

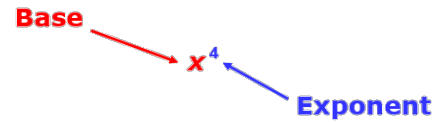
# 6.1 Integer Exponents

**Objectives:** 1. Evaluate expressions containing zero and integer exponents.  
2. Simplify expressions containing zero and integer exponents.

You have seen positive exponents. Recall that to simplify  $3^2$ , use 3 as a factor 2 times:  $3^2 = 3 \cdot 3 = 9$ .

But what does it mean for an exponent to be negative or 0? You can use a table and look for a pattern to figure it out.

<b>Power</b>	$5^5$	$5^4$	$5^3$	$5^2$	$5^1$	$5^0$	$5^{-1}$	$5^{-2}$
<b>Value</b>	3125	625	125	25	5			



Integer Exponents		
WORDS	NUMBERS	ALGEBRA
Zero exponent—Any nonzero number raised to the zero power is 1.	$3^0 = 1$ $123^0 = 1$ $(-16)^0 = 1$ $\left(\frac{3}{7}\right)^0 = 1$	If $x \neq 0$ , then $x^0 = 1$ .
Negative exponent—A nonzero number raised to a negative exponent is equal to 1 divided by that number raised to the opposite (positive) exponent.	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$	If $x \neq 0$ and $n$ is an integer, then $x^{-n} = \frac{1}{x^n}$ .

## Simplify this expression.

- One cup is  $2^{-4}$  gallons.
- A sand fly may have a wingspan up to  $5^{-3}$  m.

## Simplify.

a.  $4^{-3}$

b.  $7^0$

c.  $(-5)^{-4}$

d.  $-5^{-4}$

e.  $10^{-4}$

f.  $(-2)^{-4}$

g.  $(-2)^{-5}$

h.  $-2^{-5}$

## Evaluate the expression for the given value of the variables.

a.  $x^{-2}$  for  $x = 4$

b.  $-2a^0b^{-4}$  for  $a = 5$  and  $b = -3$

c.  $p^{-3}$  for  $p = 4$

d.  $8a^{-2}b^0$  for  $a = -2$  and  $b = 6$

What if you have an expression with a negative exponent in a denominator, such as  $\frac{1}{x^{-n}}$  ?

$$x^{-n} = \frac{1}{x^n}, \text{ or } \frac{1}{x^n} = x^{-n}$$

An expression that contains negative or zero exponents is not considered to be simplified. Expressions should be rewritten with only positive exponents.

**Simplify.**

a.  $7w^{-4}$

b.  $\frac{-5}{k^{-2}}$

c.  $\frac{a^0 b^{-2}}{c^{-3} d^6}$

d.  $2r^0 m^{-3}$

e.  $\frac{r^{-3}}{7}$

f.  $\frac{g^4}{h^{-6}}$

## 6.2 Rational Exponents

**Objective:** 1. Evaluate and simplify expressions containing rational exponents.

Recall that the radical symbol  $\sqrt{\quad}$  is used to indicate roots. The index is the small number to the left of the radical symbol that tells which root to take. For example  $\sqrt[3]{\quad}$  represents a cubic root. Since  $2^3 = 2 \cdot 2 \cdot 2 = 8$ ,  $\sqrt[3]{8} = 2$ .

Another way to write  $n$ th roots is by using fractional exponents. For example, for  $b > 1$ , suppose  $\sqrt{b} = b^k$ .

$$\begin{aligned} \sqrt{b} &= b^k \\ (\sqrt{b})^2 &= (b^k)^2 \\ b^1 &= b^{2k} \end{aligned}$$

$$1 = 2k$$

$$\frac{1}{2} = k$$

So for all  $b > 1$ ,  $\sqrt{b} = b^{\frac{1}{2}}$ .

### Definition of $b^{\frac{1}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{1}{n}$ is equal to the $n$ th root of that number.	$3^{\frac{1}{2}} = \sqrt{3}$ $5^{\frac{1}{4}} = \sqrt[4]{5}$ $2^{\frac{1}{7}} = \sqrt[7]{2}$	If $b > 1$ and $n$ is an integer, where $n \geq 2$ , then $b^{\frac{1}{n}} = \sqrt[n]{b}$ . $b^{\frac{1}{2}} = \sqrt{b}$ , $b^{\frac{1}{3}} = \sqrt[3]{b}$ , $b^{\frac{1}{4}} = \sqrt[4]{b}$ , and so on.

