7.1: Factors and Greatest Common Factors

Objectives.
1. Write the prime factorization of numbers.
2. Find the GCF of monomials.

A representation of number or a polynomial as a product of primes is a <u>prime factorization</u>. (write the product in prime # only)

Factorizations of 12

$1 \cdot 12$ $2 \cdot 6$ $3 \cdot 4$ $1 \cdot 4 \cdot 3$ $2 \cdot 2 \cdot 3$

Write the prime factorization of the following.

98 $= 2 \cdot 7 \cdot 7 = 2 \cdot 7^2$

2 49

7 7

40

4 10

$2 \cdot 2 \cdot 2 \cdot 5$

$\boxed{2^3 \cdot 5}$

33

$3 \cdot 11$

$\boxed{3 \cdot 11}$

49

7 7

$\boxed{7^2}$

19 $= \boxed{1 \cdot 19}$

1 19

Factors that are shared by two or more whole numbers are called common factors. The greatest of these common factors is called the greatest Common Factor, or GCF.

*Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 32: 1, 2, 4, 8, 16, 32

Common factors: 1, 2, 4

The greatest of the common factors is 4.

Find the GCF of each pair of numbers.

Method 1 List the factors.

100 and 60

100: 1, 2, 4, 5, 10, 20, 25, 50, 100

60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

GCF: 20

12 and 16

12: 1, 2, 3, 4, 6, 12

16: 1, 2, 4, 8, 16

GCF: 4

Method 2 Prime factorization.

26 and 52

26 = 2 · 13

52 = 2 · 26 = 2 · 2 · 13

GCF: 26

15 and 25

15 = 3 · 5

25 = 5 · 5

GCF: 5

Find the GCF of each pair of monomials. $4 \cdot 4$

$15x^3$ and $9x^2$

$15x^3: 3 \cdot 5 \cdot x \cdot x \cdot x$

$9x^2: 3 \cdot 3 \cdot x \cdot x$

GCF: $3x^2$

$16a^6$ and $9b$

$16a^6: 2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

$9b: 3 \cdot 3 \cdot b$

$\boxed{\text{GCF: } 1}$

$8x^2$ and $7y^3$

$8x^2: 2 \cdot 2 \cdot 2 \cdot x \cdot x$

$7y^3: 7 \cdot y \cdot y \cdot y$

GCF: 1

$8x$ and $7v^2$

$8x: 2 \cdot 2 \cdot 2 \cdot x$

$7v^2: 7 \cdot v \cdot v$

GCF: 1

$18g^2$ and $27g^3$

$18g^2: 2 \cdot 3 \cdot 3 \cdot g \cdot g$

$27g^3: 3 \cdot 3 \cdot 3 \cdot g \cdot g \cdot g$

$9g^2$: GCF

$3x^3$ and $6x^2$

$3x^3: 3 \cdot x \cdot x \cdot x$

$6x^2: 3 \cdot 2 \cdot x \cdot x$

$3x^2$: GCF

A cafeteria has 18 chocolate-milk cartons and 24 regular-milk cartons. The cook wants to arrange the cartons with the same number of cartons in each row. Chocolate and regular milk will not be in the same row. How many rows will there be if the cook puts the greatest possible number of cartons in each row?

18: 3, $\boxed{6}$; 6: 3, 2

24: $\boxed{6}$, 4; 6: 3, 2; 4: 2, 2

GCF: 6

18 chocolate milk: 6 in each
3 rows

24 regular-milk: 6 in each
4 rows

rows total
7

Adrianne is shopping for a CD storage unit. She has 36 CDs by pop music artists and 48 CDs by country music artists. She wants to put the same number of CDs on each shelf without putting pop music and country music CDs on the same shelf. If Adrianne puts the greatest possible number of CDs on each shelf, how many shelves does her storage unit need?

GCF: 12 on a shelf

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

36 Pop: 3 shelves

48 Country: 4 shelves

$\boxed{\text{7 shelves total}}$

Section 7.2 Factoring by GCF

Objectives. **1. Factor by using the greatest common factor.**

Recall that the Distributive Property states that $ab + ac = a(b + c)$.

Factor each polynomial. <u>Check your answer</u>.

