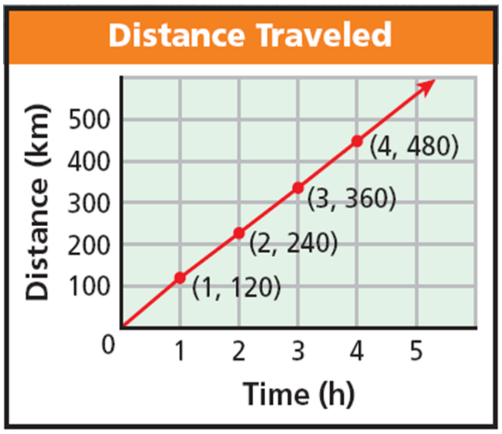
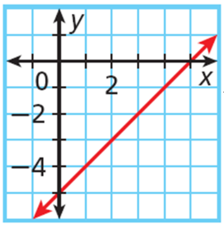
4.1 A: Identifying Linear Functions

The graph represents a function because each domain value (*x*-value) is paired with exactly one range value (*y*-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a Linear function.

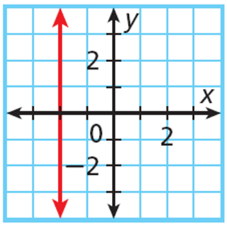


Identify whether the graph represents a function. Explain. If the graph does represent a function, is the function linear?

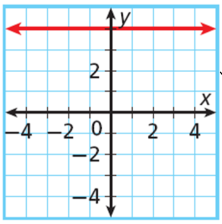


Yes a function because it passes the vertical line test.

Yes, it's linear.

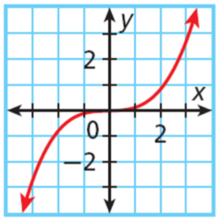


Not a function because it fails the vertical line test.



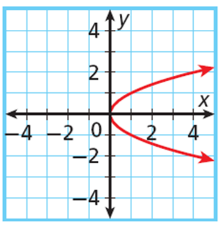
Yes a function it passes the vertical line test.

Yes it's linear.

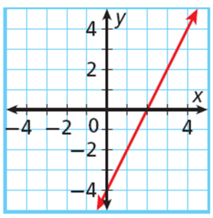


Yes a function it passes the vertial line test.

Not a linear function.



Not a function it fails the vertical line test



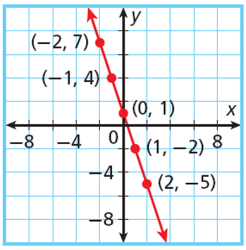
Yes a function passes the Vert. line test.

It is linear.

You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in *x* corresponds to a constant change in *y*.

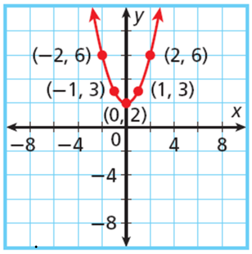
	x	*y*	
	−2	7	
+1	−1	4	−3
+1	0	1	−3
+1	1	−2	−3
+1	2	−5	−3

Yes it's a linear function, x and y change at a constant rate.



	x	*y*	
	−2	6	
+1	−1	3	−3
+1	0	2	−1
+1	1	3	+1
+1	2	6	+3

Yes it's a function, but <u>not</u> linear because y isn't constant change.



Tell whether the set of ordered pairs satisfies a linear function. Explain.

1. {(0, −3), (4, 0), (8, 3), (12, 6), (16, 9)}

Yes because x and y change at a constant rate.

	x	**y**	
	0	**−3**	
+4	**4**	**0**	+3
+4	**8**	**3**	+3
+4	**12**	**6**	+3
+4	**16**	**9**	+3

2. {(−4, 13), (−2, 1), (0, −3), (2, 1), (4, 13)}

Not a linear function because the y isn't constant change.

x	y
−4	13
−2	1
0	−3
2	1
4	13

3. **{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)}**

Yes a linear function because x and y have constant change.

	x	y	
	3	5	
	5	4	−1
+2	7	3	
	9	2	
	11	1	

4.1 B: Identifying Linear Functions

Objectives: 1. Identify linear functions and linear equations.
2. Graph linear functions that represent real-world situations and give their domain and range.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation.* A linear equation is any equation that can be written in the *standard form* shown below.

Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0

Notice that when a linear equation is written in standard form

- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

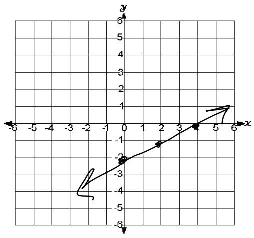
Linear		Not Linear	
$3x + 2y = 10$	*Standard form*	$3xy + x = 1$	*x and y are multiplied.*
$y - 2 = 3x$	*Can be written as* $3x - y = -2$	$x^3 + y = -1$	*x has an exponent other than 1.*
$-y = 5x$	*Can be written as* $5x + y = 0$	$x + \frac{6}{y} = 12$	*y is in a denominator.*

Tell whether the function is linear. If so, graph the function.

$x = 2y + 4$

$-2y \quad -2y$

$x - 2y = 4$



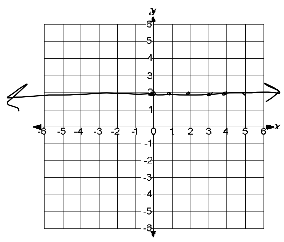
y	$x = 2y + 4$	(x, y)
0	$2(0)+4 = 4$	$(4, 0)$
−1	$2(-1)+4 = -2+4$	$(2, -1)$
−2	$2(-2)+4 = -4+4$	$(0, -2)$

$xy = 4$

Not Linear because it's multiplied.

$y = 2$

Yes

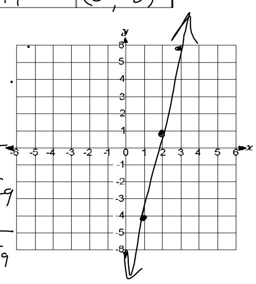


$y = 5x - 9$

Yes

$1(5)$
$5 - 9$

x	y	
1	−4	
2	1	$5(2) = 10 - 9$
3	6	$5(3) = 15 - 9$



$y = 2^x$ Not linear because of the exponent.

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

Domain: x-value – input

Range: y-value – output

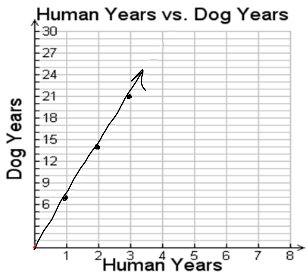
An approximate relationship between human years and dog years is given by the function *y* = 7*x*, where *x* is the number of human years. Graph this function and give its domain and range.

x	f(x) = 7x	
1	7(1)	(1,7)
2	7(2)	(2,14)
3	7(3)	(3,21)

D ↓ R

D: $x \geq 0$

R: $y \geq 0$

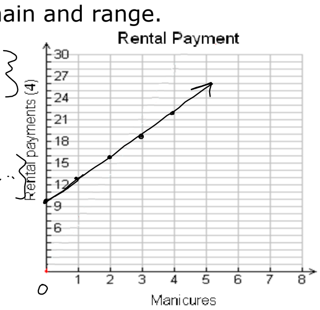


At a salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by *f*(*x*) = 3*x* + 10, where *x* is the number of manicures. Graph this function and give its domain and range.

x	f(x)=3x+10	y
1	3(1) = 3+10	13
2	3(2) = 6+10	16
3	3(3) = 9+10	19
4	3(4) = 12+10	22

D: {0, 1, 2, 3, 4, ...}

R: {10, 13, 16, 19, 22, ...}

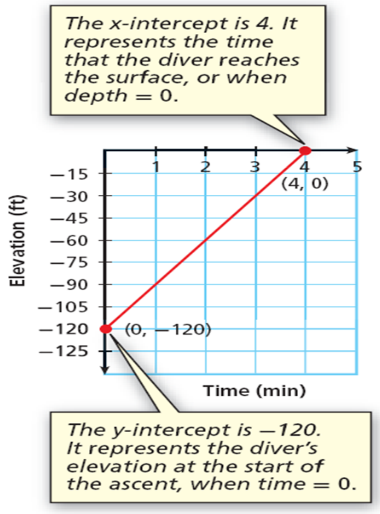


4.2A: Using Intercepts

Objectives: 1. Find x and y intercepts and interpret their meanings in real world situations.
2. Use x and y intercepts to graph lines.

