5.1 Solving Systems of Equations by Graphing

Objectives: 1. Identify solutions of linear equations in two variables.
2. Solve systems of linear equations in two variables by graphing.

A ____________________________ is a set of two or more linear equations containing two or more variables. A ________________ ______________________ with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

Tell whether the ordered pair is a solution of the given system.

1. (5, 2); $\begin{cases} \frac{2}{5}x - y = 0 \\ 3x - y = 13 \end{cases}$

2. (−2, 2); $\begin{cases} x + 3y = 4 \\ -x + y = 2 \end{cases}$

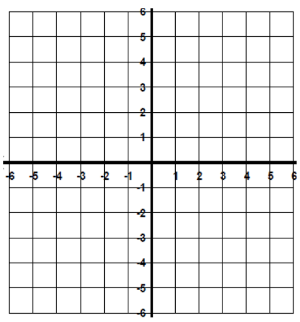
3. (1, 3); $\begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$

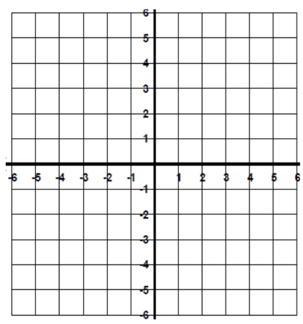
All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their _____ ______________________.

Solve the system by graphing. Check your answer.

1. $\begin{cases} y = x \\ y = -2x - 3 \end{cases}$

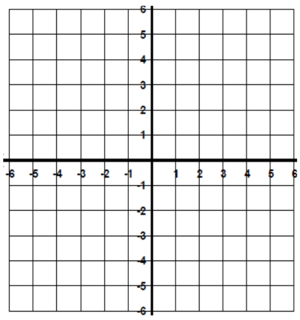
2. $\begin{cases} y = x - 6 \\ y + \frac{1}{3}x = -1 \end{cases}$

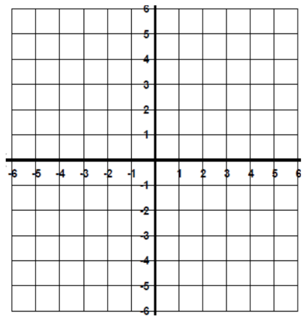




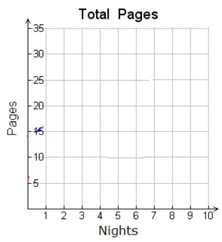
1. $\begin{cases} y = \frac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$

2. $\begin{cases} y + 2x = 9 \\ y = 4x - 3 \end{cases}$

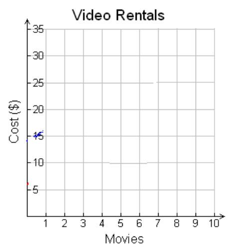




Wren and Jenni are reading the same book. Wren is on page 14 and reads 2 pages every night. Jenni is on page 6 and reads 3 pages every night. After how many nights will they have read the same number of pages? How many pages will that be?



Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost?



Joy has 5 collectable stamps and will buy 2 more each month. Ronald has 25 collectable stamps and will sell 3 each month. After how many months will they have the same number of stamps? How many will that be?

5.2 Solving Systems by Substitution

Objective: Solve system of linear equations in one variable using substitution.

Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called ___________________.

Solving Systems of Equations by Substitution	
Step 1	
Step 2	
Step 3	
Step 4	
Step 5	

Solve the system by substitution.

1. $\begin{cases} y = 3x \\ y = x - 2 \end{cases}$

2. $\begin{cases} y = x + 1 \\ 4x + y = 6 \end{cases}$

3. $\begin{cases} x + 2y = -1 \\ x - y = 5 \end{cases}$

4. $\begin{cases} 2x + y = -4 \\ x + y = -7 \end{cases}$

5. $\begin{cases} x = 6y - 11 \\ 3x - 2y = -1 \end{cases}$

6. $\begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases}$

7. $\begin{cases} y + 6x = 11 \\ 3x + 2y = -5 \end{cases}$

8. $\begin{cases} -3x + y = -1 \\ x - y = 4 \end{cases}$

Jenna is deciding between two cell-phone plans. The first plan has a $50 sign-up fee and costs $20 per month. The second plan has a $30 sign-up fee and costs $25 per month. After how many months will the total costs be the same? What will the costs be? If Jenna has to sign a one-year contract, which plan will be cheaper? Explain.

