

No real Solutions

Ms. Pirzada is building a retaining wall along one of the long sides of her rectangular garden. The garden is twice as long as it is wide. It also has an area of 578 square feet. What will be the length of the retaining wall?



$A = 578$

$lw = 578$

$(2x)(x) = 578$

$\frac{2x^2}{2} = \frac{578}{2}$

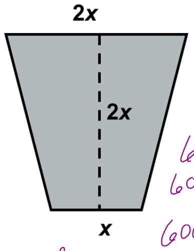
$\sqrt{x^2} = \sqrt{289}$

$x = \pm 17$ ft

Can't have neg.

$x = 17$ ft

A house is on a lot that is shaped like a trapezoid. The solid lines show the boundaries, where *x* represents the width of the front yard. Find the width of the front yard, given that the area is 6000 square feet. Round to the nearest foot. ***(Hint: Use*** $A = \frac{1}{2}h(b_1 + b_2)$***)***



front

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(2x)(x + 2x)$

$6000 = \frac{1}{2}(2x)(x + 2x)$

$6000 = \frac{1}{2}(2x)(3x)$

$6000 = \frac{1}{2}(6x^2)$

$\frac{6000}{3} = \frac{3x^2}{3}$

$\sqrt{2000} = \sqrt{x^2}$

$\sqrt{2000}$

$\sqrt{1000} \quad \sqrt{2}$

$\sqrt{100} \quad \sqrt{10}$

$\sqrt{10} \quad \sqrt{10} \quad \sqrt{5} \quad \sqrt{2}$

$x = \pm 20\sqrt{5}$

$x = 20\sqrt{5}$

$x = 44.7213$

$\boxed{x = 45 \text{ ft}}$

8.9 The Quadratic Formula

Objectives: 1. Solve quadratic equations by using the Quadratic Formula.
2. Determine the number of soultions of a quadratic equation by using the discriminant.

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Remember: To add fractions, you need a common denominator.

Solve using the Quadratic Formula.

$6x^2 + 5x - 4 = 0$

$a=6 \quad b=5 \quad c=-4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-5 \pm \sqrt{(5)^2 - 4(6)(-4)}}{2(6)}$

$\dfrac{-5 \pm \sqrt{25+96}}{12}$

$\dfrac{-5 \pm \sqrt{121}}{12}$

$\dfrac{-5 \pm 11}{12}$

$\dfrac{-5+11}{12} \qquad \dfrac{-5-11}{12}$

$x = \dfrac{6}{12} \qquad \dfrac{-16}{12}$

$\boxed{x = \frac{1}{2} \quad x = -\frac{4}{3}}$

$x^2 = x + 20$

$-x^2 \quad -x^2$

$a=-1$
$b=1$
$c=20$

$0 = -x^2 + x + 20$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(-1)(20)}}{2(-1)}$

$x = \dfrac{-1 \pm \sqrt{1+80}}{-2}$

$\dfrac{-1 \pm \sqrt{81}}{-2}$

$x = \dfrac{-1 \pm 9}{-2}$

$x = \dfrac{-1+9}{-2} = \dfrac{8}{-2} = -4$

$x = \dfrac{-1-9}{-2} = \dfrac{-10}{-2} = 5$

$\boxed{\begin{array}{l} x = -4 \\ x = 5 \end{array}}$

$-3x^2 + 5x + 2 = 0$

$a=-3 \quad b=5 \quad c=2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-5 \pm \sqrt{(5)^2 - 4(-3)(2)}}{2(-3)}$

$\dfrac{-5 \pm \sqrt{25+24}}{-6}$

$\dfrac{-5 \pm \sqrt{49}}{-6}$

$x = \dfrac{-5 \pm 7}{-6}$

$\dfrac{-5+7}{-6} \qquad \dfrac{-5-7}{-6}$

$x = \dfrac{2}{-6} \qquad x = \dfrac{-12}{-6}$

$\boxed{x = -\frac{1}{3} \quad x = 2}$

$2 - 5x^2 = -9x$

$+9x \quad +9x$

$-5x^2 + 9x + 2 = 0$

$a=-5 \quad b=9 \quad c=2$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-9 \pm \sqrt{(9)^2 - 4(-5)(2)}}{2(-5)}$

$\dfrac{-9 \pm \sqrt{81+40}}{-10}$

$\dfrac{-9 \pm \sqrt{121}}{-10}$

$\dfrac{-9 \pm 11}{-10}$

$x = \dfrac{-9+11}{-10} \qquad x = \dfrac{-9-11}{-10}$

$\dfrac{2}{-10} \qquad \dfrac{-20}{-10}$

$\boxed{x = -\frac{1}{5} \quad x = 2}$

Solve using the Quadratic Formula. Simplify your answer and then round to the nearest hundredth.

