Chapter 6 Notes – Quadrilaterals
Basic Geometry

Name Kel

6.1 Polygons

Goal: Identify and describe polygons. Find angle measures of quadrilaterals.

Polygon: a plane figure that is formed by 3 or more segments called sides. Each side intersects exactly two other sides at each of its endpoints.

Vertex of a polygon: the endpoints of a side

Diagonal of a polygon: a segment that joins two nonconsecutive vertices of a polygon

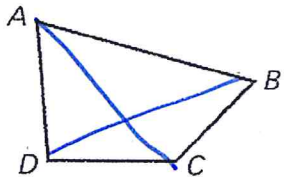
Classifying Polygons by Sides	
Number of Sides	**Name**
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
n	n-gon

Use polygon ABCD at the right.

Name all vertices: A, B, C, D

Name all sides: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$

Name all diagonals: $\overline{AC}$, $\overline{BD}$

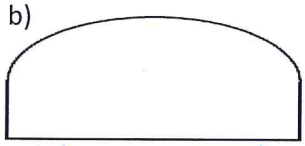


Determine if each figure is a polygon. If it is, classify it by its sides. If not, explain your reasoning.

a)

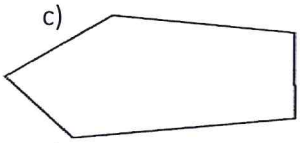
NO → not a closed figure

b)



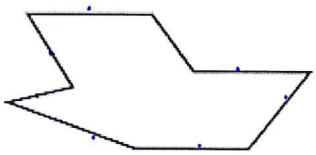
NO → round side

c)



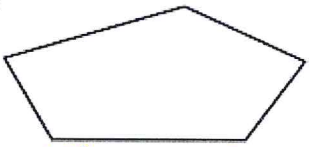
Yes - pentagon

d)



Yes - octagon

e)

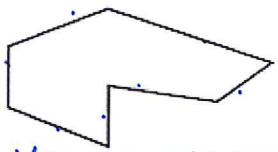


yes - pentagon

f)

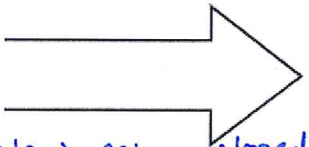
NO → curved sides

h)



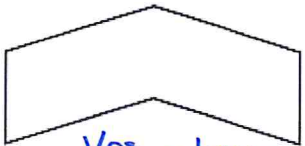
Yes - heptagon

i)



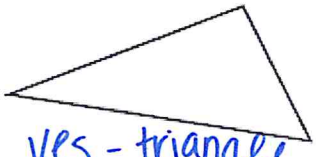
NO → not a closed figure

j)



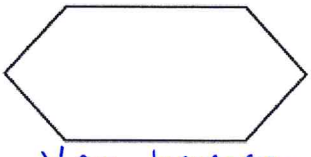
Yes - hexagon

k)



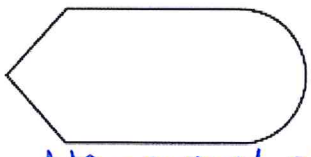
yes - triangle

l)



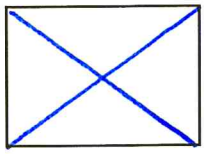
Yes - hexagon

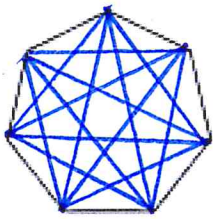
m)

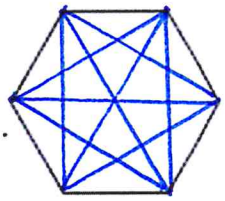


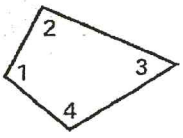
NO - curved side

Draw all diagonals for the figures below.





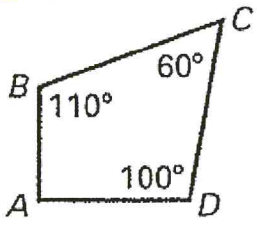


Quadrilateral Interior Angles Theorem: The sum of the measures of the angles in a quadrilateral is 360°	 m∠1 + m∠2 + m∠3 + m∠4 = 360°

Find $m\angle A$.

