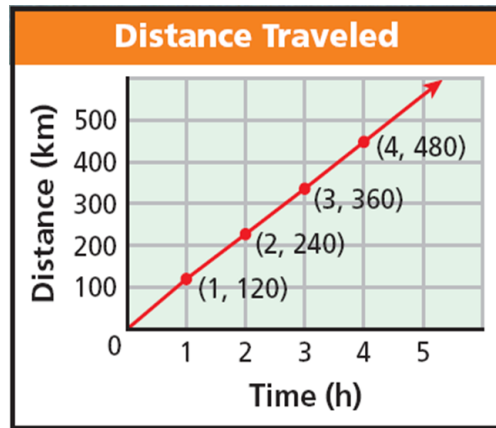
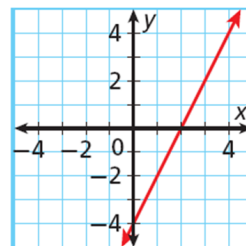
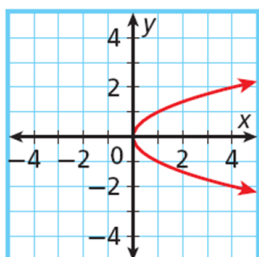
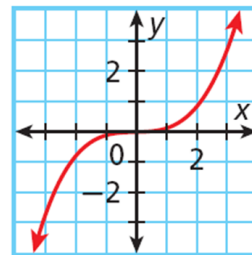
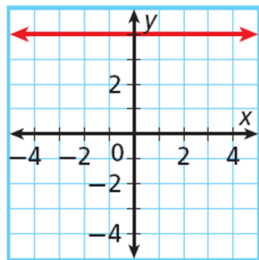
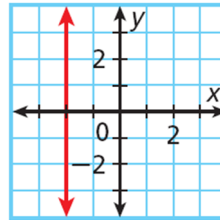
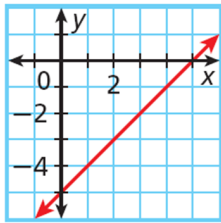


4.1 A: Identifying Linear Functions

The graph represents a function because each domain value (x -value) is paired with exactly one range value (y -value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a _____.



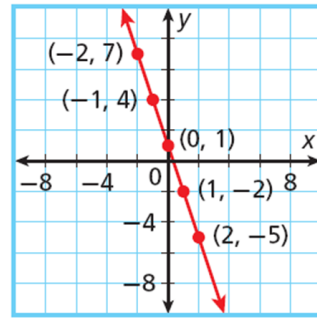
Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?



You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in x corresponds to a constant change in y .

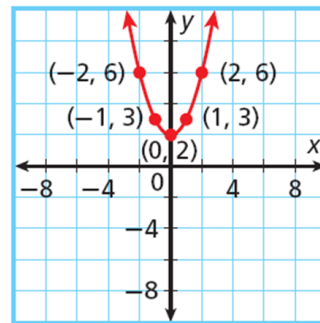
x	y
-2	7
-1	4
0	1
1	-2
2	-5

Arrows on the left indicate a constant change of +1 in x for each row. Arrows on the right indicate a constant change of -3 in y for each row.



x	y
-2	6
-1	3
0	2
1	3
2	6

Arrows on the left indicate a constant change of +1 in x for each row. Arrows on the right indicate changes in y : -3 from $x=-2$ to $x=-1$, -1 from $x=-1$ to $x=0$, +1 from $x=0$ to $x=1$, and +3 from $x=1$ to $x=2$.



Tell whether the set of ordered pairs satisfies a linear function. Explain.

1. $\{(0, -3), (4, 0), (8, 3), (12, 6), (16, 9)\}$

x	y
0	-3
4	0
8	3
12	6
16	9

2. $\{(-4, 13), (-2, 1), (0, -3), (2, 1), (4, 13)\}$

x	y

3. $\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}$

4.1 B: Identifying Linear Functions

- Objectives:** 1. Identify linear functions and linear equations.
2. Graph linear functions that represent real-world situations and give their domain and range.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation*. A _____ is any equation that can be written in the *standard form* shown below.

