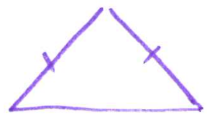
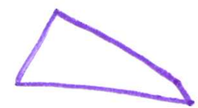
Chapter 4 Notes – Parallel and Perpendicular Lines Name Key
Basic Geometry

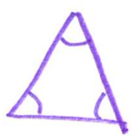
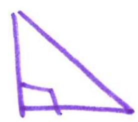
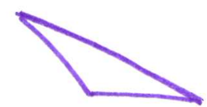
4.1 Classifying Triangles

Goal: Classify triangles by their sides and by their measures.

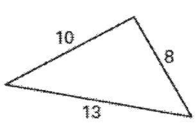
Triangle: a figure formed by three Segments joining three noncolinear points

Vertex: a point that joins two Sides of the triangle

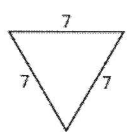
Classification of Triangles by Sides		
Equilateral	**Isosceles**	**Scalene**
		
3 congruent sides	2 congruent sides	No congruent sides

Classification of Triangles by Angles			
Equiangular	**Acute**	**Right**	**Obtuse**
			
3 congruent angles	3 acute angles	1 right angle	1 obtuse angle

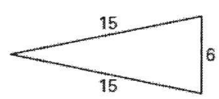
Classify each triangle by its sides.



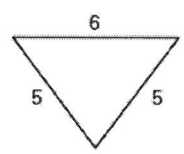
scalene



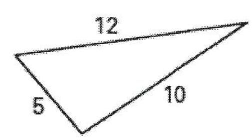
equilateral



isosceles

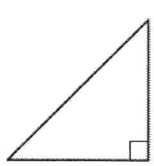


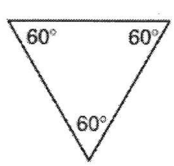
isosceles



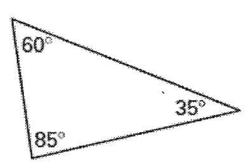
scalene

Classify the triangle by its angles.

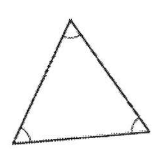

right

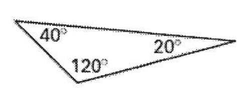


equiangular



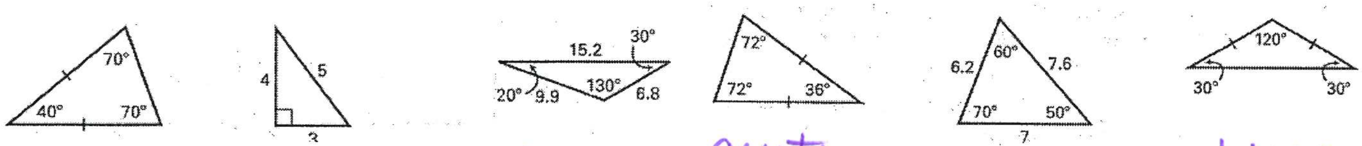
acute


equiangular



obtuse

Classify each triangle by its angles AND sides.



acute isosceles

right Scalene

obtuse Scalene

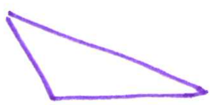
acute isosceles

acute scalene

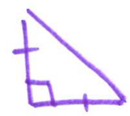
obtuse isosceles

Determine if a triangle can be both classifications. If possible, draw the type of triangle.

Obtuse and scalene? Yes



Right and isosceles? Yes



Equilateral and acute?

Not possible

If equilateral then equiangular

Right and acute?

Not possible

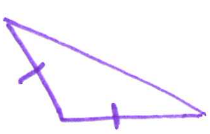
Scalene and isosceles?

Not possible

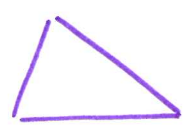
Obtuse and equilateral?

Not possible

Obtuse and isosceles? Yes

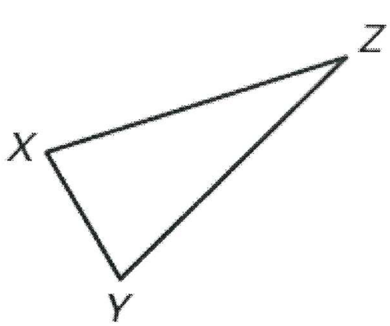


Scalene and acute?

