### 6.1 Integer Exponents

Objectives: 1. Evaluate expressions containing zero and integer exponents.
2. Simplify expressions containing zero and integer exponents

You have seen positive exponents. Recall that to simplify $3^{2}$, use 3 as a factor 2 times: $3^{2}=3 \bullet 3=9$.
But what does it mean for an exponent to be negative or 0 ? You can use a table and look for a pattern to figure it out.

| Power | $5^{5}$ | $5^{4}$ | $5^{3}$ | $5^{2}$ | $5^{1}$ | $5^{0}$ | $5^{-1}$ | $5^{-2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value | 3125 | 625 | 125 | 25 | 5 | 1 | $1 / 5$ | $1 / 25$ |

Base

Integer Exponents


Simplify this expression.

1. One cup is $2^{-4}$ gallons. 2. A sand fly may have a wingspan up to $5^{-3} \mathrm{~m}$.


## Simplify.

a. $4^{-3}$

b. $7^{0}$
C. $(-5)^{-4}$
d. $-5^{-4}$

$\frac{1}{-5^{4}} \frac{1}{-625}$
e. $10^{-4}$

f. $(-2)^{-4}$
g. $(-2)^{-5}$
$\frac{1}{(-2)^{5}} \frac{1}{-32}$
h. $-2^{-5}$


Evaluate the expression for the given value of the variables.
a. $x^{-2}$ for $x=4$

b. $-2 a^{0} b^{-4}$ for $a=5$ and $b=-3$

C. $\boldsymbol{p}^{-3}$ for $p=4$

d. $8 a^{-2} b^{0}$ for $a=-2$ and $b=6$


What if you have an expression with a negative exponent in a denominator, such as $\frac{1}{x^{-n}}$ ? $\quad x^{-n}=\frac{1}{x^{n}}$, or $\frac{1}{x^{n}}=x^{-n}$
An expression that contains negative or zero exponents is not considered to be simplified. Expressions should be rewritten with only positive exponents.
Simplify.
a. $7 w^{-4}$
$\frac{7}{1 w^{4}}$
b. $\frac{-5}{k^{-2}}$
C. $\frac{a^{0} b^{-2}}{c^{-3} d^{6}}$
$-5 K^{2}$
$\frac{c^{3}}{b^{2} d^{6}}$
d. $2 r^{-0} m^{-3}$
$\frac{2}{m^{3}}$
e. $\frac{r^{-3}}{7}$
$\frac{1}{7 r^{3}}$

$$
\begin{aligned}
& \mathrm{f} \cdot \frac{\boldsymbol{g}^{4}}{\boldsymbol{h}^{-6}} \\
& g^{4} h^{6}
\end{aligned}
$$

### 6.2 Rational Exponents <br> Objective: 1. Evaluate and simplify expressions containing rational exponents.

Recall that the radical symbol $\sqrt{ }$ is used to indicate roots. The index is the small number to the left of the radical symbol that tells which root to take. For example $\sqrt[3]{ }$ represents a cubic root. Since $2^{3}=$ $2 * 2 * 2=8, \sqrt[3]{8}=2$.

Another way to write nth roots is by using fractional exponents. For example, for $b>1$, suppose $\sqrt{b}=b^{k}$.

$$
\begin{aligned}
\sqrt{b} & =b^{k} \\
(\sqrt{b})^{2} & =\left(b^{k}\right)^{2} \\
\boldsymbol{b}^{\mathbf{1}} & =\boldsymbol{b}^{\mathbf{2} k} \\
\mathbf{1} & =\mathbf{2 k} \\
\frac{1}{2} & =k \\
\text { So for all } \boldsymbol{b} & >\mathbf{1}, \sqrt{b}=b^{\frac{1}{2}} .
\end{aligned}
$$

| Definition of $b^{\frac{7}{n}}$ |  |  |
| :--- | :--- | :---: |
| WORDS | NUMBERS | ALGEBRA |
| A number raised to the | $3^{\frac{1}{2}}=\sqrt{3}$ | If $b>1$ and $n$ is an <br> integer, where $n \geq 2$, <br> power of $\frac{1}{n}$ is equal to the |
| $n$th root of that number. | $5^{\frac{1}{4}}=\sqrt[4]{5}$ | then $b^{\frac{1}{n}}=\sqrt[4]{b}$. <br> $b^{\frac{1}{2}}=\sqrt{b}, b^{\frac{1}{3}}=\sqrt[3]{b}$, <br> $b^{\frac{1}{4}}=\sqrt[4]{b}$, and so on. |