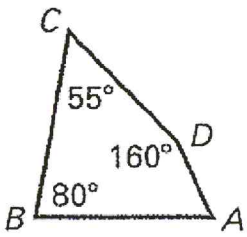
a) 90°

m∠A + 110 + 60 + 100 = 360



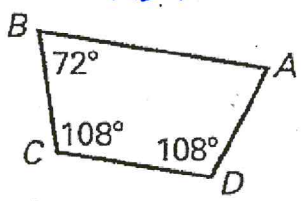
b) 65°

m∠A + 55 + 160 + 80 = 360



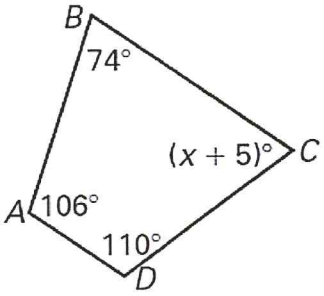
c) 72°

m∠A + 72 + 108 + 108 = 360



Find the value of x.

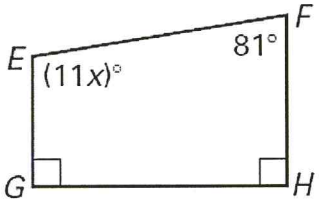
a) x = 65



106 + 110 + 74 + x + 5 = 360

295 + x = 360

b) x = 9

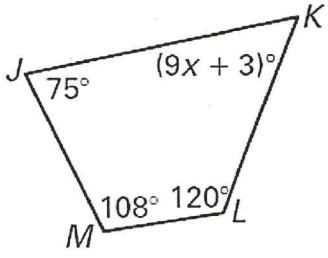


11x + 81 + 90 + 90 = 360

11x + 261 = 360

11x = 99

c) x = 6



75 + 108 + 120 + 9x + 3 = 360

306 + 9x = 360

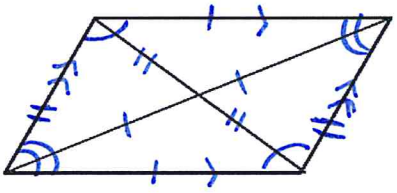
9x = 54

6.2 Properties of Parallelograms

Goal: Use properties of parallelograms to find missing angles and lengths.

Parallelogram: a quadrilateral in which both pairs of opposite sides are parallel

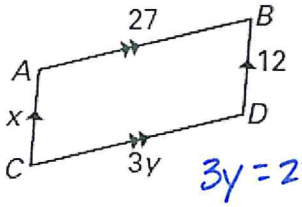
Properties of Parallelograms



- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other. (cut in half)

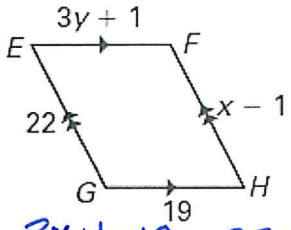
Use properties of parallelogram to find the values of x and y.

a) x = 12 y = 9



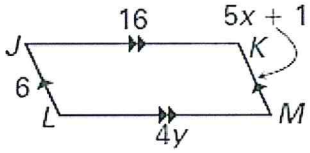
3y = 27

b) x = 23 y = 6



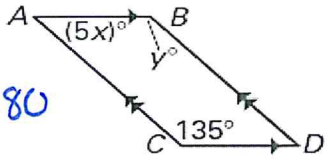
3x + 1 = 19 22 = x - 1

c) x = 1 y = 4



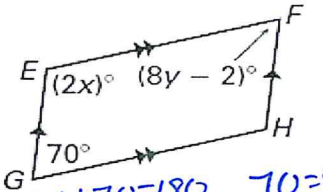
6 = 5x + 1 16 = 4y

e) x = 9 y = 135



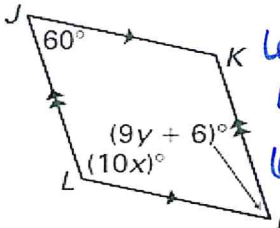
5x + 135 = 180

f) x = 55 y = 9



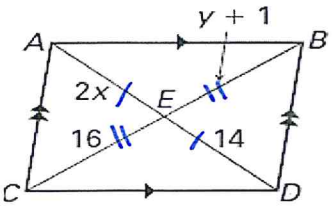
2x + 70 = 180 70 = 8y - 2
2x = 110 72 = 8y

g) x = 12 y = 6



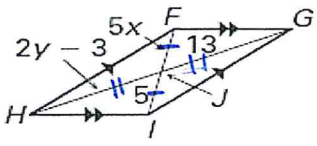
60 + 10x = 180
10x = 120
60 = 9y + 6
54 = 9y