$2x^2 - 4$

$2(x^2 - 2)$

$8x^3 - 4x^2 - 16x$

$4x(2x^2 - x - 4)$

$-14x - 12x^2$

$-2x(7 + 6x)$

$3x^3 + 2x^2 - 10$

Cannot be factored by GCF.

$-18y^3 - 7y^2$

$y^2(-18y-7)$

$-y^2(18y+7)$

$9d^2 - 8^2$

Can't be Factored by GCF

$9d^2-64$

$5b + 9b^3$

$b(5+9b^2)$

$8x^4 + 4x^3 - 2x^2$

$2x^2(4x^2+2x-1)$

To write expressions for the length and width of a rectangle with area expressed by a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

The area of a court for the game squash is $(9x^2 + 6x)$ square meters. Factor this polynomial to find possible expressions for the dimensions of the squash court.

$9x^2 + 6x$

$\underbrace{\boxed{3x}}_{l}\underbrace{\boxed{(3x+2)}}_{w}$

what's the length? $3x$

what's the width? $3x+2$

label

Mandy's calculator is powered by solar energy. The area of the solar panel is $(7x^2+x)$ cm^2. Factor this polynomial to find possible expression for the dimensions of the solar panel.

$7x^2+x$

$\boxed{x}\boxed{(7x+1)}$

length: $\underline{x}$ width: $\underline{7x+1}$

Sometimes the GCF of terms is a binomial. This GCF is called a common binomial factor. You factor out a common binomial factor the same way you factor out a monomial factor.

Factor each expression.

$5\cancel{(x+2)} + 3x\cancel{(x+2)}$

$(x+2)(5+3x)$

$-2b\cancel{(b^2+1)} + \cancel{(b^2+1)}$

$(b^2+1)(-2b+1)$

$4z(z^2-7) + 9(2z^3+1)$

Cannot be Factored by GCF

$4s\cancel{(s+6)} - 5\cancel{(s+6)}$

$(s+6)(4s-5)$

Factor each expression.

$7x(2x + 3) + 1(2x + 3)$

$(2x+3)(7x+1)$

$3x(y + 4) - 2y(x + 4)$

Cannot be factored by GCF

$5x(5x - 2) - 2(5x - 2)$

$(5x-2)(5x-2)$

$(5x-2)^2$

$3y(2y + 3) - 5(2y + 3)$

$(2y+3)(3y-5)$

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can make two groups and factor out the GCF from each group.

Factor each polynomial by grouping. Check your answer.

$(6h^4 - 4h^3) + (12h - 8)$

$2h^3(3h-2) + 4(3h-2)$

$\boxed{(3h-2)(2h^3+4)}$

$2 \cdot 3 \cdot h \cdot h \cdot h \cdot h$ $\quad 2 \cdot 2 \cdot 3 \cdot h$

$2 \cdot 2 \cdot h \cdot h \cdot h$ $\quad 2 \cdot 2 \cdot 2$

$(6b^3 + 8b^2) + (9b + 12)$

$2b^2(3b+4) + 3(3b+4)$

$\boxed{(3b+4)(2b^2+3)}$

$(5y^4 - 15y^3) + (y^2 - 3y)$

$5y^3(y-3) + y(y-3)$

$(y-3)(5y^3+y)$

$(5y^3+y)(y-3) = y(5y^2+1)(y-3)$

$(4r^3 + 24r) + (r^2 + 6)$

$4r(r^2+6) + 1(r^2+6)$

$\boxed{(r^2+6)(4r+1)}$

Recognizing opposite binomials can help you factor polynomials. The binomials (5 – *x*) and (*x* – 5) are opposites. Notice (5 – *x*) can be written as –1(*x* – 5).

$(-x+5) = -1(x-5)$

Factor each polynomial by grouping.

$(2x^3 - 12x^2) + (18 - 3x)$

$2x^2(x-6) - 3(-6+x)$

$2x^2(x-6) - 3(x-6)$

$(x-6)(2x^2-3)$

$(15x^2 - 10x^3) + (8x - 12)$

$5x^2(3-2x) - 4(-2x+3)$

$(3-2x)(5x^2-4)$

$(8y - 8)(-x + xy)$

$8(y-1) + x(-1+y)$

$(8+x)(y-1)$

$(2x^3 + x^2)(-6x - 3)$

$x^2(2x+1) - 3(2x+1)$

$(2x+1)(x^2-3)$

7.3 Factoring $x^2 + bx + c$

Objective: **1. Factor by quadratic trinomial in the form $x^2 + bx + c$**

In Chapter 6, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binominals.