One cable television provider has a $60 setup fee and charges $80 per month, and the second has a $160 equipment fee and charges $70 per month.

5.3 Solving Systems by Elimination

Objectives: 1. Solve system of linear equations in two variables by elimination.

2. Compare and choose an appropriate method for solving systems of linear equations.

Another method for solving systems of equations is __________. Like substitution, the goal of elimination is to get one equation that has only one variable.

Solving Systems of Equations by Elimination
Step 1
Step 2
Step 3
Step 4

Solve the system by elimination.

1. $\begin{cases} 3x - 4y = 10 \\ x + 4y = -2 \end{cases}$

2. $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$

3. $\begin{cases} 2x + y = -5 \\ 2x - 5y = 13 \end{cases}$

4. $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$

5. $\begin{cases} x + 2y = 11 \\ -3x + y = -5 \end{cases}$

6. $\begin{cases} -5x + 2y = 32 \\ 2x + 3y = 10 \end{cases}$

7. $\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$

8. $\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$

Paige has $7.75 to buy 12 sheets of felt and card stock for her scrapbook. The felt costs $0.50 per sheet, and the card stock costs $0.75 per sheet. How many sheets of each can Paige buy?

Harlan has $44 to buy 7 pairs of socks. Athletic socks cost $5 per pair. Dress socks cost $8 per pair. How many pairs of each can Harlan buy?

5.4 Solving Special Systems

Objectives: 1. Solve special systems of linear equations in two variables.
2. Classify systems of linear equations and determine the number of solutions.

Systems with at least one solution are called ________________. When the two lines in a system do not intersect they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an ______________________.

Show that the system has no solution.

1. $\begin{cases} y = x - 4 \\ -x + y = 3 \end{cases}$

2. $\begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases}$

If two linear equations in a system have the same graph, the graphs are the same line. There are ___________________________ of the system because every point on the line represents a solution of both equations.

Show that the system has infinitely many solutions.

1. $\begin{cases} y = 3x + 2 \\ 3x - y + 2 = 0 \end{cases}$

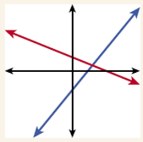
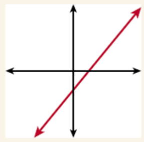
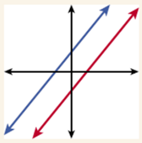
2. $\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}$

Consistent systems can either be independent or dependent.

An _______________**system** has exactly one solution. The graph of an independent system consists of two intersecting lines.

A ________________**system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines.

Classification of Systems of Linear Equations

CLASSIFICATION	CONSISTENT AND INDEPENDENT	CONSISTENT AND DEPENDENT	INCONSISTENT
Number of Solutions	Exactly one	Infinitely many	None
Description	Different slopes	Same slope, same y-intercept	Same slope, different y-intercepts
Graph	Intersecting lines 	Coincident lines 	Parallel lines 

Classify the system. Give the number of solutions.

1. $\begin{cases} 3y = x + 3 \\ -\frac{1}{3}x + y = 1 \end{cases}$

2. $\begin{cases} x + y = 5 \\ 4 + y = -x \end{cases}$

3. $\begin{cases} y = 4(x + 1) \\ y - 3 = x \end{cases}$

4. $\begin{cases} x + 2y = -4 \\ -2(y + 2) = x \end{cases}$

5. $\begin{cases} y = -2(x - 1) \\ y = -x + 3 \end{cases}$

6. $\begin{cases} 2x - 3y = 6 \\ y = \frac{2}{3}x \end{cases}$

Jared and David both started a savings account in January. If the pattern of savings in the table continues, when will the amount in Jared's account equal the amount in David's account?