h) x = 7 y = 15



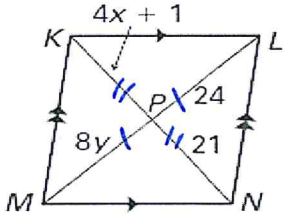
16 = y + 1 2x = 14

i) x = 1 y = 8



5x = 5 2y - 3 = 13
2y = 16

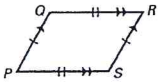
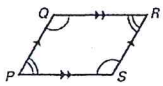
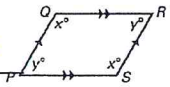
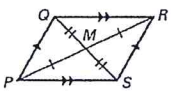
j) x = 5 y = 3



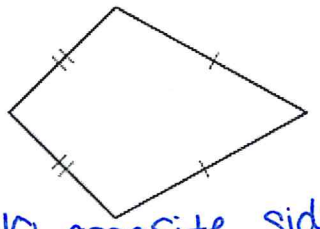
4x + 1 = 21 8y = 24

6.3 Showing Quadrilaterals are Parallelograms

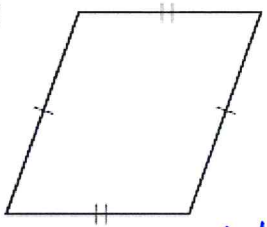
Goal: Use properties to determine whether a quadrilateral is a parallelogram

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram	If $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$, then PQRS is a parallelogram 
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram	If $\angle Q \cong \angle S$ and $\angle P \cong \angle R$, then PQRS is a parallelogram 
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral Is a parallelogram	If $m\angle P + m\angle Q = 180°$ and $m\angle P + m\angle S = 180°$ then PQRS is a parallelogram 
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	If $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$, then PQRS is a parallelogram 

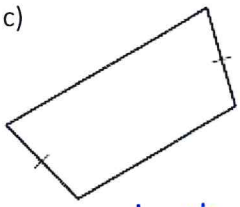
Tell whether each quadrilateral is a parallelogram. *Explain your reasoning*.

a) 

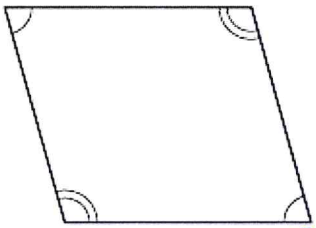
NO- opposite sides are NOT $\cong$

b) 

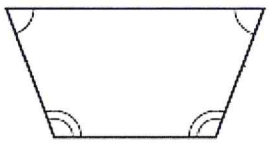
Yes- opp sides are $\cong$

c) 

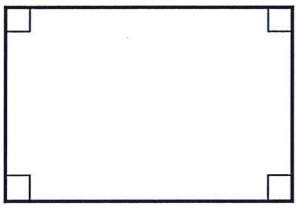
NO- both pairs of opp sides are NOT $\cong$

e) 

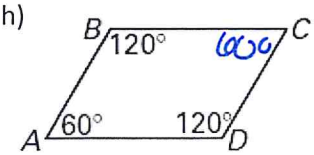
Yes - opp $\angle$'s are $\cong$

f) 

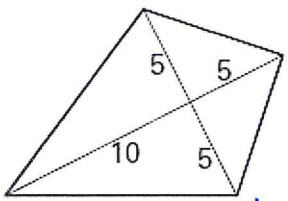
NO - opp $\angle$s are NOT $\cong$

g) 

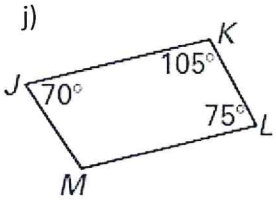
Yes - opp $\angle$'s $\cong$

h) 

Yes - opp $\angle$s are $\cong$

i) 

No - diagonals do not bisect each other

j) 

NO - $\angle J \ncong \angle L$

If a quadrilateral is a parallelogram, then the opposite sides are parallel. On coordinate plane, two lines are parallel if they have the same Slope.