## Simplify each expression.

1. 

$343^{\frac{1}{3}}$
$\sqrt[3]{343}$
7
2. $32^{\frac{1}{5}}+9^{\frac{1}{2}}$

3. $81^{\frac{1}{4}}$
4. $121^{\frac{1}{2}}+256^{\frac{1}{4}}$

$\sqrt{121}+\sqrt[4]{256}$
$11+4=15$

| Definition of $b^{\frac{m}{7 \prime}}$ |  |  |
| :---: | :---: | :---: |
| WORDS | NUMBERS | ALGEBRA |
| A number raised to the power of $\frac{m}{n}$ is equal to the $n$th root of the number raised to the $m$ th power. | $\begin{aligned} & 8^{\frac{2}{3}}=(\sqrt[2]{8})^{2}=2^{2}=4 \\ & 8^{\frac{2}{3}}=\sqrt[3]{8^{2}}=\sqrt[3]{64}=4 \end{aligned}$ | If $b>1$ and $m$ and $n$ are integers, where $m \geq 1$ and $n \geq 2$, then $b^{\frac{m}{n}}=(\sqrt[n]{b})^{m}=\sqrt[m]{b^{m}}$ |

## Simplify each expression.



Given a cube with surface area $S, 3$ the volume $V$ of the cube can be found by using the formula $\boldsymbol{v}=\left(\frac{\boldsymbol{s}}{\mathbf{6}}\right)^{\frac{3}{2}}$
Find the volume of a cube with surface area $54 \mathrm{~m}^{2}$.


$(9)^{3 / 2}$
$3^{3}=27 m^{3}$

The approximate number of Calories $C$ that an animal needs each day is given by $c=72 \mathrm{~m}^{\frac{3}{4}}$, where m is the animal's mass in kilograms. Find the number of Calories that an 81 kg panda needs each day.

$$
\left.C=72 m^{3 / 4}=72(8)\right)^{3 / 4} \quad-72(\sqrt[4]{81})^{3}
$$

$72(3)^{3}$
$72(2)=1944$

When $n$ is even, you must simplify $\sqrt[n]{x^{n}}$ to $|x|$, because you do not know whether $x$ is positive or negative. When $n$ is odd, simplify $\sqrt[n]{x^{n}}$ to $x$. When you are told that all variables represent nonnegative numbers, you do not need to use absolute values in your answer.

| When $x$ is... | and $n$ is... | $x^{n}$ is... | and $\sqrt[n]{x^{n}}$ is... |
| :---: | :---: | :---: | :---: |
| Positive | Even | Positive | Positive |
| Negative | Even | Positive | Positive |
| Positive | Odd | Positive | Positive |
| Negative | Odd | Negative | Negative |

## Simplify. All variables represent nonnegative numbers.

## 1. $\sqrt[4]{a^{4} b^{20}}$ <br> 

2. $\left(x^{6} y^{\frac{1}{4}}\right)^{\frac{1}{2}} \sqrt{y^{2}}$

$x^{3} y^{3}$
3. $\sqrt[4]{x^{4} y^{12}}$
$x y^{3}$
4. $\frac{\left(x y^{\frac{1}{2}}\right)^{2}}{\sqrt[5]{x^{5}}}$
5. $\left[a^{4} b^{\frac{1}{4}}\right] 4 \sqrt[3]{b^{3}}$
6. $\sqrt[5]{x^{10} z^{5}}$


$a^{16} b^{2}$

### 6.3 Polynomials

Objectives: 1. Classify polynomials and write polynomials in standard form.
2. Evaluate polynomial expressions.