$a=2 \quad b=-8 \quad c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$x^2 - 2x - 4 = 0$

$a=1 \quad b=-2 \quad c=-4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$\frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm \sqrt{20}}{2}$$

$\sqrt{20}$, $\sqrt{4}\sqrt{5}$, $\sqrt{2}\sqrt{2}$

$$\frac{2 \pm 2\sqrt{5}}{2} = \frac{2}{2} + \frac{2\sqrt{5}}{2}$$

$$\frac{2(1 \pm \sqrt{5})}{2}$$

$1 + \sqrt{5} \approx \boxed{3.24}$

$1 - \sqrt{5} \approx \boxed{-1.24}$

$2x^2 - 8x + 1 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(1)}}{2(2)} = \frac{8 \pm \sqrt{64-8}}{4}$$

$$x = \frac{8 \pm \sqrt{56}}{4} = \frac{8 \pm 2\sqrt{14}}{4} = \frac{2(4 \pm \sqrt{14})}{4}$$

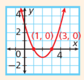
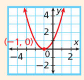
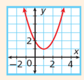
$\sqrt{56}$, $\sqrt{7}\sqrt{8}$, $\sqrt{4}\sqrt{2}$, $\sqrt{2}\sqrt{2}$

$$x = \frac{4 \pm \sqrt{14}}{2} \qquad \frac{4+\sqrt{14}}{2} \qquad \frac{4-\sqrt{14}}{2}$$

$\boxed{\approx 3.87 \qquad \approx .13}$

If the quadratic equation is in standard form, the discriminant of a quadratic equation is $b^2 - 4ac$, the part of the equation under the radical sign. Recall that quadratic equations can have two, one, or no real solutions.

You can determine the number of solutions of a quadratic equation by evaluating its discriminant.

Equation	$x^2 - 4x + 3 = 0$	$x^2 + 2x + 1 = 0$	$x^2 - 2x + 2 = 0$
Discriminant	$a = 1, b = -4, c = 3$ $b^2 - 4ac$ $(-4)^2 - 4(1)(3)$ $16 - 12$ 4 The discriminant is positive.	$a = 1, b = 2, c = 1$ $b^2 - 4ac$ $2^2 - 4(1)(1)$ $4 - 4$ 0 The discriminant is zero.	$a = 1, b = -2, c = 2$ $b^2 - 4ac$ $(-2)^2 - 4(1)(2)$ $4 - 8$ -4 The discriminant is negative.
Graph of Related Function	Notice that the related function has two x-intercepts. 	Notice that the related function has one x-intercept. 	Notice that the related function has no x-intercepts. 
Number of Solutions	two real solutions	one real solution	no real solutions

The Discriminant of Quadratic Equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$, the equation has two real solutions. Positive then 2 real sol.

If $b^2 - 4ac = 0$, the equation has one real solution. $= 0$ then 1 real sol.

If $b^2 - 4ac < 0$, the equation has no real solutions. neg. then no real sol.

Find the number of solutions of each equation using the discriminant. $b^2 - 4ac$

$3x^2 - 2x + 2 = 0$

$a=3 \quad b=-2 \quad c=2$

$(-2)^2 - 4(3)(2)$

$4 - 24$

-20

No Real Solutions

$2x^2 + 11x + 12 = 0$

$a=2 \quad b=11 \quad c=12$

$(11)^2 - 4(2)(12)$

$121 - 96$

25 2 real Solutions

$x^2 + 8x + 16 = 0$

$a=1 \quad b=8 \quad c=16$

$8^2 - 4(1)(16)$

$64 - 64 = 0$

1 real Solution

$2x^2 - 2x + 3 = 0$

$a=2 \quad b=-2 \quad c=3$

$(-2)^2 - 4(2)(3)$

$4 - 24$

-20

No Real Solutions

$x^2 + 4x + 4 = 0$

$a=1 \quad b=4 \quad c=4$

$4^2 - 4(1)(4)$

$16 - 16$

0

1 real Solution

$x^2 - 9x + 4 = 0$

$a=1 \quad b=-9 \quad c=4$

$(-9)^2 - 4(1)(4)$

$81 - 16$

65

2 real Solutions

The height h in feet of an object shot straight up with initial velocity v in feet per second is given by $h = -16t^2 + vt + c$, where c is the initial height of the object above the ground. The ringer on a carnival strength test is 2 feet off the ground and is shot upward with an initial velocity of 30 feet per second. Will it reach a height of 20 feet? Use the discriminant to explain your answer.

$30 \Rightarrow v$
$2 \Rightarrow c$
$20 \Rightarrow h$

$$\boxed{\begin{array}{l} b^2 - 4ac \\ a = -16 \\ b = 30 \\ c = -18 \end{array}}$$

$h = -16t^2 + vt + c$

$h = -16t^2 + 30t + 2$

$20 = -16t^2 + 30t + 2$
$-20 \qquad\qquad\qquad -20$

$0 = -16t^2 + 30t - 18$

$(30)^2 - 4(-16)(-18)$

$900 - 1152$

-252

$\boxed{\text{No it won't reach 20}}$