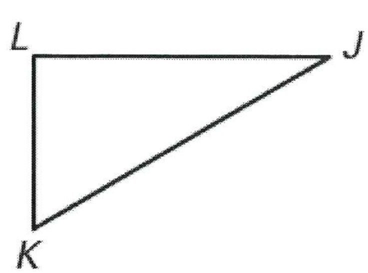


Identify which side is opposite each angle.

$\angle X \rightarrow \overline{YZ}$ $\angle Y \rightarrow \overline{XZ}$ $\angle Z \rightarrow \overline{XY}$



$\angle L \rightarrow \overline{JK}$ $\angle K \rightarrow \overline{LJ}$ $\angle J \rightarrow \overline{LK}$

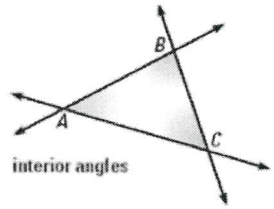


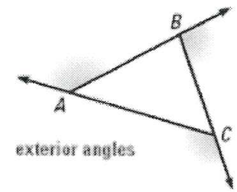
4.2 Angle Measures of Triangles

Goal: Find interior and exterior angle measures in triangles.

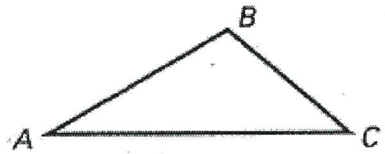
Interior Angles: when the sides of a triangle are extended, the three original angles are the interior angles.

Exterior Angles: when the sides of a triangle are extended, the angles that are adjacent to the interior angles are the exterior angles.





Triangle Sum Theorem: The sum of the measures of the angles of a triangle is 180°

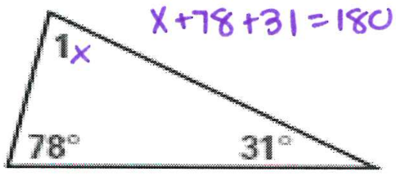


$m\angle A$ + $m\angle B$ + $m\angle C$ = $180°$

Find the measure of $\angle 1$.

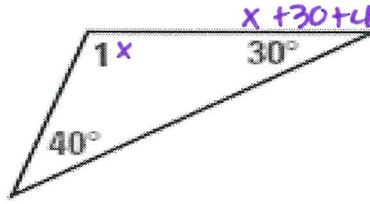
a) 71°

$x+78+31=180$



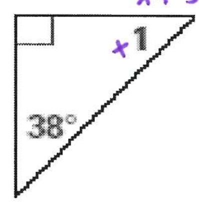
b) 110°

$x+30+40=180$



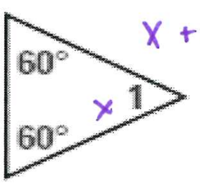
c) 52°

$x+38+90=180$



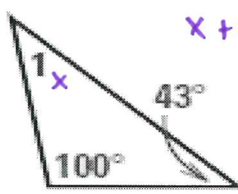
d) 60°

$x+60+60=180$

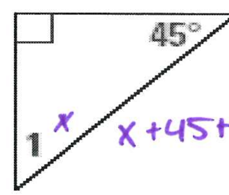


e) 37°

$x+43+100=180$



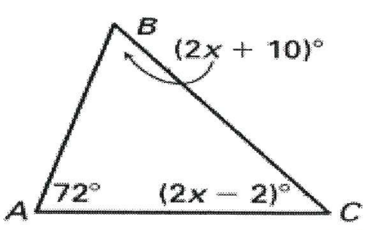
f) 45°



$x+45+90=180$

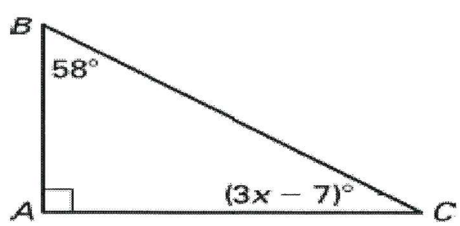
Find the value of x. Then find the measure of each angle.