The y-intercept is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0. $(0, \square)$

The x-intercept is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0. $(\square, 0)$



Find the x- and y-intercepts.

1. $-3x + 5y = 30$

x-int $(-10, 0)$

y-int $(0, 6)$

$-3x + 5(0) = 30$

$\frac{-3x}{-3} = \frac{30}{-3}$

$x = -10$

$-3(0) + 5y = 30$

$\frac{5y}{5} = \frac{30}{5}$

$y = 6$

2. $5x - 2y = 10$

x-int $(2, 0)$

y-int $(0, -5)$

$5x - 2(0) = 10$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

$5(0) - 2y = 10$

$\frac{-2y}{-2} = \frac{10}{-2}$

$y = -5$

3. $4x + 2y = 16$

x-int $(4, 0)$

y-int $(0, 8)$

$4x + 2(0) = 16$

$\frac{4x}{4} = \frac{16}{4}$

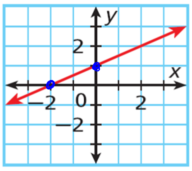
$x = 4$

$4(0) + 2y = 16$

$\frac{2y}{2} = \frac{16}{2}$

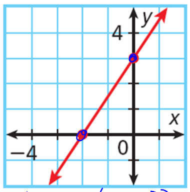
$y = 8$

4.



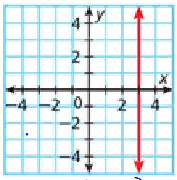
x-int: (-2, 0)
y-int: (0, 1)

5.



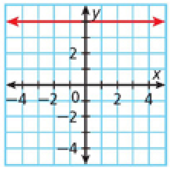
x-int (-2, 0)
y-int (0, 3)

6.



x-int: (3, 0)
y-int: none

7.



x-int: none
y-int: (0, 4)

Trish can run the 200 m dash in 25 s. The function $f(x) = 200 - 8x$ gives the distance remaining to be run after x seconds. Graph this function and find the intercepts. What does each intercept represent?

x	0	3	6	9	12
f*(*x*) = 200 − 8*x	200	176	152	128	104

200 − 8(0)
200 − 0

200 − 8(3)
200 − 24

200 − 8(6)
200 − 48

200 − 8(9)
200 − 72

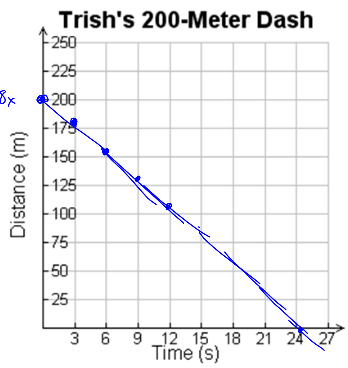
200 − 8(12)
200 − 96

$0 = 200 - 8x$ +8x

$\frac{8x}{8} = \frac{200}{8}$

x-int: (25, 0)
Time @ the end of the race.

y-int: (0, 200)
Starting point of the race. 200m to go.



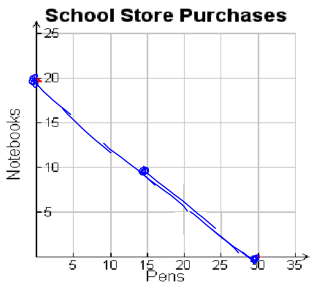
The school sells pens for $2.00 and notebooks for $3.00. The equation $2x + 3y = 60$ describes the number of pens x and notebooks y that you can buy for $60. Graph the function and find its intercepts.

x-int: (30, 0)
y-int: (0, 20)

$2x + 3y = 60$
$2x + 3(0) = 60$
$\frac{2x}{2} = \frac{60}{2}$
$x = 30$

$2x + 3y = 60$
$2(0) + 3y = 60$
$\frac{3y}{3} = \frac{60}{3}$
$y = 20$

$2(15) + 3y = 60$
$30 + 3y = 60$
$-30 \qquad -30$
$\frac{3y}{3} = \frac{30}{3}$
$y = 10$



4.2B: Using Intercepts

Objectives: 1. Find x and y intercepts and interpret their meanings in real world situations.
2. Use x and y intercepts to graph lines.

Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

Helpful Hint

You can use a third point to check your line. Either choose a point from your graph and check it in the equation, or use the equation to generate a point and check that it is on your graph.

Use intercepts to graph the line described by the equation.

$3x - 7y = 21$

(7,0) x-int

(0,-3) y-int

$3x - 7(0) = 21$

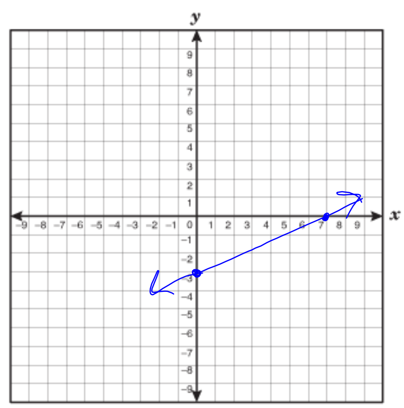
$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

$3(0) - 7y = 21$

$\frac{-7y}{-7} = \frac{21}{-7}$

$y = -3$



$y = -x + 4$

(4,0) x-int

(0,4) y-int

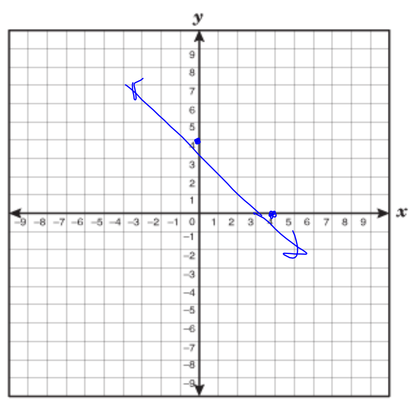
$y = 0 + 4$

$y = 4$

$0 = -x + 4$

$+x \quad +x$

$x = 4$



$$-3x + 4y = -12$$

x-int $(4, 0)$

y-int $(0, -3)$

$-3x + 4(0) = -12$

$\frac{-3x}{-3} = \frac{-12}{-3}$

$x = 4$

$-3(0) + 4y = -12$

$\frac{4y}{4} = \frac{-12}{4}$

$y = 3$

$$y = \frac{1}{3}x - 2$$

x-int $(6, 0)$

y-int $(0, -2)$

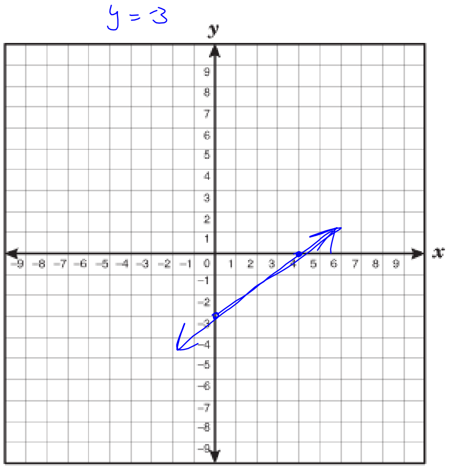
$y = \frac{1}{3}(0) - 2$

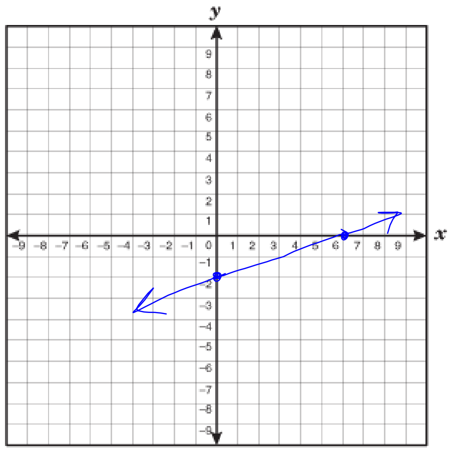
$y = -2$

$0 = \frac{1}{3}x - 2$
$+2 \quad +2$

$(3)2 = \frac{1}{3}x(3)$

$6 = x$





4.3 Rate of Change and Slope

Objectives: 1. Find rates of change and slope.
2. Relate a constant rate of change to the slope of a line.