	Jan	Feb	Mar	Apr	May
Jared	25	30	35	40	45
David	40	45	50	55	60

Matt has $100 in a checking account and deposits $20 per month. Ben has $80 in a checking account and deposits $30 per month. Will the accounts ever have the same balance? Explain.

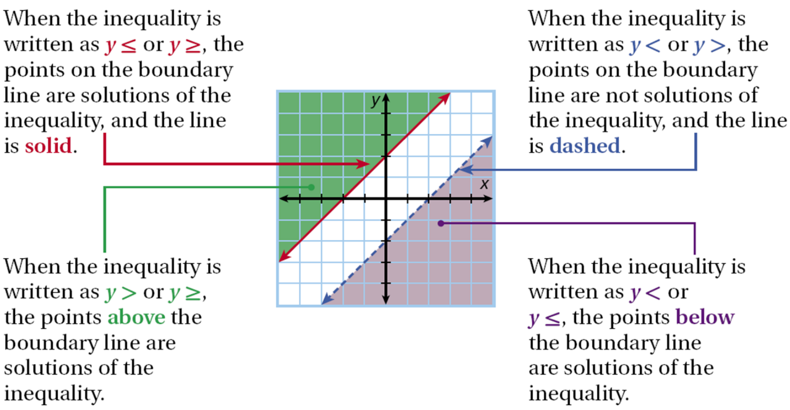
5.5 Solving Linear Inequalities

Objective: Graph and solve linear inequalities in two variables.

A ____________________ is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A ____________________ is any ordered pair that makes the inequality true.

Tell whether the ordered pair is a solution of the inequality.

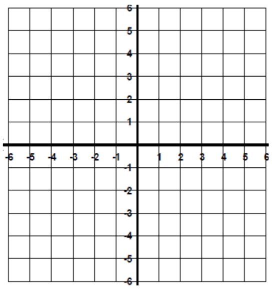
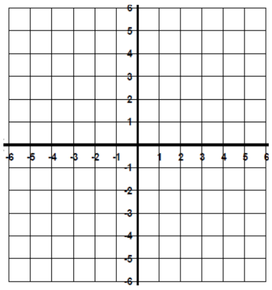
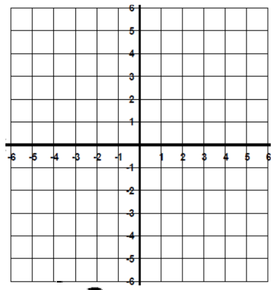
1. **(–2, 4); $y < 2x + 1$** **2.** **(3, 1); $y > x - 4$** **3.** **(4, 5); $y < x + 1$**



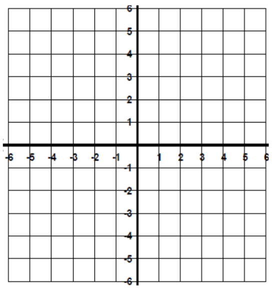
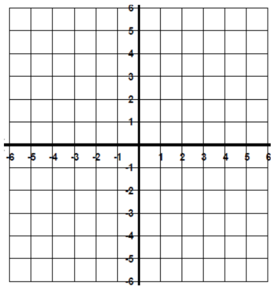
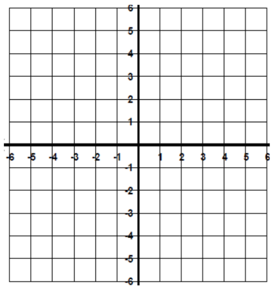
Graphing Linear Inequalities	
Step 1	
Step 2	
Step 3	

Graph the solutions of the linear inequality.