a) x = 25



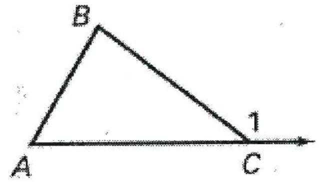
$72+2x+10+2x-2=180$
$4x+80=180$
$4x=100$
$x=25$

b) x = 13



$90+58+3x-7=180$
$3x=39$
$x=13$

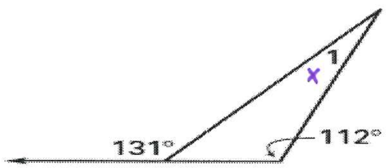
Exterior Angle Theorem: the measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles



$m\angle 1 = m\angle A + m\angle B$

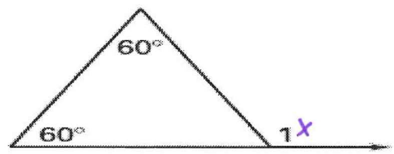
Find the measure of each missing angle.

a) 19°



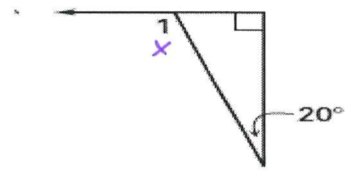
$131 = x + 112$

b) 120°



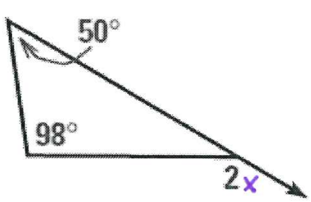
$x = 60 + 60$

c) 110°



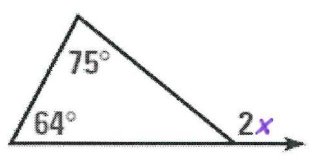
$x = 90 + 20$

d) 148°



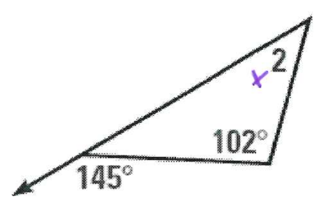
$x = 50 + 98$

e) 139°



$x = 75 + 64$

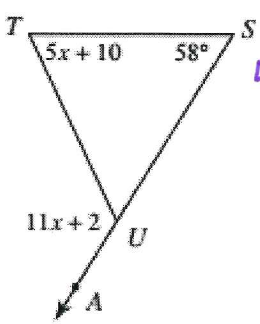
f) 43°



$145 = x + 102$

Find the value of x.

a) x = 11

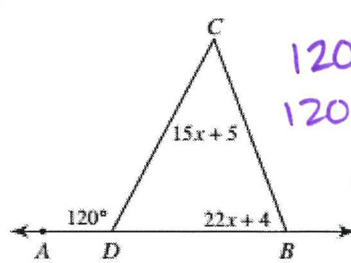


$5x + 10 + 58 = 11x + 2$

$5x + 68 = 11x + 2$

$\frac{66}{6} = \frac{6x}{6}$

b) x = 3



$120 = 15x + 5 + 22x + 4$

$120 = 37x + 9$

$\frac{111}{37} = \frac{37x}{37}$

4.3 Isosceles and Equilateral Triangles

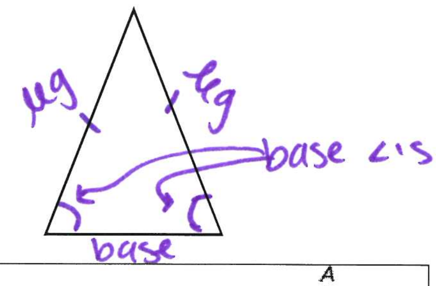
Goal: Use properties of isosceles and equilateral triangles to find side lengths and angle measures.