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

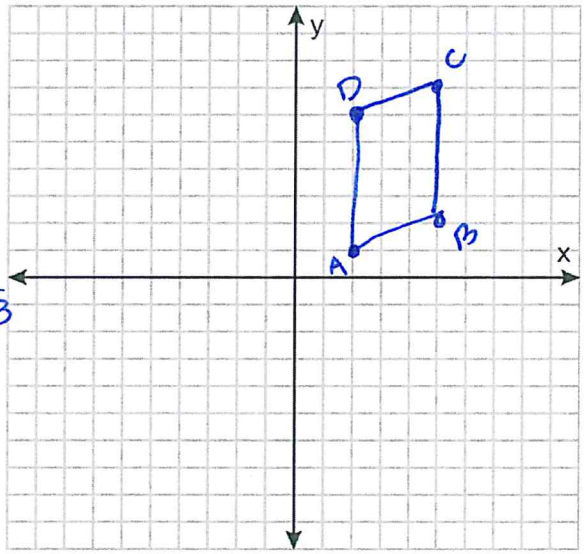
Graph each quadrilateral, then use slope to determine if it is a parallelogram.

a) $A(2,1)$ $B(5,2)$ $C(5,7)$ $D(2,6)$

Slope AB: 1/3 Slope CD: 1/3

Slope BC: undefined Slope AD: undefined

Parallelogram? Yes



Slope AB: $\frac{2-1}{5-2} = \frac{1}{3}$ Slope CD: $\frac{6-7}{2-5} = \frac{-1}{-3} = \frac{1}{3}$

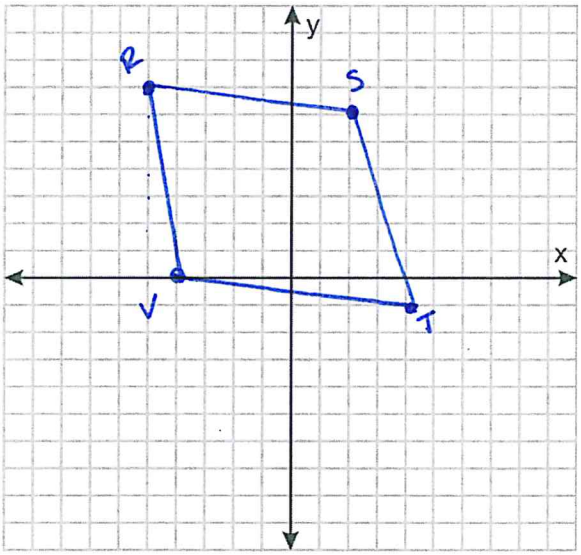
Slope BC: $\frac{7-2}{5-5} = \frac{5}{0}$ = undefined Slope AD: $\frac{6-1}{2-2} = \frac{5}{0}$

b) $R(-5,7)$ $S(2,6)$ $T(4,-1)$ $V(-4,0)$

Slope RS: -1/7 Slope TV: -1/8

Slope ST: -1/2 Slope RV: -7

Parallelogram? NO



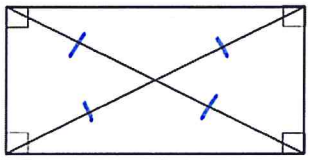
Slope RS: $\frac{6-7}{2-5} = \frac{-1}{7}$ Slope TV: $\frac{0-1}{-4-4} = \frac{1}{-8}$

Slope ST: $\frac{-1-6}{4-2} = \frac{-7}{2}$ Slope RV: $\frac{0-7}{-4-5} = \frac{-7}{1}$

6.4 Rhombuses, Rectangles, and Squares

Goal: Use properties of special types of parallelograms

Properties of Rectangles

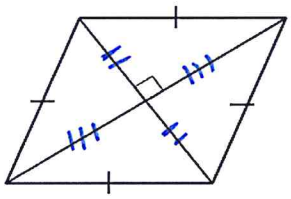


Rectangles have the same properties as parallelograms, plus:

- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary
- Diagonals bisect each other.

- All right angles.
- Diagonals are congruent

Properties of Rhombi → Rhombus

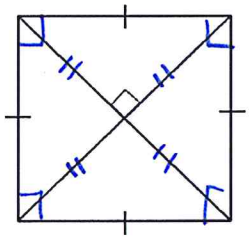


Rhombi have the same properties as parallelograms, plus:

- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary
- Diagonals bisect each other.

- All sides are congruent.
- Diagonals are perpendicular.
- Diagonals bisect opposite angles.

Properties of Squares

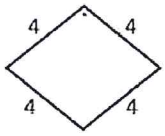


Squares have ALL the properties of parallelograms, rectangles, and rhombi.