A rate of change is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

The table shows the average temperature (°F) for five months in a certain city. Find the rate of change for each time period. During which time period did the temperature increase at the fastest rate?

Month	2	3	5	7	8
Temp. (°F)	56	56	63	71	72

$\frac{56-56}{3-2} = \frac{0}{1} = 0$; $\frac{63-56}{5-3} = \frac{7}{2}$; $\frac{71-63}{7-5} = \frac{8}{2} = 4$; $\frac{72-71}{8-7} = \frac{1}{1} = 1$

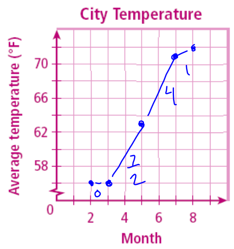
The table shows the balance of a bank account on different days of the month. Find the rate of change during each time interval. During which time interval did the balance decrease at the greatest rate?

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

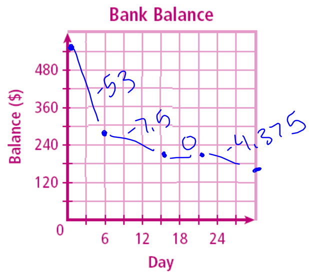
$\frac{285-550}{6-1} = \frac{-265}{5} = -53$; $\frac{210-285}{16-6} = \frac{-75}{10} = -7.5$; $\frac{210-210}{22-16} = \frac{0}{6} = 0$;

$\frac{175-210}{30-22} = \frac{-35}{8} = -4.375$

Graph the data from example 1 and show the rates of change.



Graph the data from example 2 and show the rates of change.



Slope of a Line

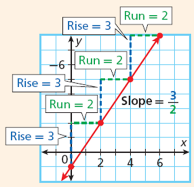
The **rise** is the difference in the *y-values* of two points on a line.

The **run** is the difference in the *x-values* of two points on a line.

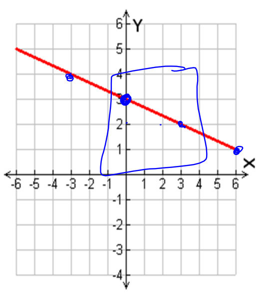
The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

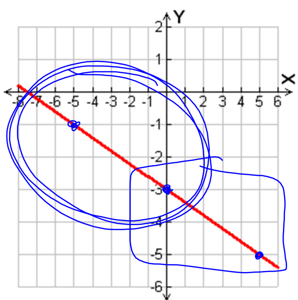
(Remember that *y* is the **dependent variable** and *x* is the **independent variable**.)



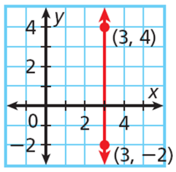
Find the slope of the line.



$\frac{\Delta y}{\Delta x} = \frac{-1}{3} = \text{slope}$

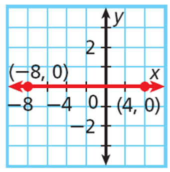


$\frac{\Delta y}{\Delta x} = \frac{-2}{5}$ $\frac{2}{-5}$

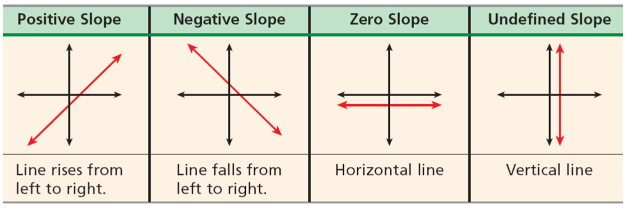


$\frac{y\ -6}{x\ 0}$

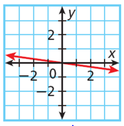
Undefined.



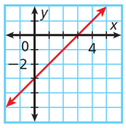
$\frac{y\ 0}{x\ 12} = 0$



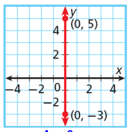
Tell whether the slope of each line is positive, negative, zero or undefined.