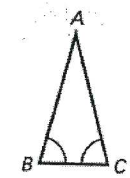
Isosceles Triangle Vocabulary

Legs: the congruent sides

Base: the side that is not a leg

Base angles: the two angles at the base

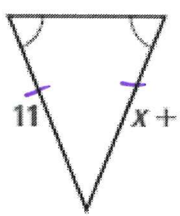


Base Angles Theorem: If two sides of a triangle are congruent, then the angles opposite of them are congruent	If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$	
Converse of the Base Angles Theorem: If two angles are congruent, then the sides opposite of them are congruent	If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$	

Find the value of x. If x is an angle, also find the measure of each angle.

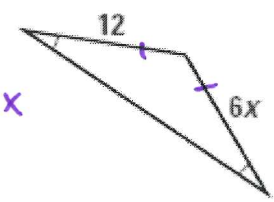
a) x = 7

$x+4=11$



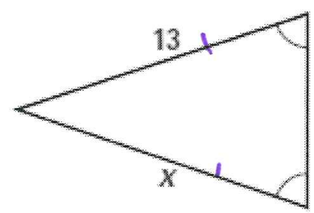
b) x = 2

$12=6x$



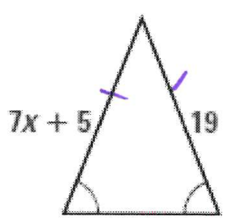
c) x = 13

$x=13$

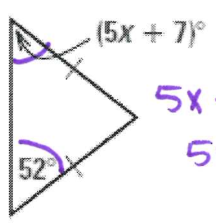


d) x = 2

$7x+5=19$
$\frac{7x}{7}=\frac{14}{7}$

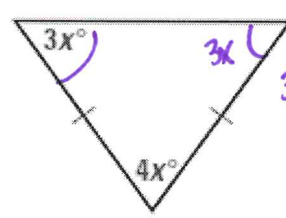


e) x = 9



$5x+7=52$
$5x=45$

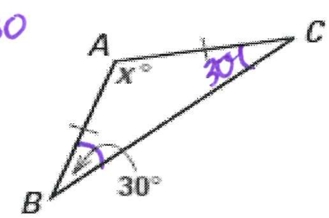
f) x = 18



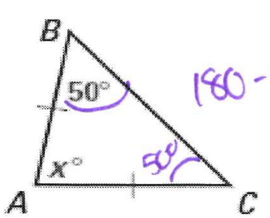
$3x+3x+4x=180$
$10x=180$

g) x = 120°

$180-30-30$

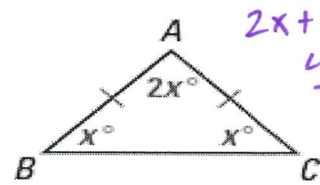


h) x = 80

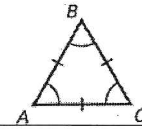
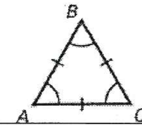


$180-50-50$

i) x = 45



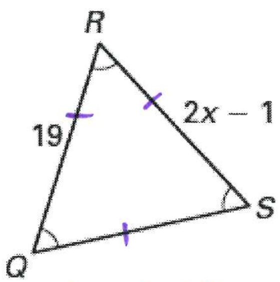
$2x+x+x=180$
$\frac{4x}{4}=\frac{180}{4}$

Equilateral Theorem: If a triangle is equilateral, then it is equiangular	If $\overline{AB} \cong \overline{AC} \cong \overline{BC}$, then $\angle A \cong \angle B \cong \angle C$	
Equiangular Theorem: If a triangle is equiangular, then it is equilateral	If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC} \cong \overline{BC}$	

**Each angle in an equiangular triangle will always be 60°

Find the value of each variable.