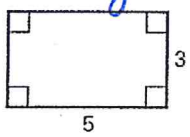
- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- All right angles.
- All sides are congruent.
- Diagonals are congruent.
- Diagonals are perpendicular.
- Diagonals bisect opposite angles.

Use the information in the diagram to name the special quadrilateral.

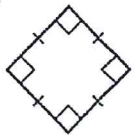
a) rhombus



b) rectangle



c) square



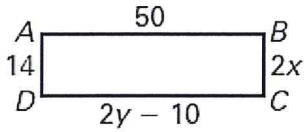
Find the values of the variables.

a) x = 7 y = 30

b) x = 2 y = 4

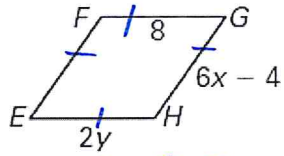
c) x = 3 y = 13

rectangle *ABCD*



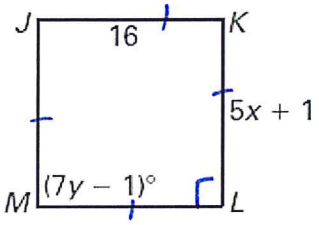
$14 = 2x$ $50 = 2y - 10$ $60 = 2y$

rhombus *EFGH*



$8 = 6x - 4$ $8 = 2y$ $12 = 6x$

square *JKLM*

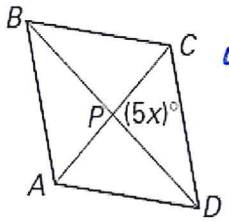


$16 = 5x + 1$ $7y - 1 = 90$ $7y = 91$ $15 = 5x$

Find the value of x in each rhombus and rectangle below.

a) x = 18

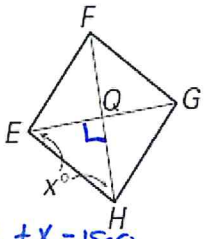
rhombus *ABCD*



$5x = 90$

b) x = 45

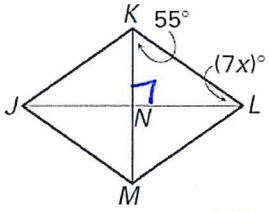
rhombus *EFGH*



$90 + x + x = 180$ $2x = 90$

c) x = 5

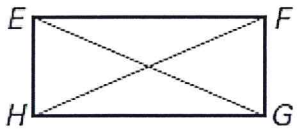
JKLM is a rhombus.



$55 + 90 + 7x = 180$ $7x = 35$

d) x = 8

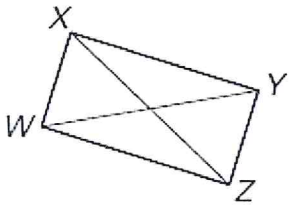
rectangle *EFGH*, $EG = 48$, $HF = 6x$



$48 = 6x$

e) x = 7

rectangle *WXYZ*, $XZ = 37$, $WY = 5x + 2$



$37 = 5x + 2$ $35 = 5x$

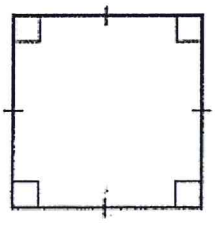
ABCD has the properties shown in the picture. Is the statement true or false? Explain.

T ABCD is a rhombus → all sides ≅

T ABCD is a parallelogram → opp sides ≅

T ABCD is a rectangle → 4 right ∠'s

T The diagonals are congruent → It is a rectangle

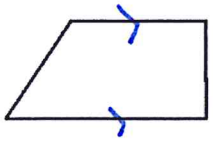


6.5 Trapezoids

Goal: Use properties of trapezoid.

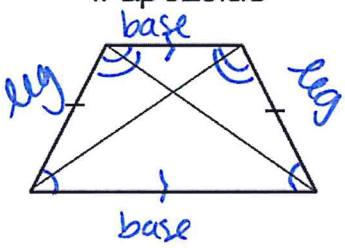
Properties of Trapezoids

Non-Isosceles Trapezoids



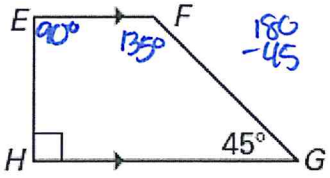
- Only one pair of opposite sides are parallel.
- Consecutive (non-base) angles are supplementary. → 180°