A monomial is a number, a variable, or a product of numbers and variables with whole-number exponents.

| Monomials |  | Not Monomials |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | $x$ | $-7 x y$ | $0.5 x^{4}$ | $-0.3 x^{-2}$ | $4 x-y \quad \frac{2}{x^{3}}$.

The degree of monomial is the sum of the exponents of the variables. A constant has degree 0 .
Find the degree of each monomial.

1. $\mathbf{4} \boldsymbol{p}^{4} \boldsymbol{q}^{3}$
clegree: 7
2. Ted
degree: 2


## 4. $1.5 k^{2} m$

5. $4 x$
deg: 3
deg:1
6. $2 c^{3}$


A polynomial monomials.

The degree of a polynomial is the degree of the term with the greatest degree.
Find the degree of each polynomial.

1. $11 x^{7}+3 x^{3}$

2. $\frac{1}{3} w^{2} z+\frac{1}{2} z^{4}-5$
3. $5 x-6$
deg: 1
4. $x^{3} y^{2}+x^{2} y^{3}-x^{4}+2$
$\operatorname{deg}: 5$
deg: 5
5. $7 a^{3} b^{2}-2 a^{4}+4 b-15$
6. $25 x^{2}-3 x^{4}$
$\operatorname{deg}: 4$

The standardiof op polynomial $\qquad$ that contains one variable is written with the terms io order from greatest degree to least degree. When written in standard form, the coefficient of the first term is called the leading Coefficient.

## Write the polynomial in standard form. Then give the leading coefficient.

1. $6 x-7 x^{5}+4 x^{2}+9$
std: $-7 x^{5}+4 x^{2}+6 x+9$
$\ell_{c}:-7$
2. $16-4 x^{2}+x^{5}+9 x^{3}$
$x^{5}+9 x^{3}-4 x^{2}+16$
$\ell c: 1$
3. $y^{2}+y^{6}-3 y$
$y^{6}+y^{2}-3 y$
lc: 1
4. $18 y^{5}-3 y^{8}+14 y$
$-3 y^{8}+18 y^{5}+14 y$
lc:-3

Some polynomials have special names based on their degree and the number of terms they have.

| Terms | Name |
| :---: | :---: |
| 1 | Monomial |
| 2 | Binomial |
| 3 | Trinomial |
| dor | Polynomial |


| Degree | Name |
| :---: | :--- |
| 0 | Constant |
| 1 | linear |
| 2 | Quadratic |
| 3 | Cubic |
| 4 | Quartic |
| 5 | Quintic |
| 6 or more | 6 th $7^{\text {th }, ~ 8 t h ~ c t r . ~}$ |

Classify each polynomial according to its degree \& number of terms.

1. $5 n^{3}+4 n$
deg: cubic
binomial
2. $4 y^{6}-5 y^{3}+2 y-9$
$6^{\text {th }}$ deg. polynomial
3. $-2 x$
linear
monomial
```
4. }\mp@subsup{x}{}{3}+\mp@subsup{x}{}{2}-x+
cubic
    polynomial
```

5. $-3 y^{8}+18 y^{5}+14 y$
6. 6

Constant monomial

A tourist accidentally drops her lip balm off the Golden Gate Bridge. The bridge is 220 feet from the water of the bay. The height of the lip balm is given by the polynomial $-16 t^{2}+220$, where $t$ is time in seconds. How far above the water will the lip balm be after 3 seconds? $-16(3)^{2}+220$
$-16(9)+226$
$-144+220=76 f+$
What if...? Another firework with a 5 -second fuse is launched from the same platform at a speed of 400 feet per second. Its height is given by $-16 t^{2}$ $+400 t+6$. How high will this firework be when it explodes?