**Simplify each expression.**

1.  $343^{\frac{1}{3}}$

2.  $32^{\frac{1}{5}} + 9^{\frac{1}{2}}$

3.  $81^{\frac{1}{4}}$

4.  $121^{\frac{1}{2}} + 256^{\frac{1}{4}}$

**Definition of  $b^{\frac{m}{n}}$** 

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{m}{n}$ is equal to the $n$ th root of the number raised to the $m$ th power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$ $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$	If $b > 1$ and $m$ and $n$ are integers, where $m \geq 1$ and $n \geq 2$ , then $b^{\frac{m}{n}} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$ .

**Simplify each expression.**

1.  $81^{\frac{5}{4}}$

2.  $3125^{\frac{2}{5}}$

3.  $16^{\frac{3}{4}}$

4.  $1^{\frac{3}{5}}$

5.  $27^{\frac{4}{3}}$

Given a cube with surface area  $S$ , the volume  $V$  of the cube can be found by using the formula  $V = \left(\frac{S}{6}\right)^{\frac{3}{2}}$

Find the volume of a cube with surface area  $54 \text{ m}^2$ .

The approximate number of Calories  $C$  that an animal needs each day is given by  $C = 72m^{\frac{3}{4}}$ , where  $m$  is the animal's mass in kilograms. Find the number of Calories that an 81 kg panda needs each day.

When  $n$  is even, you must simplify  $\sqrt[n]{x^n}$  to  $|x|$ , because you do not know whether  $x$  is positive or negative. When  $n$  is odd, simplify  $\sqrt[n]{x^n}$  to  $x$ . When you are told that all variables represent nonnegative numbers, you do not need to use absolute values in your answer.

When $x$ is...	and $n$ is...	$x^n$ is...	and $\sqrt[n]{x^n}$ is...
Positive	Even	Positive	Positive
Negative	Even	Positive	Positive
Positive	Odd	Positive	Positive
Negative	Odd	Negative	Negative

**Simplify. All variables represent nonnegative numbers.**

1.  $\sqrt[4]{a^4 b^{20}}$

2.  $(x^6 y^4)^{\frac{1}{2}} \sqrt{y^2}$

3.  $\sqrt[4]{x^4 y^{12}}$

4.  $\left(\frac{xy^{\frac{1}{2}}}{\sqrt[5]{x^5}}\right)^2$

5.  $\left(a^4 b^{\frac{1}{4}}\right)^4 \sqrt[3]{b^3}$

6.  $\sqrt[5]{x^{10} z^5}$

## 6.3 Polynomials

**Objectives:** 1. Classify polynomials and write polynomials in standard form.  
2. Evaluate polynomial expressions.

A \_\_\_\_\_ is a number, a variable, or a product of numbers and variables with whole-number exponents.

Monomials	Not Monomials
5 $x$ $-7xy$ $0.5x^4$	$-0.3x^{-2}$ $4x - y$ $\frac{2}{x^3}$

The \_\_\_\_\_ is the sum of the exponents of the variables. A constant has degree 0.

**Find the degree of each monomial.**

1.  $4p^4q^3$

2.  $7ed$

3.  $3$

4.  $1.5k^2m$

5.  $4x$

6.  $2c^3$

A \_\_\_\_\_ is a monomial or a sum or difference of monomials.

The \_\_\_\_\_ is the degree of the term with the greatest degree.

**Find the degree of each polynomial.**

1.  $11x^7 + 3x^3$

2.  $\frac{1}{3}w^2z + \frac{1}{2}z^4 - 5$

3.  $5x - 6$

4.  $x^3y^2 + x^2y^3 - x^4 + 2$

5.  $7a^3b^2 - 2a^4 + 4b - 15$

6.  $25x^2 - 3x^4$

The \_\_\_\_\_ that contains one variable is written with the terms in order from greatest degree to least degree. When written in standard form, the coefficient of the first term is called the \_\_\_\_\_.

**Write the polynomial in standard form. Then give the leading coefficient.**

1.  $6x - 7x^5 + 4x^2 + 9$

2.  $y^2 + y^6 - 3y$

3.  $16 - 4x^2 + x^5 + 9x^3$

4.  $18y^5 - 3y^8 + 14y$

Some polynomials have special names based on their degree and the number of terms they have.