When you multiply two binomials, multiply:

First terms

Outer terms

Inner terms

Last terms

Factor by ~~guess and check.~~

$x^2 + 15x + 36$

$(x+12)(x+3)$

$x^2 + 10x + 24$

$(x+6)(x+4)$

$x^2 + 7x + 12$

$(x+3)(x+4)$

$x^2 - 11x + 30$

$(x-5)(x-6)$

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.

Factors of 18	Sum		
1 and 18	19	✗	
2 and 9	11	✗	
3 and 6	9	✓	$x^2 + 9x + 18 = (x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive and when b is negative, the factors are negative.

Factor each trinomial. Check your answer.

$x^2 + 6x + 5$

$(x+5)(x+1)$

$x^2 + 6x + 9$

$(x+3)(x+3)$

$x^2 + 8x + 15$

$(x+5)(x+3)$

$x^2 + 8x + 12$

$(x+6)(x+2)$

$x^2 + 7x + 10$

$(x+5)(x+2)$

$x^2 - 9x + 18$

$(x-6)(x-3)$

$x^2 + 13x + 42$

$(x+7)(x+6)$

$x^2 + 10x + 16$

$(x+8)(x+2)$

$x^2 + 15x + 50$

$(x+10)(x+5)$

$x^2 – 5x + 6$

$(x-3)(x-2)$

7.3B Factoring $x^2 + bx + c$

When *c* is negative, its factors have opposite signs. The sign of *b* tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as *b.*

Factor each trinomial.

$x^2 + x - 20$

$(x-4)(x+5)$

$x^2 - 3x - 18$

$(x+3)(x-6)$

$x^2 + 2x - 15$

$(x+5)(x-3)$

$x^2 - 6x + 8$

$(x-4)(x-2)$

$x^2 - 8x - 20$

$(x+2)(x-10)$

$x^2 - 9x + 20$

$(x-5)(x-4)$

$x^2 - 16x + 28$

$(x-14)(x-2)$

$x^2 - 16x + 48$

$(x-12)(x-4)$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

Factor $y^2 + 10y + 21$. Show that the original polynomial and the factored form have the same value for y = 0, 1, 2, 3, and 4.

$(y+7)(y+3)$

y	$(y+7)(y+3)$
0	21
1	32
2	45
3	60
4	77

y	$y^2 + 10y + 21$
0	21
1	32
2	45
3	60
4	77

Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for n = 0, 1, 2, 3, and 4.

$(n-5)(n-2)$

y	$(n-5)(n-2)$
0	10
1	4
2	0
3	-2
4	-2

y	$n^2-7n+10$
0	10
1	4
2	0
3	-2
4	-2

7.4 Factoring $ax^2 + bx + c$

Objective: 1. Factor by quadratic trinomials in the form $ax^2 + bx + c$

When you multiply $(3x + 2)(2x + 5)$, the coefficient of the x^2-term is the product of the coefficients of the x-terms. Also, the constant term in the trinomial is the product of the constants in the binomials.

$$(3x + 2)(2x + 5) = 6x^2 + 19x + 10$$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, write two sets of parentheses (__ x + __)(__ x + __).

To factor $a^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b.

Product = *a*

Product = *c*

$$(\quad x + \quad)(\quad x + \quad) = ax^2 + bx + c$$

Sum of outer and inner products = *b*

Factor each trinomial. Check your answer.