$$
\begin{array}{r}
-16(5)^{2}+400(5)+6 \\
-16(25)+2000+6 \\
-400+2000+6 \\
1600+6 \\
1,606 \mathrm{ft}
\end{array}
$$

The polynomial $3.675 v+0.096 v^{2}$ is used to estimate the stopping distance in feet for a car whose speed is $v$ miles per hour on flat dry pavement. What is the stopping distance for a car traveling at 70 miles per hour?

$$
\begin{aligned}
& 3.675(70)+0.096(70)^{2} \\
& 257.25+0.096(4900) \\
& 257.25+470.4=727.65 \text { feet }
\end{aligned}
$$

### 6.4 Adding \& Subtracting Polynomials

Objective: 1. Add or subtract polynomials
Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

## Add or subtract.

1. $12 p^{3}+11 p^{2}+8 p^{3}$

2. $5 x^{2}-6-3 x+8$ $5 x^{2}-3 x+2$
3. $t^{2}+2 s^{2}-4 t^{2}-s^{2}$

4. $10 m^{2} n+4 m^{2} n-8 m^{2} n$
$14 m^{2} n-8 m^{2} n$ $6 m^{2} n$
5. $2 s^{2}+3 s^{2}+s$
$5 s^{2}+s$
6. $9 b^{3} c^{2}+5 b^{3} c^{2}-13 b^{3} c^{2}$
$14 b^{3} c^{2}-13 b^{3} c^{2}$


Add.

1. $\left(4 m^{2}+5\right)+\left(m^{2}-m+6\right)$
$5 m^{2}-m+11$
2. $(10 x y+x)+(-3 x y+y)$



To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of each term in the polynomial:

$$
+\left(-2 x^{3}+3 x+-7\right)=-2 x^{3}+3 x-7
$$

## Subtract.

1. $\left(x^{3}+4 y\right)+\left(-2 x^{3}\right)$
2. $\left(7 m^{4}-2 m^{2}\right)+\left(5 m^{4}+5 m^{2}+8\right)$

3. $\left(-10 x^{2}-3 x+7\right)+\left(x^{2}+9\right)$

4. $\left(9 q^{2}-3 q\right)+\left(-q^{2}+5\right)$

5. $\left(2 x^{2}-3 x^{2}+1\right)+\left(x^{2}+x+-1\right)$ $-x^{2}+1-x^{2}-x-1$
6. $(2.5 a b+14 b)+(+1.5 a b+4 b)$


A farmer must add the areas of two plots of land to determine the amount of seed to plant. The area of plot $A$ can be represented by $3 x^{2}+7 x-5$ and the area of plot B can be represented by $5 x^{2}-4 x+$ 11. Write a polynomial that represents the total area of both plots of land.


The profits of two different manufacturing plants can be modeled as shown, where $x$ is the number of units produced at each plant. Use the information above to write a polynomial that represents the total profits from both plants.
$\left(-.03 x^{2}+25 x-1500\right)+\left(-.02 x^{2}+21 x-1700\right)$



### 6.5 Multiplying Polynomials

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

Multiply.

1. $\left(6 y^{3}\right)\left(3 y^{5}\right)$

2. $\left(3 m n^{2}\right)\left(9 m^{2} n\right)$ $27 m^{3} n^{3}$

$$
\begin{aligned}
& \text { 3. }\left[\frac{1}{4} s^{2} t^{2}\right](s t)\left(-12 s t^{2}\right) \\
& -3 s^{4} t^{5}
\end{aligned}
$$

4. $\left(3 x^{3}\right)\left(6 x^{2}\right)$
$18 x^{5}$
5. $\left(2 r^{2} t\right)\left(5 t^{3}\right)$

6. $\left(\frac{1}{3} x^{2} y\right)\left(12 x^{3} z^{2}\right)\left(y^{4} z^{5}\right)$


## Multiply.

1. $4\left(3 x^{2}+4 x-8\right)$
2. $6 p q(2 p-q)$
$12 x^{2}+16 x-32 \quad 12 p^{2} q-6 p q^{2}$
3. $\frac{1}{2} x^{2} y\left(6 x y+8 x^{2} y^{2}\right)$