Terms	Name
1	
2	
3	
4 or more	

Degree	Name
0	
1	
2	
3	
4	
5	
6 or more	

**Classify each polynomial according to its degree & number of terms.**

1.  $5n^3 + 4n$

2.  $4y^6 - 5y^3 + 2y - 9$

3.  $-2x$

4.  $x^3 + x^2 - x + 2$

5.  $-3y^8 + 18y^5 + 14y$

6.  $6$

A tourist accidentally drops her lip balm off the Golden Gate Bridge. The bridge is 220 feet from the water of the bay. The height of the lip balm is given by the polynomial  $-16t^2 + 220$ , where  $t$  is time in seconds. How far above the water will the lip balm be after 3 seconds?

**What if...?** Another firework with a 5-second fuse is launched from the same platform at a speed of 400 feet per second. Its height is given by  $-16t^2 + 400t + 6$ . How high will this firework be when it explodes?

The polynomial  $3.675v + 0.096v^2$  is used to estimate the stopping distance in feet for a car whose speed is  $v$  miles per hour on flat dry pavement. What is the stopping distance for a car traveling at 70 miles per hour?

## 6.4 Adding & Subtracting Polynomials

**Objective:** 1. Add or subtract polynomials.

Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

**Add or subtract.**

1.  $12p^3 + 11p^2 + 8p^3$

2.  $5x^2 - 6 - 3x + 8$

3.  $t^2 + 2s^2 - 4t^2 - s^2$

4.  $10m^2n + 4m^2n - 8m^2n$    5.  $2s^2 + 3s^2 + s$    6.  $9b^3c^2 + 5b^3c^2 - 13b^3c^2$

**Add.**

1.  $(4m^2 + 5) + (m^2 - m + 6)$

2.  $(10xy + x) + (-3xy + y)$

3.  $(6x^2 - 4y) + (3x^2 + 3y - 8x^2 - 2y)$

4.  $\left(\frac{1}{2}a^2 + b + 2\right) + \left(\frac{3}{2}a^2 - 4b + 5\right)$

To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of *each* term in the polynomial:

$$-(2x^3 - 3x + 7) = -2x^3 + 3x - 7$$

**Subtract.**

1.  $(x^3 + 4y) - (2x^3)$

2.  $(7m^4 - 2m^2) - (5m^4 - 5m^2 + 8)$

3.  $(-10x^2 - 3x + 7) - (x^2 - 9)$

4.  $(9q^2 - 3q) - (q^2 - 5)$

5.  $(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$

6.  $(2.5ab + 14b) - (-1.5ab + 4b)$



A farmer must add the areas of two plots of land to determine the amount of seed to plant. The area of plot A can be represented by  $3x^2 + 7x - 5$  and the area of plot B can be represented by  $5x^2 - 4x + 11$ . Write a polynomial that represents the total area of both plots of land.

The profits of two different manufacturing plants can be modeled as shown, where  $x$  is the number of units produced at each plant. Use the information above to write a polynomial that represents the total profits from both plants.



## 6.5 Multiplying Polynomials

**Objective:** 1. Multiply polynomials.

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

**Multiply.**

1.  $(6y^3)(3y^5)$

2.  $(3mn^2)(9m^2n)$

3.  $\left(\frac{1}{4} s^2 t^2\right)(st)(-12 s t^2)$

4.  $(3x^3)(6x^2)$

5.  $(2r^2t)(5t^3)$

6.  $\left(\frac{1}{3} x^2 y\right)(12 x^3 z^2)(y^4 z^5)$

## Multiply.

1.  $4(3x^2 + 4x - 8)$     2.  $6pq(2p - q)$

3.  $\frac{1}{2}x^2y(6xy + 8x^2y^2)$

4.  $2(4x^2 + x + 3)$

5.  $3ab(5a^2 + b)$

6.  $5r^2s^2(r - 3s)$

To multiply a binomial by a binomial, you can apply the Distributive Property.

$$\begin{aligned}(x+3)(x+2) &= x(x+2) + 3(x+2) && \text{Distribute.} \\ &= x(x+2) + 3(x+2) && \text{Distribute again.} \\ &= x(x) + x(2) + 3(x) + 3(2) && \text{Multiply.} \\ &= x^2 + 2x + 3x + 6 && \text{Combine like terms.} \\ &= x^2 + 5x + 6\end{aligned}$$

Another method for multiplying binomials is called the **FOIL** method.