Standard Form of a Linear Equation

$Ax + By = C$ where A , B , and C are real numbers and A and B are not both 0

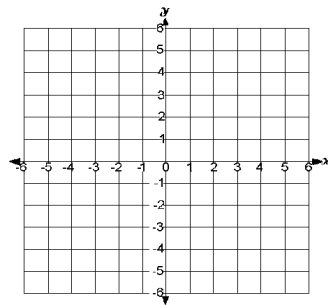
Notice that when a linear equation is written in standard form

- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

Linear	Not Linear
$3x + 2y = 10$ <i>Standard form</i>	$3xy + x = 1$ <i>x and y are multiplied.</i>
$y - 2 = 3x$ <i>Can be written as</i> $3x - y = -2$	$x^3 + y = -1$ <i>x has an exponent other than 1.</i>
$-y = 5x$ <i>Can be written as</i> $5x + y = 0$	$x + \frac{6}{y} = 12$ <i>y is in a denominator.</i>

Tell whether the function is linear. If so, graph the function.

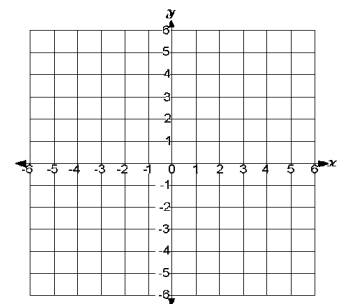
$x = 2y + 4$



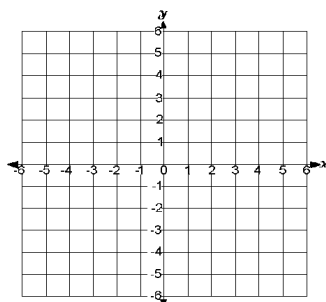
$xy = 4$

y	$x = 2y + 4$	(x, y)
0		
-1		
-2		

$y = 2$



$y = 5x - 9$



$y = 2^x$

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

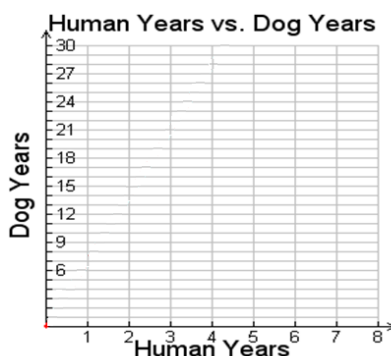
Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

Domain: _____

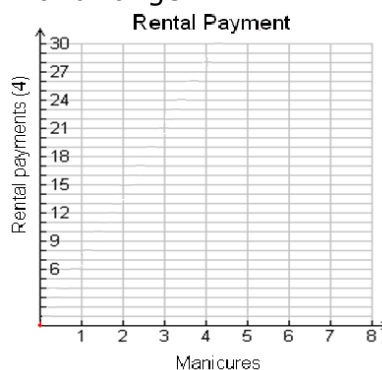
Range: _____

An approximate relationship between human years and dog years is given by the function $y = 7x$, where x is the number of human years. Graph this function and give its domain and range.

x	$f(x) = 7x$
1	
2	
3	



At a salon, Sue can rent a station for \$10.00 per day plus \$3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function and give its domain and range.

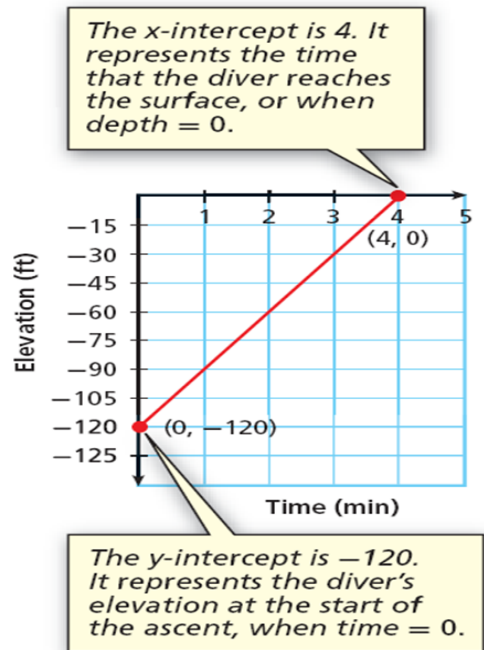


4.2A: Using Intercepts

Objectives: 1. Find x and y intercepts and interpret their meanings in real world situations.
2. Use x and y intercepts to graph lines.

The _____ is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0.

The _____ is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0.

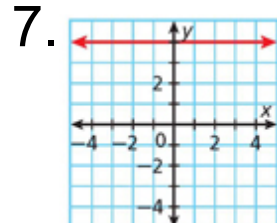
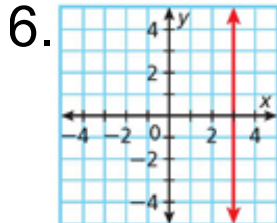
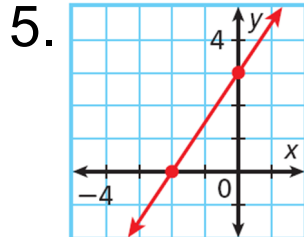
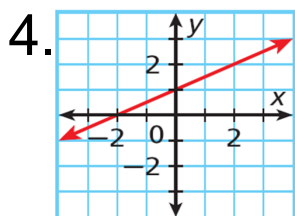


Find the x- and y-intercepts.