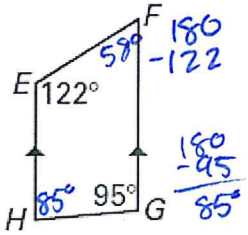
Isosceles Trapezoids

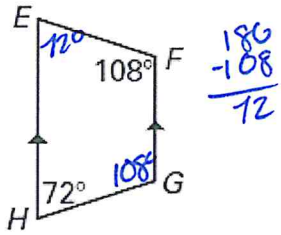


- Only one pair of opposite sides are congruent.
- Diagonals are congruent.
- Non-parallel sides (legs) are congruent.
- Opposite angles are supplementary.
- Base angles are congruent.
- Midsegment is the average of the two bases.

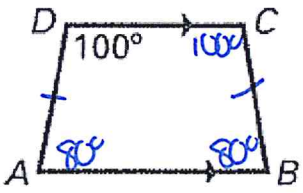
EFGH is a trapezoid. Find the missing angle measures.



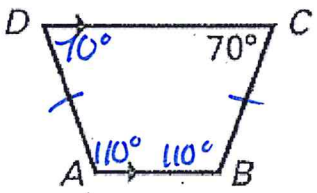




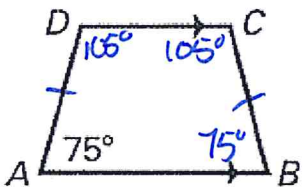
ABCD is an isosceles trapezoid. Find the missing angle measures.



180
-100
80



180
-70
110



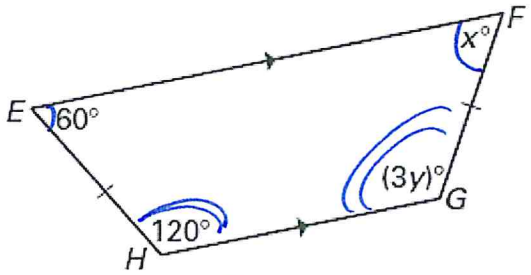
180
-75

Find the value of each variable.

a) x = 60 y = 40

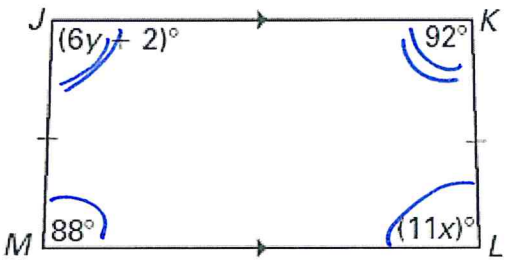
b) x = 8 y = 15

isosceles trapezoid *EFGH*



120 = 3y

isosceles trapezoid *JKLM*



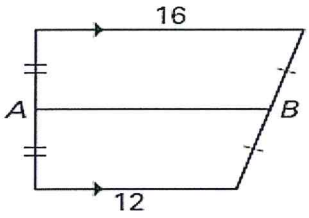
88 = 11x

6y + 2 = 92
6y = 90

base1 + base2 / 2

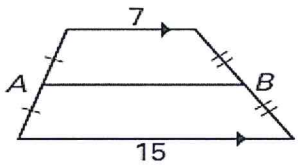
Find the length of midsegment AB. Remember that the midsegment is the average of the two bases.

a) 14



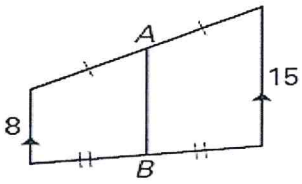
16 + 12 / 2

b) 11



7 + 15 / 2

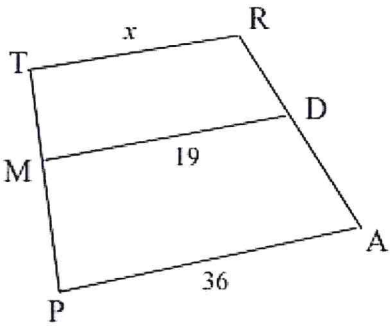
c) 11.5



8 + 15 / 2

MD is the midsegment of each trapezoid. Solve for x.