1. $y \le 2x - 3$ **2.** $5x + 2y > -8$ **3.** $4x - y + 2 \le 0$

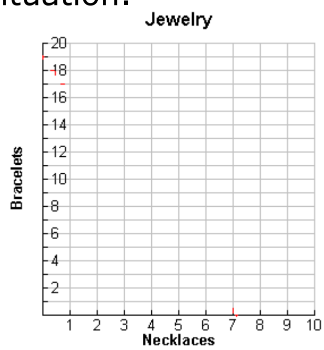




4. $4x - 3y > 12$ **5.** $2x - y - 4 > 0$ **6.** $y \ge -\frac{2}{3}x + 1$

Ada has at most 285 beads to make jewelry. A necklace requires 40 beads, and a bracelet requires 15 beads.

a. Write a linear inequality to describe the situation.

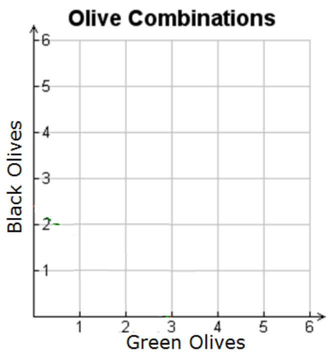


b. Graph the solutions.

c. Give two combinations of necklaces and bracelets that Ada could make.

Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than \$6. Green olives cost \$2 per pound and black olives cost \$2.50 per pound.

a. Write a linear inequality to describe the situation.



b. Graph the solutions.

c. Give two combinations of olives that Dirk could buy.

5.6 Solving Systems of Linear Inequalities

Objective: Graph and solve systems of linear inequalities in two variables.

A ________________________________ is a set of two or more linear inequalities containing two or more variables. The ________________ __________________________ are all the ordered pairs that satisfy all the linear inequalities in the system.

Tell whether the ordered pair is a solution of the given system.

1. $(-1, -3); \begin{cases} y \le -3x + 1 \\ y < 2x + 2 \end{cases}$

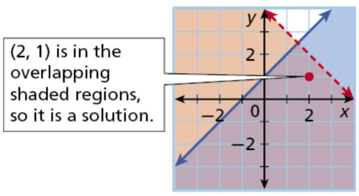
2. $(-1, 5); \begin{cases} y < -2x - 1 \\ y \ge x + 3 \end{cases}$

3. $(0, 1); \begin{cases} y < -3x + 2 \\ y \ge x - 1 \end{cases}$

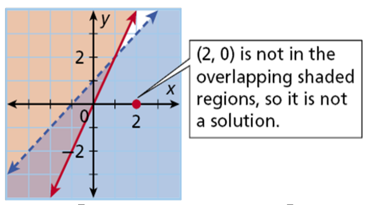
4. $(0, 0); \begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$

To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions.

Example 1A



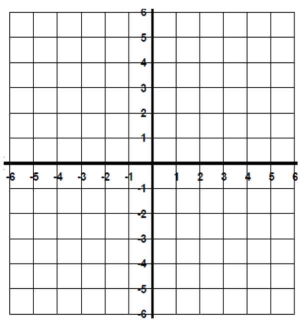
Example 1B

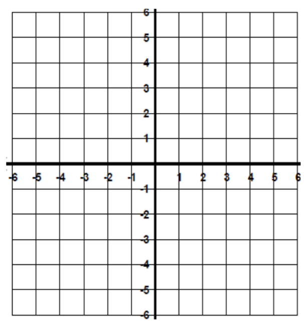


Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

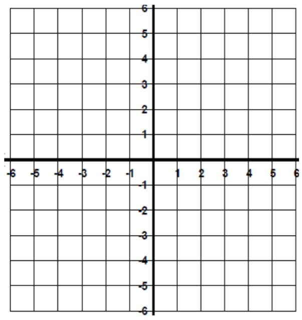
1. $\begin{cases} y \le 3 \\ y > -x + 5 \end{cases}$