1. $-3x + 5y = 30$

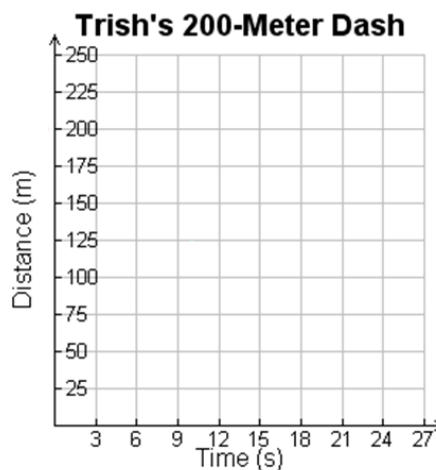
2. $5x - 2y = 10$

3. $4x + 2y = 16$



Trish can run the 200 m dash in 25 s. The function $f(x) = 200 - 8x$ gives the distance remaining to be run after x seconds. Graph this function and find the intercepts. What does each intercept represent?

x					
$f(x) = 200 - 8x$					



The school sells pens for \$2.00 and notebooks for \$3.00. The equation $2x + 3y = 60$ describes the number of pens x and notebooks y that you can buy for \$60. Graph the function and find its intercepts.



4.2B: Using Intercepts

Objectives: 1. Find x and y intercepts and interpret their meanings in real world situations.
2. Use x and y intercepts to graph lines.

Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

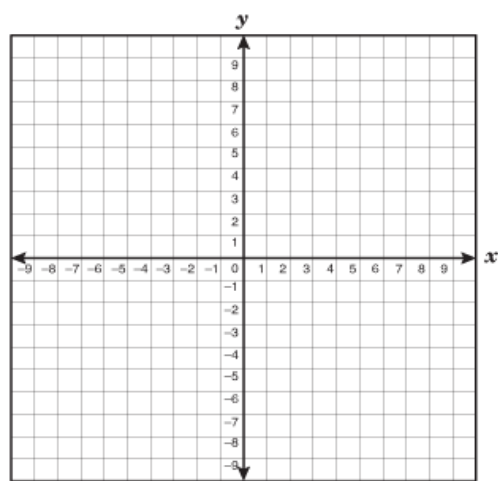
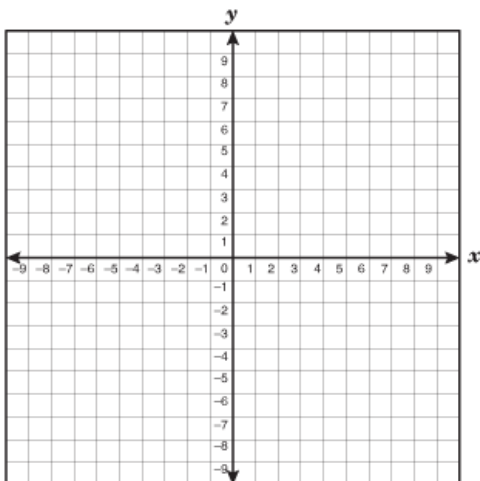
Helpful Hint

You can use a third point to check your line. Either choose a point from your graph and check it in the equation, or use the equation to generate a point and check that it is on your graph.

Use intercepts to graph the line described by the equation.