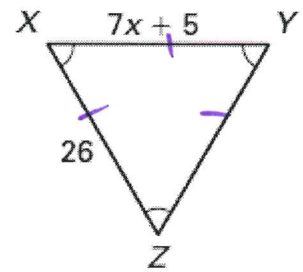
a) x = 10



$2x-1=19$

$\frac{2x}{2}=\frac{20}{2}$

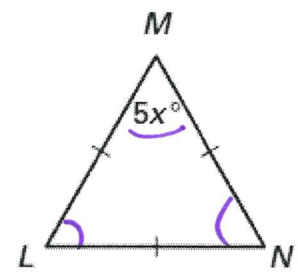
b) x = 3



$7x+5=26$

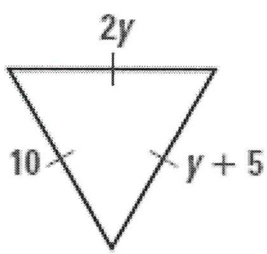
$\frac{7x}{7}=\frac{21}{7}$

c) x = 12



$\frac{5x}{5}=\frac{60}{5}$

d) y = 5

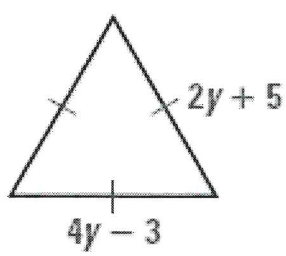


$2y=10$

OR

$y+5=10$

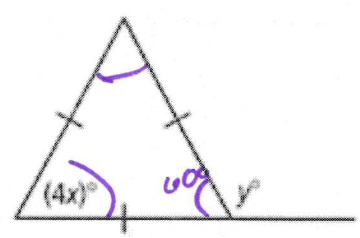
e) y = 4



$4y-3=2y+5$

$\frac{2y}{2}=\frac{8}{2}$

f) x = 15 y = 120



$\frac{4x}{4}=\frac{60}{4}$

$180-60=y$

4.4 Part A: The Distance Formula

Goal: Find the distance between two coordinates.

The Distance Formula	
For any two coordinates $A(x_1, y_1)$ and $B(x_2, y_2)$, the distance between A and B is written AB and is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	**Step 1**: plug in the coordinates **Step 2**: Simplify the parentheses **Step 3**: Square each parenthesis. Remember! Squaring a negative number makes it positive. **Step 4**: Add together the two numbers **Step 5**: Simplify the square root

Why will your answer never be a negative number?
you cannot have a negative distance + you cannot take the square root of a negative number.

Find the distance between each pair of coordinates.

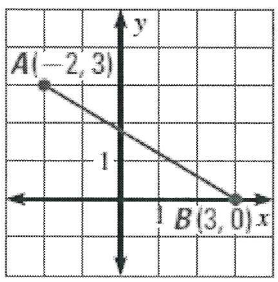
a) AB = 5.8

$\sqrt{(-2-3)^2 + (3-0)^2}$

$\sqrt{(-5)^2 + (3)^2}$

$\sqrt{25+9}$

$\sqrt{34}$



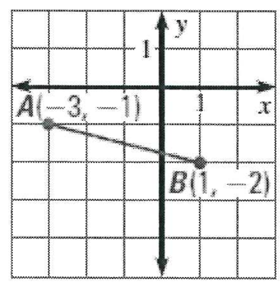
b) AB = 4.1

$\sqrt{(-3-1)^2 + (-1-{}^-2)^2}$

$\sqrt{(-4)^2 + (1)^2}$

$\sqrt{16+1}$

$\sqrt{17}$



c) CD = 9.1 $C(-3, -4)$ *and* $D(-2, 5)$

$\sqrt{(-3--2)^2 + (-4-5)^2}$

$\sqrt{(-1)^2 + (-9)^2}$

$\sqrt{1+81}$

$\sqrt{82}$

d) EF = 5.7 $E(0, -4)$ *and* $F(4, 0)$

$\sqrt{(0-4)^2 + (-4-0)^2}$

$\sqrt{(-4)^2 + (-4)^2}$

$\sqrt{16+16}$

$\sqrt{32}$

e) GH = 6.7 $G(-8, 10)$ *and* $H(-2, 7)$

$\sqrt{(-8--2)^2 + (10-7)^2}$

$\sqrt{(-6)^2 + (3)^2}$

$\sqrt{36+9}$

$\sqrt{45}$

f) IJ = 21.6 $I(11, 13)$ *and* $J(-1, -5)$

$\sqrt{(11-{}^-1)^2 + (13-{}^-5)^2}$

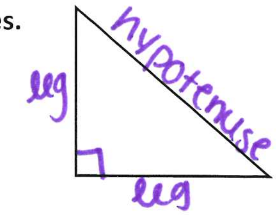
$\sqrt{(12)^2 + (18)^2}$

$\sqrt{144+324}$

$\sqrt{468}$

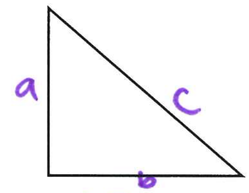
4.4 Part B: The Pythagorean Theorem

Goal: Use the Pythagorean Theorem to find missing sides of right triangles.