a) x = 2



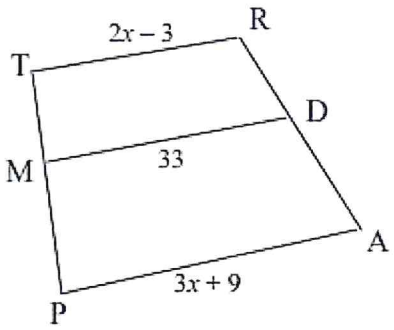
$\frac{x + 36}{2} = 19 \cdot 2$

$x + 36 = 38$

$-36 \quad -36$

$x = 2$

b) x = 12



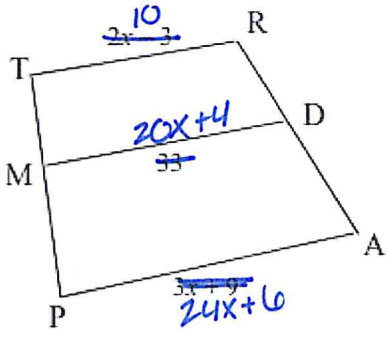
$\frac{2x - 3 + 3x + 9}{2} = 33 \cdot 2$

$5x + 6 = 66$

$-6 \quad -6$

$\frac{5x}{5} = \frac{60}{5}$

c) x = 0.5



$\frac{10 + 24x + 6}{2} = (20x + 4)2$

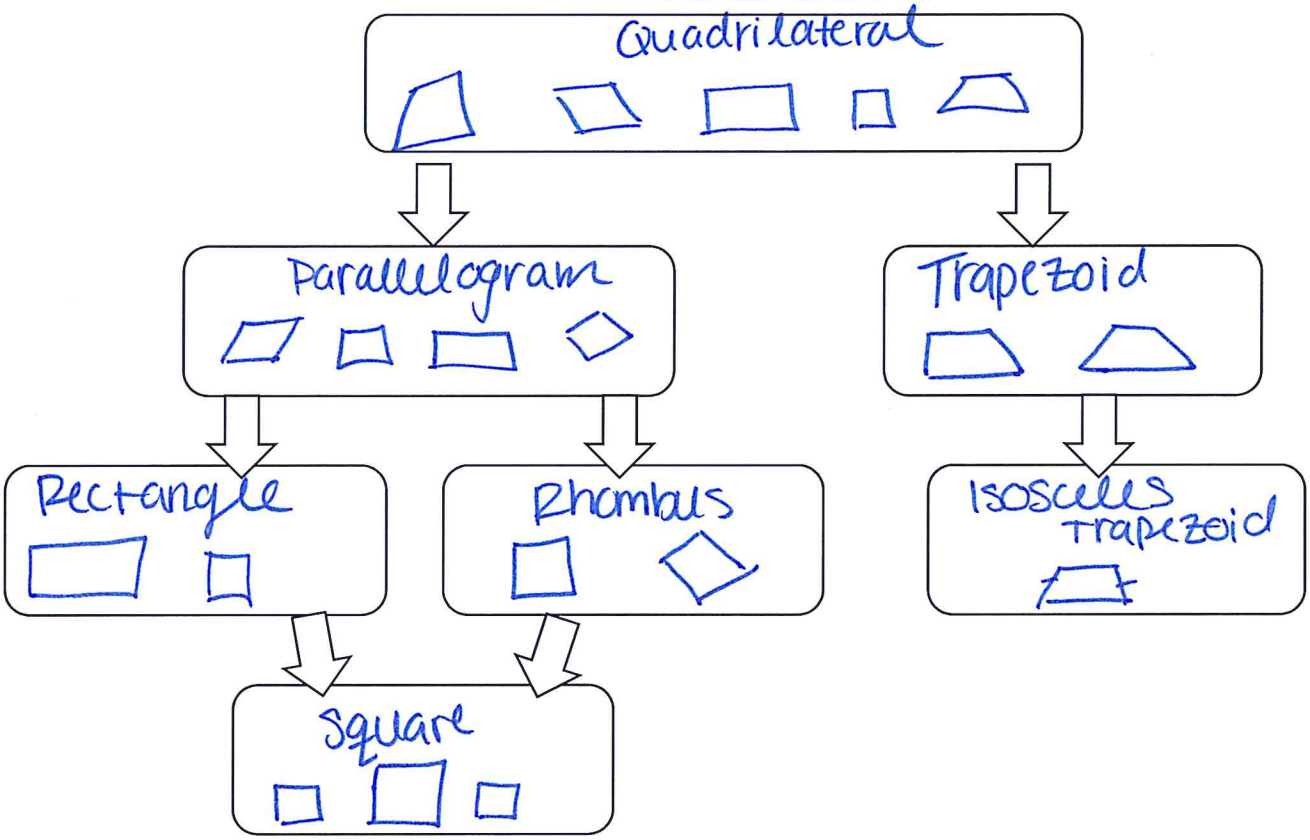
$16 + 24x = 40x + 8$

$\frac{8}{16} = \frac{16x}{16}$

6.6 Reasoning About Special Quadrilaterals

Goal: Identify special quadrilaterals based on limited information

The Quadrilateral Family Tree



A square is always a rhombus, rectangle, parallelogram, and quadrilateral.