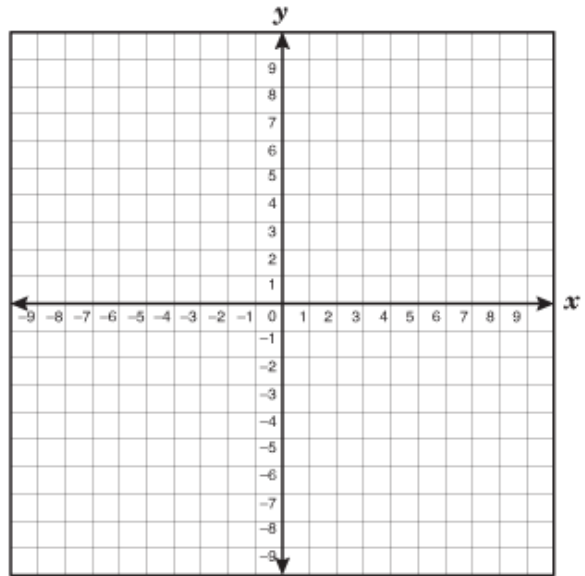
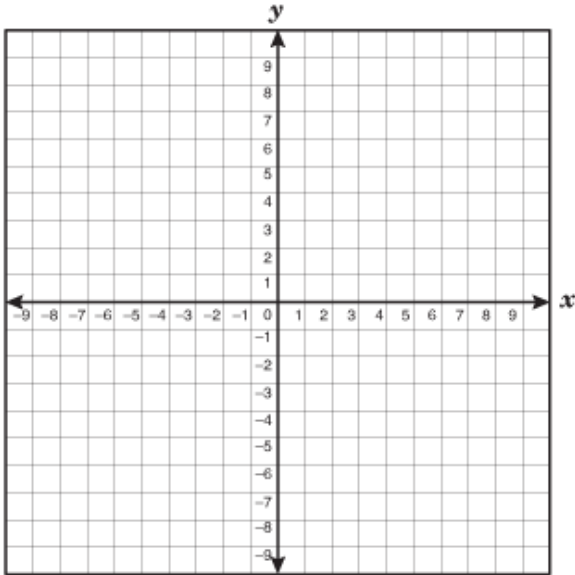
$$3x - 7y = 21$$

$$y = -x + 4$$



$$-3x + 4y = -12$$

$$y = \frac{1}{3}x - 2$$



4.3 Rate of Change and Slope

- Objectives:** 1. Find rates of change and slope.
2. Relate a constant rate of change to the slope of a line.

A _____ is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

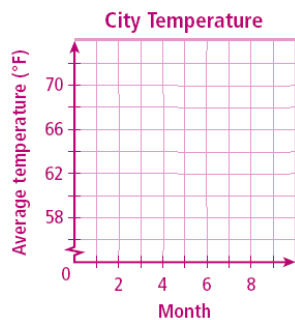
The table shows the average temperature (°F) for five months in a certain city. Find the rate of change for each time period. During which time period did the temperature increase at the fastest rate?

Month	2	3	5	7	8
Temp. (°F)	56	56	63	71	72

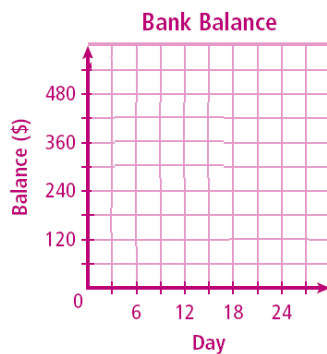
The table shows the balance of a bank account on different days of the month. Find the rate of change during each time interval. During which time interval did the balance decrease at the greatest rate?

Day	1	6	16	22	30
Balance (\$)	550	285	210	210	175

Graph the data from example 1 and show the rates of change.



Graph the data from example 2 and show the rates of change.



Slope of a Line

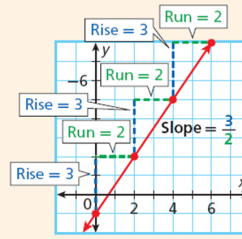
The **rise** is the difference in the **y-values** of two points on a line.

The **run** is the difference in the **x-values** of two points on a line.

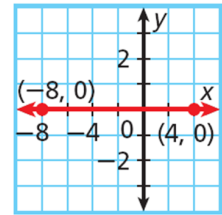
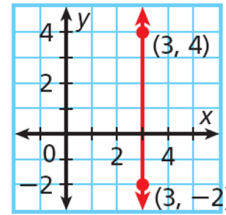
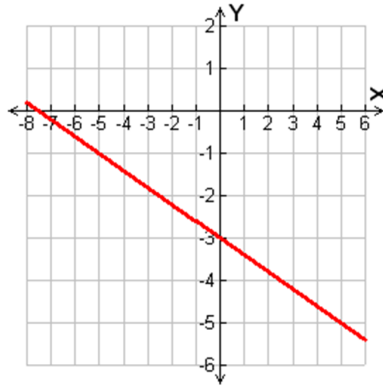
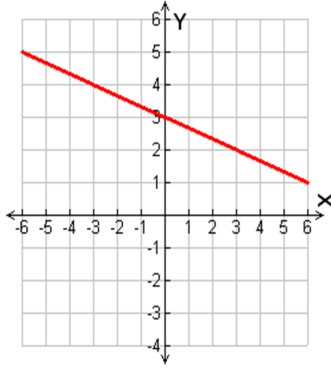
The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that **y** is the **dependent variable** and **x** is the **independent variable**.)