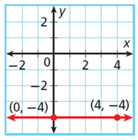
negative



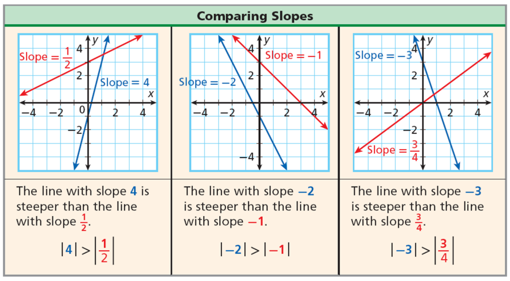
positive



undefined



zero



4.4 The Slope Formula

Objectives: 1. Find slope by using the slope formula.

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in *y*-values to the difference in *x*-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4-(-3)}{1-2} = \frac{7}{-1} = -7$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$x_1 \; y_1 \quad x_2 \; y_2$

Find the slope of the line that contains (2, 5) & (8, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{8-2} = \frac{-4 \div 2}{6 \div 2} = -\frac{2}{3}$$

Find the slope of the line that contains (−2, −2) & (7, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 + {}^{+}2}{7 + {}^{+}2} = \frac{0}{9} = 0$$

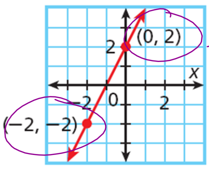
Find the slope of the line that contains (5, −7) & (6, −4).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 + {}^{+}4}{5-6} = \frac{-3}{-1} = 3$$

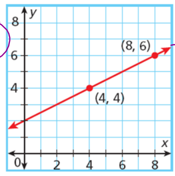
Find the slope of the line that contains $\left(\frac{3}{4}, \frac{7}{5}\right)$ **and** $\left(\frac{1}{4}, \frac{2}{5}\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{2}{5} - \frac{7}{5}}{\frac{1}{4} - \frac{3}{4}} = \frac{\frac{-5}{5}}{\frac{-2}{4}} = \frac{-1}{-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 1 \cdot \frac{2}{1} = 2$$

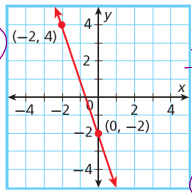
The graphs show a linear relationship. Find the slope.



$\frac{2+2}{0+2} = \frac{4}{2} = 2$



$\frac{6-4}{8-4} = \frac{2}{4} = \frac{1}{2}$



$\frac{4+2}{-2-0}$

$\frac{6}{-2}$

3

The tables show a linear relationship. Find the slope.

x	−2	−1	0	1
y	5	3	1	−1

$\frac{5-3}{-2+1} = \frac{2}{-1} = -2$

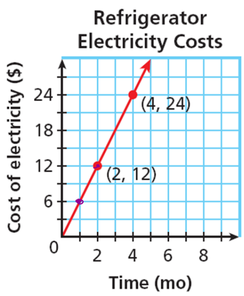
x	0	2	5	6
y	1	5	11	13

$\frac{5-11}{2-5} = \frac{-6}{-3} = 2$

x	−2	0	2	4
y	3	0	−3	−6

$\frac{3-0}{-2-0} = \frac{3}{-2}$

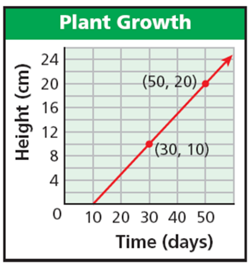
The graph shows the average electricity costs (in dollars) for operating a refrigerator for several months. Find the slope of the line. Then tell what the slope represents.



$\frac{y_2-y_1}{x_2-x_1} = m = \frac{24-12}{4-2} = \frac{12}{2} = \$6/\text{month}$

$\frac{\text{change in cost (\$)}}{\text{chang in time months}}$

The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.



$m = \frac{y_2-y_1}{x_2-x_1} = \frac{20-10}{50-30} = \frac{10}{20} = \frac{1}{2}$

$\frac{\text{change in height}}{\text{change in time}}$

Find the slope of the line described by 4*x* – 2*y* = 16.

x-int (4,0)   y-int (0,-8)

$4x - \cancel{2(0)} = 16$ $\cancel{4(0)} - 2y = 16$

$\frac{4x}{4} = \frac{16}{4}$ $x = 4$ $\frac{-2y}{-2} = \frac{16}{-2}$ $y = -8$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-0}{0-4} = \frac{-8}{-4}$