4. $2\left(4 x^{2}+x+3\right)$
$8 x^{2}+2 x+6$
5. $3 a b\left(5 a^{2}+b\right)$
$15 a^{3} b+3 a b^{2}$
6. $5 r^{2} s^{2}(r-3 s)$

$$
5 r^{3} s^{2}-15 r^{2} s^{3}
$$

To multiply a binomial by a binomial, you can apply the Distributive Property.

$$
\begin{aligned}
(x+3)(x+2) & =x(x+2)+3(x+2) & & \text { Distribute. } \\
& =x(x+2)+3(x+2) & & \text { Distribute again. } \\
& =x(x)+x(2)+3(x)+3(2) & & \text { Multiply. } \\
& =x^{2}+2 x+3 x+6 & & \text { Combine like terms. } \\
& =x^{2}+5 x+6 & &
\end{aligned}
$$

Another method for multiplying binomials is called the FOIL method. 1. Multiply the First terms. $(x+3)(x+2) \longrightarrow x \cdot x=x^{2}$
2. Multiply the Outer terms. $(x+3)(x+2) \longrightarrow x \cdot 2=2 x$
3. Multiply the Inner terms. $(x+3)(x+2)-3 \cdot x=3 x$
4. Multiply the Last terms. $(x+3)(x+2) \longrightarrow 3 \cdot 2=6$

$$
\begin{aligned}
(x+3)(x+2) & =x^{2}+2 x+3 x+6=x^{2}+5 x+6 \\
& 1
\end{aligned}
$$

## Multiply.

1. $(s+4)(s-2)$

$$
\Delta^{2}-2 \Delta+4 \Delta-8
$$

$$
s^{2}+2 s-8
$$

2. $(x-4)^{2}$

$$
\begin{aligned}
& (x-4)(x-4) \\
& x^{2}-4 x-4 x+16 \\
& x^{2}-8 x+16
\end{aligned}
$$

3. $\left(8 m^{2}-n\right)\left(m^{2}-3 n\right)$
$8 m^{4}-24 m^{2} n-m^{2} n+3 n^{2}$ $8 m^{4}-25 m^{2} n+3 n^{2}$
4. $(a+3)(a-4)$
$a^{2}-4 a+3 a-12$ $a^{2}-a-12$
5. $(x-3)^{2}$
$(x-3)(x-3)$

$$
x^{2}-3 x-3 x+9
$$

$$
x^{2}-6 x+9
$$

6. $\left(2 a-b^{2}\right)\left(a+4 b^{2}\right)$

$2 a^{2}+7 a b^{2}-4 b^{4}$

To multiply polynomials with more than two terms, you can use the Distributive Property several times.

$$
\begin{aligned}
(5 x+3) & \left(2 x^{2}+10 x-6\right)=5 x\left(2 x^{2}+10 x-6\right)+3\left(2 x^{2}+10 x-6\right) \\
& =5 x\left(2 x^{2}+10 x-6\right)+3\left(2 x^{2}+10 x-6\right) \\
& =5 x\left(2 x^{2}\right)+5 x(10 x)+5 x(-6)+3\left(2 x^{2}\right)+3(10 x)+3(-6) \\
& =10 x^{3}+50 x^{2}-30 x+6 x^{2}+30 x-18 \\
& =10 x^{3}+56 x^{2}-18
\end{aligned}
$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length $\left(2 x^{2}+10 x-6\right)$ and width $(5 x+3)$ :

$$
\begin{aligned}
& \\
&
\end{aligned}
$$

## Multiply.


3. $(x+3)^{3}$
$(x+3)(x+3)(x+3)$

$-8 x^{3}+56 x-15$
4. $(3 x+1)\left(x^{3}+4 x^{2}-7\right)$

$x^{3}+9 x^{2}+27 x+27$
5. $(x+3)\left(x^{2}-4 x+6\right)$

$+33 x^{2}-12 x$

$3 x^{3}-4 x^{2}+11 x+10$

The width of a rectangular prism is 3 feet less than the height, and the length of the prism is 4 feet more than the height.
a. Write a polynomial that represents the area of the base of the prism.