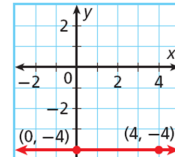
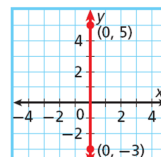
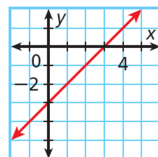
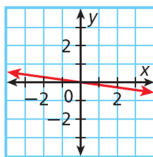


Find the slope of the line.



Positive Slope	Negative Slope	Zero Slope	Undefined Slope
Line rises from left to right.	Line falls from left to right.	Horizontal line	Vertical line

Tell whether the slope of each line is positive, negative, zero or undefined.



Comparing Slopes		
The line with slope 4 is steeper than the line with slope $\frac{1}{2}$. $ 4 > \frac{1}{2} $	The line with slope -2 is steeper than the line with slope -1. $ -2 > -1 $	The line with slope -3 is steeper than the line with slope $\frac{3}{4}$. $ -3 > \frac{3}{4} $

4.4 The Slope Formula

Objectives: 1. Find slope by using the slope formula.

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in y-values to the difference in x-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7$.

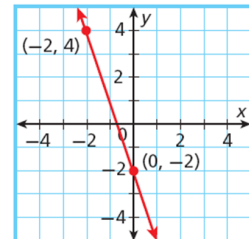
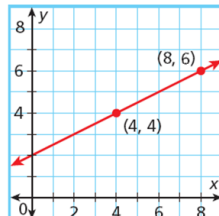
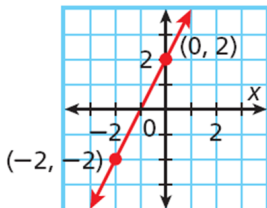
Find the slope of the line that contains $(2, 5)$ & $(8, 1)$.

Find the slope of the line that contains $(-2, -2)$ & $(7, -2)$.

Find the slope of the line that contains $(5, -7)$ & $(6, -4)$.

Find the slope of the line that contains $\left(\frac{3}{4}, \frac{7}{5}\right)$ and $\left(\frac{1}{4}, \frac{2}{5}\right)$.

The graphs show a linear relationship. Find the slope.



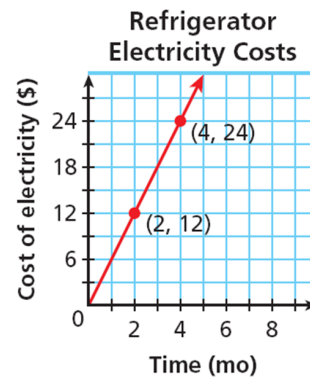
The tables show a linear relationship. Find the slope.

x	-2	-1	0	1
y	5	3	1	-1

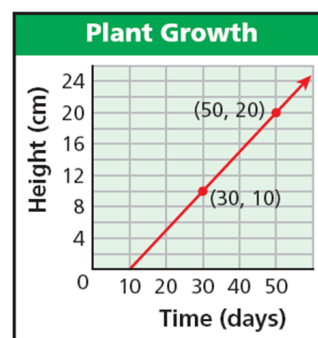
x	0	2	5	6
y	1	5	11	13

x	-2	0	2	4
y	3	0	-3	-6

The graph shows the average electricity costs (in dollars) for operating a refrigerator for several months. Find the slope of the line. Then tell what the slope represents.



The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.



Find the slope of the line described by $4x - 2y = 16$.

Find the slope of the line described by $2x + 3y = 12$.

Find the slope of the line described by $x + 2y = 8$.

4.5 Direct Variation

Objectives: 1. Identify, write, and graph a direct variation.

A _____ is a special type of linear relationship that can be written in the form $y = kx$, where k is a nonzero constant called the _____.

Tell whether the equation represents a direct variation. If so, identify the constant of variation.

$$y = 3x$$

$$3x + y = 8$$

$$-4x + 3y = 0$$

$$3y = 4x + 1$$

$$3x = -4y$$

$$y + 3x = 0$$

Tell whether the relationship is a direct variation. Explain.

x	2	4	6
y	6	12	18

Method 1

Method 2

x	1	3	7
y	-2	0	4

x	y
-3	0
1	3
3	6

x	y
2.5	-10
5	-20
7.5	-30

The value of y varies directly with x , and $y = 3$, when $x = 9$.
Find y when $x = 21$.

Method 1

Method 2

The value of y varies directly with x , and $y = 4.5$ when $x = 0.5$. Find y when $x = 10$.

A group of people are tubing down a river at an average speed of 2 mi/h. Write a direct variation equation that gives the number of miles y that the people will float in x hours. Then graph.

x		(x, y)