$m = 2$

Find the slope of the line described by 2*x* + 3*y* = 12.

x-int (6,0)   y-int (0,4)

$2x + \cancel{3(0)} = 12$ $\cancel{2(0)} + 3y = 12$

$\frac{2x}{2} = \frac{12}{2}$ $\frac{3y}{3} = \frac{12}{3}$

$x = 6$ $y = 4$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-6} = \frac{4}{-6}$

$m = \frac{4 \div 2}{-6 \div 2} = \frac{2}{-3}$

Find the slope of the line described by *x* + 2*y* = 8.

x-int (8,0)   y-int (0,4)

$x + \cancel{2(0)} = 8$ $0 + 2y = 8$

$x = 8$ $\frac{2y}{2} = \frac{8}{2}$

$y = 4$

$m = \frac{4-0}{0-8} = \frac{4}{-8} = \frac{1}{-2}$

4.5 Direct Variation

Objectives: 1. Identify, write, and graph a direct variation.

A direct variation is a special type of linear relationship that can be written in the form *y* = *kx*, where *k* is a nonzero constant called the Constant of variation.

Tell whether the equation represents a direct variation. If so, identify the constant of variation.

y* = 3*x

yes

$k = 3$

3*x* + *y* = 8

$-3x \quad -3x$

$y = -3x + 8$

No

–4*x* + 3*y* = 0

$+4x \quad +4x$

$\frac{3y}{3} = \frac{4x}{3}$

$y = \frac{4}{3}x$

yes $k = \frac{4}{3}$

3*y* = 4*x* + 1

$\frac{3y}{3} = \frac{4x}{3} + \frac{1}{3}$

$y = \frac{4}{3}x + \frac{1}{3}$

No

3*x* = –4*y*

$\frac{3x}{-4} = \frac{-4y}{-4}$

$-\frac{3}{4}x = y$

yes $k = \frac{-3}{4}$

***y* + 3*x* = 0**

$-3x \quad -3x$

$y = -3x$

yes $k = -3$

Tell whether the relationship is a direct variation. Explain.

x	2	4	6
y	6	12	18

Method 1

$y = 3x$

Yes

Method 2

$\frac{y}{x} = \frac{6}{2} = 3$ $\frac{18}{6} = 3$

$\frac{12}{4} = 3$

Yes $\frac{y}{x} = 3 = K$

x	1	3	7
y	−2	0	4

M1

$y = x - 3$

No

M2

$\frac{-2}{1} = -2$ $\frac{0}{3} = 0$

$\frac{4}{7} =$

x	y
−3	0
1	3
3	6

M2

$\frac{0}{-3} = 0$ $\frac{3}{1} = 3$

$\frac{6}{3} = 2$

No, not equal. There is no K.

x	y
2.5	−10
5	−20
7.5	−30

M2

$\frac{-10}{2.5} = -4$ $\frac{-30}{7.5} = -4$

$\frac{-20}{5} = -4$ Yes, $K = -4$.

The value of y varies directly with x, and $y = 3$, when $x = 9$. Find y when $x = 21$.

Method 1

$y = Kx$

$\frac{3}{9} = \frac{K(9)}{9}$

$\frac{1}{3} = K$

$y = Kx$

$y = \frac{1}{3}(21)$

$y = 7$

Method 2

y $\frac{3}{9} = \frac{y}{21}$

x

$\frac{9y}{9} = \frac{63}{9}$

$y = 7$

The value of y varies directly with x, and $y = 4.5$ when $x = 0.5$. Find y when $x = 10$.

1

$y = Kx$

$\frac{4.5}{.5} = \frac{K(.5)}{.5}$

$9 = K$

$y = 9(10)$

$y = 90$

2

$\frac{4.5}{.5} = \frac{y}{10}$

$(2)\frac{1}{2}y = 45(2)$

$y = 90$

A group of people are tubing down a river at an average speed of 2 mi/h. Write a direct variation equation that gives the number of miles *y* that the people will float in *x* hours. Then graph.

$y = 2x$

x	$y = 2x$	(x, y)
1	$2(1) = 2$	$(1, 2)$
2	$2(2) = 4$	$(2, 4)$
3	$2(3) = 6$	$(3, 6)$