$$
\begin{aligned}
& (h-3)(h+4) \\
& h^{2}+h-12
\end{aligned}
$$

b. Find the area of the base when the height is 5 ft .
$5^{2}+5-12$
$25+5-12$
30-12

$$
18 \mathrm{ft}^{2}
$$

The length of a rectangle is 4 meters shorter than its width.
a. Write a polynomial that represents the area of the rectangle.
$x(x-4)=x^{2}-4 x$
b. Find the area of a rectangle when the width is 6 meters.
$6^{2}-4(6)$
$36-24=12 m^{2}$
A triangle has a base that is 4 cm longer than its height.
a. Write a polynomial that represents the area of the triangle.
$\frac{1}{2}(h)(h+4) \quad \frac{1}{2}\left(h^{2}+4 h\right) \quad \frac{1}{2} h^{2}+2 h$
b. Find the area when the height is 8 cm .
$\frac{1}{2}(8)^{2}+2(8)$
$\frac{1}{2}(64)+16$
$32+16=48 \mathrm{~cm}^{2}$

### 6.6 Special Products of Binomials <br> Objective: 1. Find special products of polynomials.

A trinomial of the form $a^{2}+2 a b+b^{2}$ is called a perfect square trinomial which is a trinomial that is the result of squaring a binomial.

## Multiply.

1. $(\underset{2}{ }+\mathbf{3})^{\mathbf{2}}=(x+3)(x+3$
2. $(4 s+3 t)^{2}$ $16 s^{2}+24 s t+9 t^{2}$
3. $\left(5+m^{2}\right)^{2}$

$$
25+20 m^{4}+m^{4}
$$

4. $(x+6)^{2}$
5. $(5 a+b)^{2}$
6. $\left(1+c^{3}\right)^{2}$
$x^{2}+12 x+36$
$25 a^{2}+10 a b+b^{2}$
$1+2 c^{3}+c^{6}$

A trinomial of the form $a^{2}-a b+b^{2}$ is also a perfect square trinomial because it is the result of squaring the binomial ( $a-b$ ).

## Multiply.

1. $(x-6)^{2}$
$x^{2}-12 x+36$

2. $(4 m-10)^{2}$
3. $(2 x-5 y)^{2}$

4. $(x-7)^{2}$
5. $(3 b-2 c)^{2}$
6. $\left(a^{2}-4\right)^{2}$
$x^{2}-14 x+49$
$x^{2}-7 x-7 x+49$
$9 b^{2}-12 b c+4 c^{2}$ $9 b^{2}-6 b c-6 b c+4 c^{2}$
$a^{4}-8 a^{2}+16$

$(a+b)(a-b)=a^{2}-b^{2}$. A binomial of the form $a^{2}-b^{2}$ is called a difference of squares. Multiply.
7. $(x+4)(x-4)$
8. $\begin{aligned} & \left(p^{2}+8 q\right)\left(p^{2}-8 q\right) \\ & p^{4}-8 p^{2} q+8 p^{2} q-64 q^{2}\end{aligned}$
9. $(10+b)(10-b)$ $x^{2}-4 x+4 x-16$ $\left(x^{2}-16\right)$

10. $(x+8)(x-8)$
11. $\left(3+2 y^{2}\right)\left(3-2 y^{2}\right)$
12. $(9+r)(9-r)$
$x^{2}-8 x+8 x-64$


$$
x^{2}-64
$$



Write a polynomial that represents the area of the vars around the pool.

$$
\begin{aligned}
& (x+5)(x+5)-[(x+2)(x-2)] \\
& \left(x^{2}+10 x+25\right)+\left[-x^{2}+4\right]
\end{aligned}
$$



$$
10 x+29
$$

Write an expression that represents the area of the swimming pool.