1. Multiply the **First** terms.  $(x+3)(x+2) \rightarrow x \cdot x = x^2$
2. Multiply the **Outer** terms.  $(x+3)(x+2) \rightarrow x \cdot 2 = 2x$
3. Multiply the **Inner** terms.  $(x+3)(x+2) \rightarrow 3 \cdot x = 3x$
4. Multiply the **Last** terms.  $(x+3)(x+2) \rightarrow 3 \cdot 2 = 6$

$$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

↑    ↑    ↑    ↑  
**F**   **O**   **I**   **L**

**Multiply.**

**1.**  $(s + 4)(s - 2)$

**2.**  $(x - 4)^2$

**3.**  $(8m^2 - n)(m^2 - 3n)$

**4.**  $(a + 3)(a - 4)$

**5.**  $(x - 3)^2$

**6.**  $(2a - b^2)(a + 4b^2)$

To multiply polynomials with more than two terms, you can use the Distributive Property several times.

$$\begin{aligned}
 (5x + 3)(2x^2 + 10x - 6) &= 5x(2x^2 + 10x - 6) + 3(2x^2 + 10x - 6) \\
 &= 5x(2x^2 + 10x - 6) + 3(2x^2 + 10x - 6) \\
 &= 5x(2x^2) + 5x(10x) + 5x(-6) + 3(2x^2) + 3(10x) + 3(-6) \\
 &= 10x^3 + 50x^2 - 30x + 6x^2 + 30x - 18 \\
 &= 10x^3 + 56x^2 - 18
 \end{aligned}$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length  $(2x^2 + 10x - 6)$  and width  $(5x + 3)$ :

	$2x^2$	$+10x$	$-6$
$5x$	$10x^3$	$50x^2$	$-30x$
$+3$	$6x^2$	$30x$	$-18$

$$\begin{aligned}
 &10x^3 + 6x^2 + 50x^2 + 30x - 30x - 18 \\
 &10x^3 + 56x^2 - 18
 \end{aligned}$$

**Multiply.**

**1.**  $(x - 5)(x^2 + 4x - 6)$

**2.**  $(2x - 5)(-4x^2 - 10x + 3)$

**3.**  $(x + 3)^3$

**4.**  $(3x + 1)(x^3 + 4x^2 - 7)$

**5.**  $(x + 3)(x^2 - 4x + 6)$

**6.**  $(3x + 2)(x^2 - 2x + 5)$

The width of a rectangular prism is 3 feet less than the height, and the length of the prism is 4 feet more than the height.

**a.** Write a polynomial that represents the area of the base of the prism.

**b.** Find the area of the base when the height is 5 ft.

The length of a rectangle is 4 meters shorter than its width.

a. Write a polynomial that represents the area of the rectangle.

b. Find the area of a rectangle when the width is 6 meters.

A triangle has a base that is 4cm longer than its height.

a. Write a polynomial that represents the area of the triangle.

b. Find the area when the height is 8 cm.

## 6.6 Special Products of Binomials

**Objective:** 1. Find special products of polynomials.

A trinomial of the form  $a^2 + 2ab + b^2$  is called a \_\_\_\_\_  
\_\_\_\_\_ which is a trinomial that is the result of  
squaring a binomial.

**Multiply.**

1.  $(x + 3)^2$

2.  $(4s + 3t)^2$

3.  $(5 + m^2)^2$

4.  $(x + 6)^2$

5.  $(5a + b)^2$

6.  $(1 + c^3)^2$

A trinomial of the form  $a^2 - ab + b^2$  is also a \_\_\_\_\_ because it is the result of squaring the binomial  $(a - b)$ .

**Multiply.**

**1.  $(x - 6)^2$**

**2.  $(4m - 10)^2$**

**3.  $(2x - 5y)^2$**

**4.  $(x - 7)^2$**

**5.  $(3b - 2c)^2$**

**6.  $(a^2 - 4)^2$**

$(a + b)(a - b) = a^2 - b^2$ . A binomial of the form  $a^2 - b^2$  is called a \_\_\_\_\_.

**Multiply.**

**1.  $(x + 4)(x - 4)$**

**2.  $(p^2 + 8q)(p^2 - 8q)$**

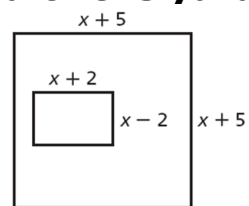
**3.  $(10 + b)(10 - b)$**

**4.  $(x + 8)(x - 8)$**

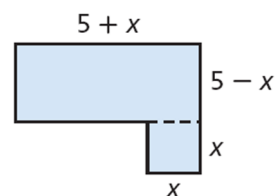
**5.  $(3 + 2y^2)(3 - 2y^2)$**

**6.  $(9 + r)(9 - r)$**

Write a polynomial that represents the area of the yard around the pool.



Write an expression that represents the area of the swimming pool.



#### Special Products of Binomials

##### Perfect-Square Trinomials

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

##### Difference of Two Squares

$$(a + b)(a - b) = a^2 - b^2$$