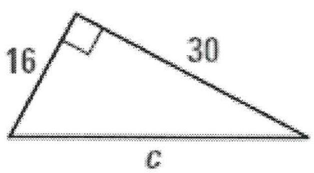
Legs of a right triangle: the sides that form the right angle

Hypotenuse: the side opposite of the right angle

The Pythagorean Theorem	
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the lengths of the legs	 a^2 + b^2 = c^2

Use the Pythagorean theorem to find each missing side length. If necessary, round to the nearest tenth.

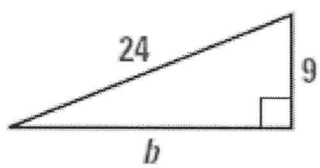
a) c = 34



$16^2 + 30^2 = c^2$

$\sqrt{1156} = \sqrt{c^2}$

b) b = 22.2

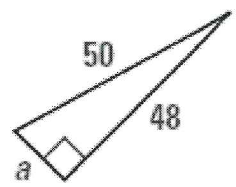


$9^2 + b^2 = 24^2$

$81 + b^2 = 576$

$\sqrt{b^2} = \sqrt{495}$

c) a = 14

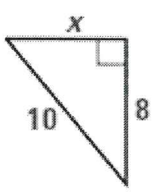


$a^2 + 48^2 = 50^2$

$a^2 + 2304 = 2500$

$\sqrt{a^2} = \sqrt{196}$

d) x = 6

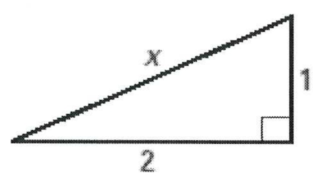


$x^2 + 8^2 = 10^2$

$x^2 + 64 = 100$

$\sqrt{x^2} = \sqrt{36}$

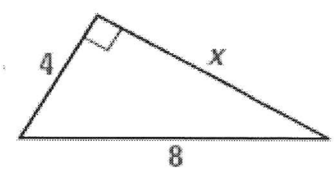
e) x = 2.2



$1^2 + 2^2 = x^2$

$\sqrt{5} = \sqrt{x^2}$

f) x = 6.9



$x^2 + 4^2 = 8^2$

$x^2 + 16 = 64$

$\sqrt{x^2} = \sqrt{48}$

Pythagorean triple: a set of three positive integers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$.

↳ Note: "c" is always the largest number

Tell whether the given side lengths form a Pythagorean triple.

a) 8, 10, 6

$6^2 + 8^2 \ ? \ 10^2$

$100 = 100$

Yes

b) 7, 8, 9

$7^2 + 8^2 \ ? \ 9^2$

$113 \neq 81$

No

c) 14, 50, 48

$14^2 + 48^2 \ ? \ 50^2$

$2500 = 2500$

Yes

Draw and label a picture of the situation, then use the Pythagorean Theorem to solve.

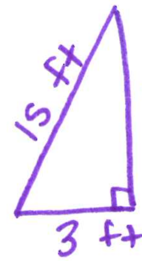
a) To hang lights on her house, Ms. Blaseg placed a 15 foot ladder 3 feet from the base of her house. How high up the house will the ladder reach?

