Chapter 4 - Linear Functions

linear - straight line

function - one domain for each range

Vertical line test - tests for functions

| | X | y | |
|---|---|---|---|
| +3 | 3 | 4 | +4 |
| | 6 | 8 | +2 |
| +3 | 9 | 10 | +2 |
| | 12 | 12 | +4 |
| | 15 | 16 | |

not linear
(not consistent)

## How to tell if an ~~equation~~ is linear.

Can't have the following:

- no exponents
- no x and y multiplied together
- no variables in denominators or radicals

Must be able to be written in standard form $Ax + By = C$

Intercepts:

x-intercept: where the line crosses @ a point on the x axis. In an equation plug 0 in for y, to find value. (x, 0)

y-int: where the line crosses @ a point on the y-axis. In an equation plug 0 in for x, to find the value. (0, y)

$3x - 2y = 12$ Find the intercepts

**x-int.**

$3x - 2(0) = 12$

$3x = 12$

$\boxed{x = 4}$

$(4, 0)$

**y-int.**

$3(0) - 2y = 12$

$-2y = 12$

$\boxed{y = -6}$

$(0, -6)$

# Rate of Change and Slope

ratio of change of the dependent variable (y) and the change of the independent variable (x).

$\frac{\text{rise}}{\text{run}}$, slope, $\frac{\text{change in } y}{\text{change in } x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$


$\frac{2}{0} \neq 2$ $\quad \frac{2}{0} \neq 0$

| | | |
|---|---|---|
| $m > 0$ | Positive | ↗ |
| $m < 0$ | Negative | ↘ |
| $m = 0$ | Horizontal | ↔ |
| $\left(\frac{2}{0}\right)$ m is undefined | Vertical | ↕ |

Find the Slope:

$\frac{y_2 - y_1}{x_2 - x_1} = m$

(3, 7) and (-2, 4)

$\frac{4-7}{-2-3} = \frac{-3}{-5} = \frac{3}{5}$ positive slope

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(-2, 4) and (-2, 2)

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(4, 7) and (-7, 7)

**Direct Variation**: linear relationship written as

$y = kx$

$k$ = Constant of Variation

| x | 2 | 3 | 4 |
|---|---|---|---|
| y | 6 | 9 | 12 |

~~$y = 2x + 1$~~

$y = 3x$

$k$ (constant of variation)

| x | 8 | 12 | 16 |
|---|---|---|---|
| y | 2 | 3 | 4 |

$$y = \frac{x}{4}$$

$$y = \frac{1}{4}x$$

↑ k (constant of variation)

Find value of y that varies directly with x.

$y = 3$ and $x = 9$ Find $y$ when $x = 18$

$\frac{y}{x} = \frac{3}{9} = \frac{y}{18}$

$\frac{3}{9} = \frac{y}{18}$

$3(18) = 9y$

$54 = 9y$

$y = 6$

$y = \frac{1}{3}(18)$

$y = 6$

$y = kx$

$\frac{3}{9} = \frac{k \cdot 9}{9}$

$\frac{3}{9} = \frac{1}{3} = k$

## Slope intercept form

$y = mx + b$

m: slope

b: y-intercept

## Point Slope Form

$y - y_1 = m(x - x_1)$

m: slope

$(x_1, y_1)$

## Standard Form

$Ax + By = C$

$m = 3 \quad b = -4$

Write in slope intercept form

$y = 3x - 4$

$3x - 2y = 12$ write in slope-intercept form

$-3x \quad -3x$

$\frac{-2y}{-2} = \frac{12 - 3x}{-2}$

$y = -6 + \frac{3}{2}x$

what is slope? $\frac{3}{2}$

what is the y-int? $-6$

what is the x-int?

Point slope Form

Slope is -3 (4, -7)

$y - y_1 = m(x - x_1)$

slope

$(x_1, y_1)$

## Parallel and perpendicular

Parallel - same slope never intersect. ex: $\frac{2}{3}$ & $\frac{2}{3}$

$y = \frac{2}{3}x + 5$

$y = \frac{2}{3}x - 10$

Perpendicular - opposite reciprocal slopes

ex: $\frac{-4}{1}$ $\frac{1}{4}$

$y = -4x + 7$

$y = \frac{1}{4}x - 2$

Write an equation of the line parallel to $y = 4x + 7$ and passes through $(3, 4)$.

$y = 4x + 7$, $m = 4$

parallel $\rightarrow m = 4$

$y = mx + b$

$4 = 4(3) + b$

$4 = 12 + b$

$-12 \quad -12$

$b = -8$

$y = 4x - 8$

Write an equation of the line perpendicular to $y = \frac{3}{4}x + 2$ and passes through $(3, 8)$.

$m = \frac{3}{4}$

perpendicular $\rightarrow \frac{-4}{3} = m$

$y = mx + b$

$8 = -\frac{4}{3}(3) + b$

$8 = -4 + b$

$12 = b$

$y = -\frac{4}{3}x + 12$

Which are parallel?

$y = 6$ $y = 6x + 2$ $y = -6x + 1$ $x = 6$

None!

Which are perpendicular?

$y = 3$ horizontal

$y = -3x + 1$ $m = -3$

$x = 4$ vertical

$y = -\frac{1}{3}x - 2$ $m = -\frac{1}{3}$

## Transformations

**translation**: y-int changes. $f(x)=2x+6$ down 9 $g(x)=2x-3$

**rotation**: slope # changes. $f(x)=2x+6$ rotate $g(x)=\frac{1}{4}x+6$

Closer to 0 : flatter

farther from 0 : steeper

**reflection**: slope sign changes. $f(x)=-7x+3$ $g(x)=7x+3$

$f(x) = 3x + 2$ transformed to $g(x) = -3x - 4$

reflection over y-axis

&

translation down 6

$g(x) = x - 7$ was translated down 4 from $f(x)$. What was the original function $f(x)$?

$$f(x) = x - 3$$

$f(x) = -x + 3$ transformed to $g(x) = x + 3$

reflection over y-axis

$f(x) = \frac{1}{2}x + 4$ transformed to $g(x) = \frac{5}{3}x - 7$

rotation around (0,4) (steeper)

+

translation down 11

$\therefore$