2. $\begin{cases} -3x + 2y \ge 2 \\ y < 4x + 3 \end{cases}$

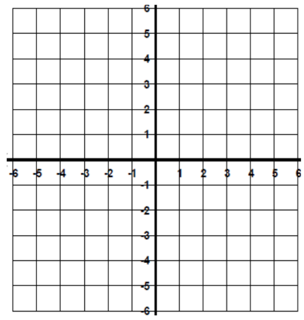




3. $\begin{cases} y \le x + 1 \\ y > 2 \end{cases}$

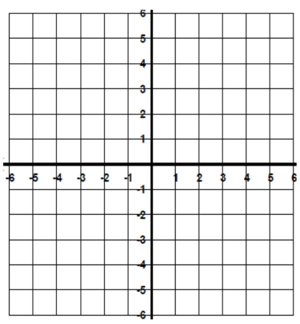


4. $\begin{cases} y > x - 7 \\ 3x + 6y \le 12 \end{cases}$

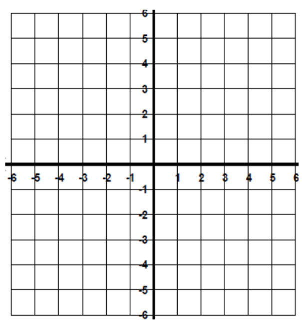


Graph the system of linear inequalities. Describe the solutions.

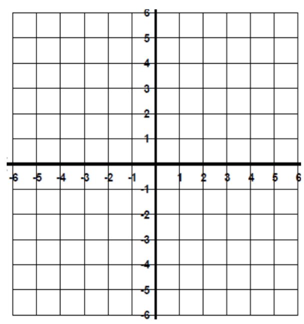
1. $\begin{cases} y \le -2x - 4 \\ y > -2x + 5 \end{cases}$



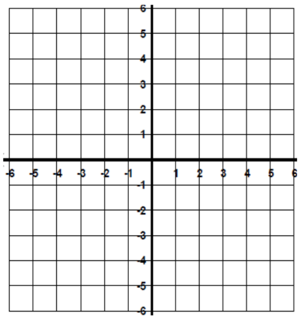
2. $\begin{cases} y < 3x + 6 \\ y > 3x - 2 \end{cases}$



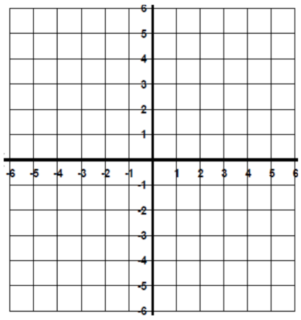
3. $\begin{cases} y \ge 4x + 6 \\ y \ge 4x - 5 \end{cases}$



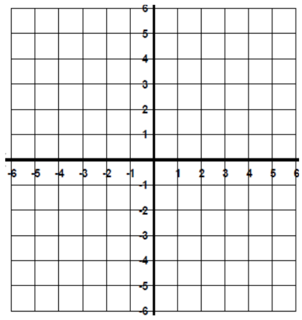
4. $\begin{cases} y > x + 1 \\ y \le x - 3 \end{cases}$



5. $\begin{cases} y \ge 4x - 2 \\ y \le 4x + 2 \end{cases}$

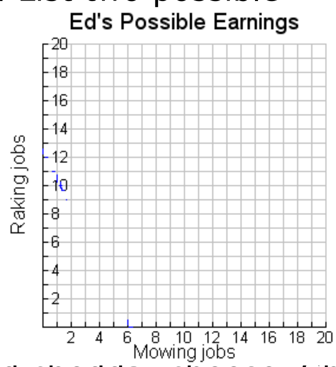


6. $\begin{cases} y > -2x + 3 \\ y > -2x \end{cases}$



In one week, Ed can mow at most 9 times and rake at most 7 times. He charges $20 for mowing and $10 for raking. He needs to make more than $125 in one week. Show and describe all the possible combinations of mowing and raking that Ed can do to meet his goal. List two possible combinations.

Earnings per Job ($)	
Mowing	20
Raking	10



At her party, Alice is serving pepper jack cheese and cheddar cheese. She wants to have at least 2 pounds of each. Alice wants to spend at most $20 on cheese. Show and describe all possible combinations of the two cheeses Alice could buy. List two possible combinations.

Price per Pound ($)	
Pepper Jack	4
Cheddar	2