A rectangle is always a parallelogram and sometimes a square.

A rhombus is always a parallelogram and sometimes a square.

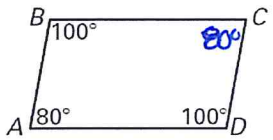
A parallelogram is always a quadrilateral.

An isosceles trapezoid is always a trapezoid and a quadrilateral.

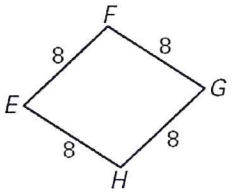
A trapezoid is always a quadrilateral.

Determine whether the quadrilateral is a trapezoid, isosceles trapezoid, parallelogram, rectangle, rhombus, or square.

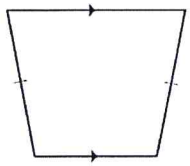
a) parallelogram



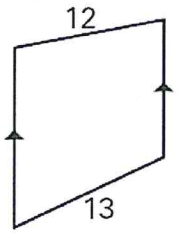
b) rhombus



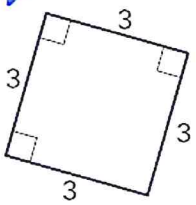
c) Isosceles trapezoid



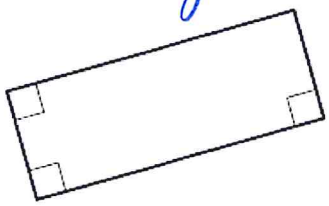
d) trapezoid



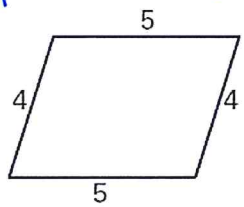
e) square



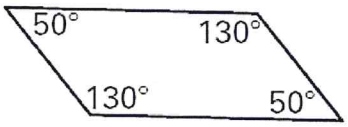
f) rectangle



g) parallelogram

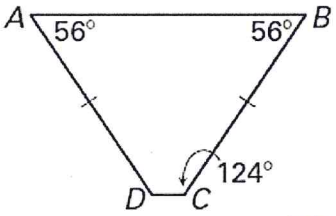


h) parallelogram



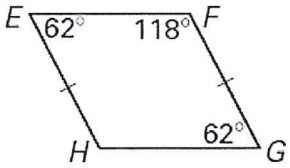
Are you given enough information to conclude that the figure is the given type of special quadrilateral? Explain your reasoning.

a) An isosceles trapezoid?



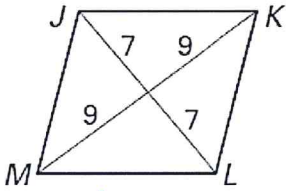
yes → consecutive ∠'s supp & 1 pair ≅ sides

b) A rhombus?



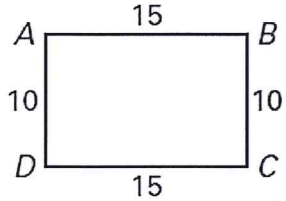
NO → we don't know that $\overline{EF} \cong \overline{HG}$

c) A parallelogram?



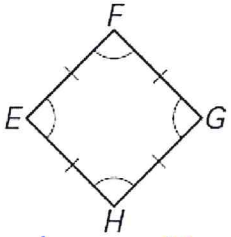
Yes → diagonals bisect each other

d) A rectangle?



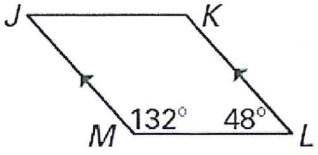
NO → we do not know anything about the ∠'s

e) A square?



yes → all sides ≅ & all ∠s must be 90°

f) A parallelogram?



NO → we do not know if $\overline{JK} \parallel \overline{ML}$