$2x^2 + 17x + 21$

$(2x + 3)(1x + 7)$

Factors of 2	Factors of 21	Outer + Inner
1·2	1·21	2·7 +
	3·7	3·1

Check $(2x+3)(x+7)$

$2x^2+14x+3x+21$

$\boxed{= 2x^2+17x+21}$

$3x^2 - 16x + 16$

$(3x + -4)(1x + -4)$

Factors of 3	Factors of 16	Outer + Inner
3·1	-1·-16	3·-4
	-2·-8	1·-4
	-4·-4	

Check $(3x-4)(x-4)$

$3x^2-4x-12x+16$

$\boxed{= 3x^2-16x+16}$

$6x^2 + 17x + 5$

6	5	outer + inner
1·6	1·5	2·1
2·3		3·5

$\boxed{(2x+5)(3x+1)}$

$9x^2 - 15x + 4$

9	4	outer + inner
1·9	-1·-4	3·-4
3·3	-2·-2	3·-1

$(3x + -4)(3x + -1)$

When *c* is negative.

Factor each trinomial. Check your answer.

$3n^2 + 11n - 4$

3	−4	outer + inner
1·3	−1·4	3·4
	−4·1	1·−1
	−2·2	

$\boxed{(3x-1)(x+4)}$

$2x^2 + 9x - 18$

2	−18	outer + inner
1·2	−1·18	2·6
	−18·1	1·−3
	−2·9	
	−9·2	
	−6·3	
	−3·6	

$\boxed{(2x-3)(x+6)}$

$4x^2 - 15x - 4$

4	−4	outer + inner
1·4	−1·4	4·−4
2·2	−4·1	1·1
	−2·2	

$\boxed{(4x+1)(x-4)}$

$6x^2 + 7x - 3$

6	−3	outer + inner
1·6	−1·3	3·3
2·3	−3·1	2·−1

$\boxed{(3x-1)(2x+3)}$

When a is negative.

Factor each trinomial. Check your answer.

$-2x^2 - 5x - 3$

-2	-3	outer + inner
-2·1	-1·3	-2·1
-1·2	-3·1	1·-3

$\boxed{(-2x-3)(x+1)}$

$-6x^2 - 17x - 12$

-6	-12	outer + inner
-2·3	-1·12	3·-3
-3·2	-12·1	-2·4
-1·6	-2·6	
-6·1	-6·2	
	-3·4	
	-4·3	

$\boxed{(-2x-3)(3x+4)}$

$-3x^2 - 17x - 10$

-3	-10	outer + inner
-1·3	-1·10	-3·5
-3·1	-10·1	1·-2
	-2·5	
	-5·2	

$\boxed{(-3x-2)(x+5)}$

$-2x^2 - 15x - 7$

-2	-7	outer + inner
-1·2	-1·7	2·-7
-2·1	-7·1	-1·1

$\boxed{(-x-7)(2x+1)}$

7.5: Factoring Special Products

Objectives: 1. Factor perfect square trinomials.
2. Factor difference of squares.

A trinomial is a perfect square if:

-The first and last terms are perfect squares.

- The middle term is two times one factor from the first term and one factor from the last term.

Perfect-Square Trinomials

PERFECT-SQUARE TRINOMIAL	EXAMPLES
$a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2$	$x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$
$a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2$	$x^2 - 2x + 1 = (x-1)(x-1) = (x-1)^2$

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$x^2 - 16x + 64$

$(x)^2 \quad 2(8)(x) \quad (8)^2$

$(x-8)(x-8)$

$(x-8)^2$

$x^2 + 10x + 25$

$(x)^2 \quad 2(5)(x) \quad (5)^2$

$(x+5)(x+5)$

$(x+5)^2$

$x^2 - 14x + 49$

$(x)^2 \quad 2(7)(x) \quad (7)^2$

$(x-7)(x-7)$

$(x-7)^2$

$x^2 + 4x + 4$

$(x)^2 \quad 2(2)(x) \quad (2)^2$

$(x+2)(x+2)$

$(x+2)^2$

$x^2 - 12x + 36$

$(x)^2 \quad 2(6)(x) \quad (6)^2$

$(x-6)(x-6)$

$(x-6)^2$

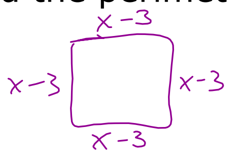
$x^2 - 6x + 4$

No, the middle term is not two times ~~one~~ from the first and one from the last.

A square piece of cloth must be cut to make a tablecloth. The area needed is $(x^2 - 6x + 9)$ in^2. The dimensions of the cloth are of the form $cx - d$, where c and d are whole numbers. Find an expression for the perimeter of the cloth. Find the perimeter when $x = 11$ inches.