$3^2 + x^2 = 15^2$

$9 + x^2 = 225$

$\sqrt{x^2} = \sqrt{216}$

$\boxed{x = 14.7 \text{ ft}}$

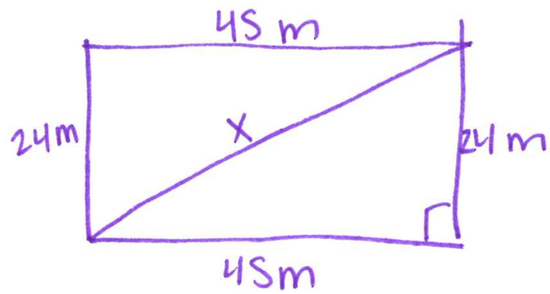


b) Steve is turning half of his backyard into a chicken pen. His backyard is a 24 meter by 45 meter rectangle. He wants to put a chicken wire fence that stretches diagonally from one corner to the opposite corner. How many meters of fencing will Steve need?

$24^2 + 45^2 = x^2$

$\sqrt{2601} = \sqrt{x^2}$

$\boxed{x = 51 \text{ m}}$

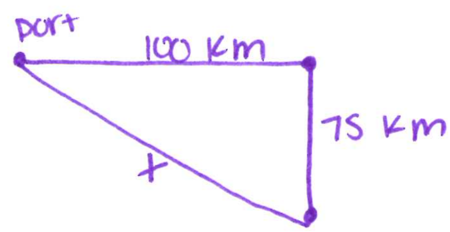


c) A ship leaves port and travels 100 km east then turns south and travels 75 km. How far from the port is the ship?

$100^2 + 75^2 = x^2$

$\sqrt{15625} = \sqrt{x^2}$

$\boxed{x = 125 \text{ km}}$

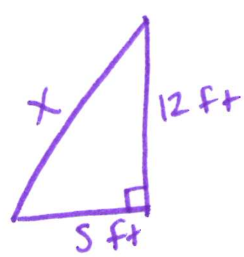


d) A wire is stretched from the top of a 12 foot pole to a stake 5 feet from the base of the pole. How long is the wire?

$5^2 + 12^2 = x^2$

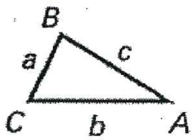
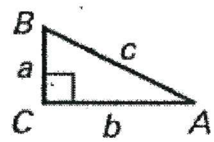
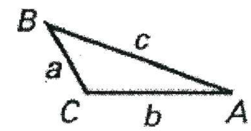
$\sqrt{169} = \sqrt{x^2}$

$\boxed{x = 13 \text{ ft}}$



4.5 The Converse of the Pythagorean Theorem

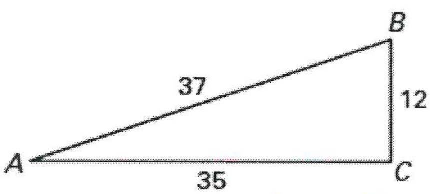
Goal: Use side lengths to determine whether triangles are acute, obtuse, or right.

Classifying Triangles Using the Converse of the Pythagorean Theorem		
In ΔABC with longest side C….		
If $c^2 < a^2 + b^2$	If $c^2 = a^2 + b^2$	If $c^2 > a^2 + b^2$
		
then the triangle is **acute**.	then the triangle is **right**.	then the triangle is **obtuse.**

Classify the triangle with the given side lengths as acute, right, or obtuse.