$x^2 - 6x + 9$

$(x-3)(x-3)$



$4(x-3) = 4x-12$

$x-3+x-3+x-3+x-3 = 4x-12$

$4(11)-12$

$44-12$

32 in

A polynomial is a difference of two squares if:

- There are two terms, one subtracted from the other.
- Both terms are perfect squares.

Difference of Two Squares

DIFFERENCE OF TWO SQUARES	EXAMPLE
$a^2 - b^2 = (a+b)(a-b)$	$x^2 - 9 = (x+3)(x-3)$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$p^2 - 9$

$(p+3)(p-3)$

$x^2 - 4$

$(x+2)(x-2)$

$x - 25$

No, because x isn't a perfect square.

$1 - x^2$

$(1-x)(1+x)$

$p^8 - q^6$

$(p^4 - q^3)(p^4 + q^3)$

$x^2 - y^4$

$(x+y^2)(x-y^2)$

7.6: Choosing a Factoring Method

Objectives:
1. Choose and appropriated method for factoring a polynomial.
2. Combine methods for factoring a polynomial.

Tell whether each expression is completely factored. If not, factor it.

$3x^2(6x - 4)$

$3x^2 2(3x-2)$

$6x^2(3x-2)$

$(x^2 + 1)(x - 5)$

Complete

$(4x + 4)(x + 1)$

$4(x+1)(x+1)$

$4(x+1)^2$

$5x^2(x - 1)$

Complete

Factoring Polynomials	
Step 1	Check for a greatest common factor.
Step 2	Check for a pattern that fits the difference of two squares or a perfect-square trinomial.
Step 3	To factor $x^2 + bx + c$, look for two numbers whose sum is b and whose product is c. To factor $ax^2 + bx + c$, check factors of a and factors of c in the binomial factors. The sum of the products of the outer and inner terms should be b.
Step 4	Check for common factors.

Factor completely and check your answer.

$8x^6y^2 - 18x^2y^2$

$2x^2y^2(4x^4 - 9)$

$\boxed{2x^2y^2(2x^2+3)(2x^2-3)}$

$4x^3 + 16x^2 + 16x$

$4x(x^2 + 4x + 4)$

$4x(x+2)(x+2)$

$4x(x+2)^2$

$2x^2y - 2y^3$

$2y(x^2 - y^2)$

$2y(x+y)(x-y)$

$12b^3 + 48b^2 + 48b$

$12b(b^2 + 4b + 4)$

$12b(b+2)(b+2)$

$12b(b+2)^2$

bigger #

diff.

$4y^2 \boxed{+} 12y \boxed{-} 72$

$4(y^2 + 3y - 18)$

$4(y+6)(y-3)$

$(x^4 - x^2)$

$x^2(x^2 - 1)$

$x^2(x-1)(x+1)$

$2p^5 + 10p^4 - 12p^3$

$2p^3(p^2 + 5p - 6)$

$2p^3(p+6)(p-1)$

$2x^4 + 18$

$2(x^4 + 9)$

Methods to Factor Polynomials

Any Polynomial—Look for the greatest common factor.	
$ab - ac = a(b - c)$	$6x^2y + 10xy^2 = 2xy(3x + 5y)$
Binomials—Look for a difference of two squares.	
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9y^2 = (x + 3y)(x - 3y)$
Trinomials—Look for perfect-square trinomials and other factorable trinomials.	
$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 4x + 4 = (x + 2)^2$ $x^2 - 2x + 1 = (x - 1)^2$
$x^2 + bx + c = (x + \square)(x + \square)$ $ax^2 + bx + c = (\square x + \square)(\square x + \square)$	$x^2 + 3x + 2 = (x + 1)(x + 2)$ $6x^2 + 7x + 2 = (2x + 1)(3x + 2)$
Polynomials of Four or More Terms—Factor by grouping.	
$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (x + y)(a + b)$	$2x^3 + 4x^2 + x + 2 = (2x^3 + 4x^2) + (x + 2)$ $= 2x^2(x + 2) + 1(x + 2)$ $= (x + 2)(2x^2 + 1)$