* put c^2 first when doing in class *

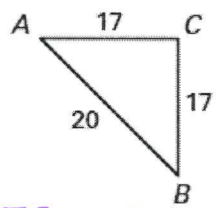
a) right



$12^2 + 35^2 \ ? \ 37^2$

$1369 = 1369$

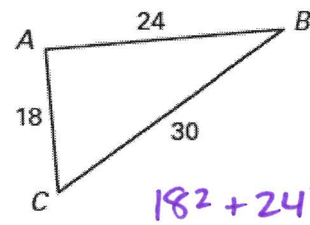
b) acute



$17^2 + 17^2 \ ? \ 20^2$

$578 > 400$

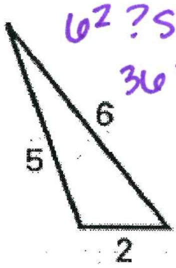
c) right



$18^2 + 24^2 \ ? \ 30^2$

$900 = 900$

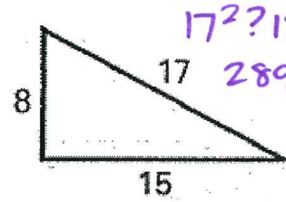
d) obtuse



$6^2 \ ? \ 5^2 + 2^2$

$36 > 29$

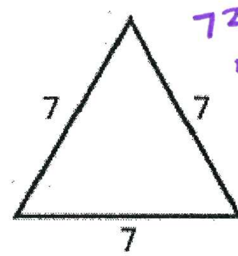
e) right



$17^2 \ ? \ 15^2 + 8^2$

$289 = 289$

f) acute



$7^2 \ ? \ 7^2 + 7^2$

$49 < 98$

g) right

$1, 8, \sqrt{65}$

$\sqrt{65}^2 \ ? \ 1^2 + 8^2$

$65 = 65$

h) acute

$\sqrt{33}, 9, 7$

$9^2 \ ? \ 7^2 + \sqrt{33}^2$

$81 < 82$

i) obtuse

$4, 9, \sqrt{104}$

$\sqrt{104} \ ? \ 9^2 + 4^2$

$104 > 97$

4.6 Medians of Triangles

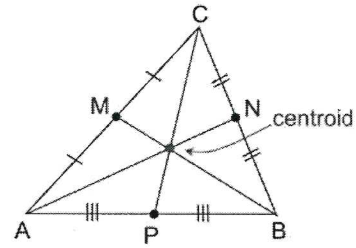
Goal: Draw medians and centroids. Use properties of medians to find missing lengths.

Median: a segment from the vertex to the

midpoint______ of the opposite side.

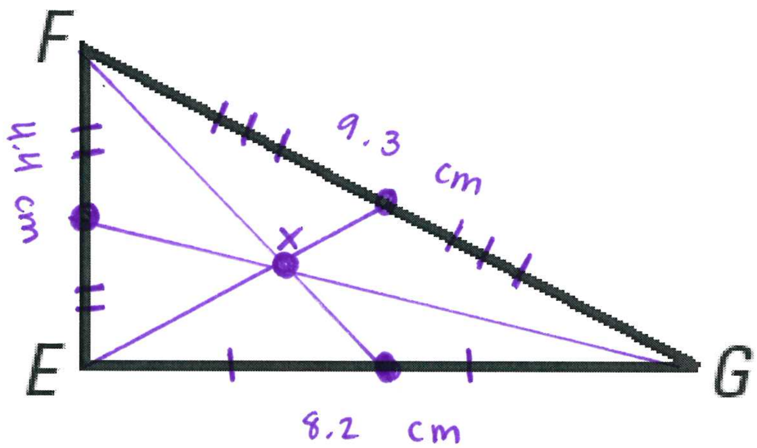
Centroid: the point at with the medians of the triangle

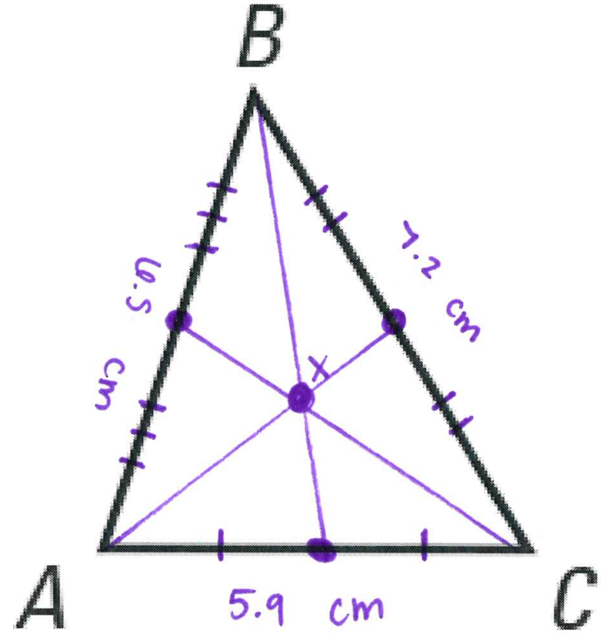
intersect______

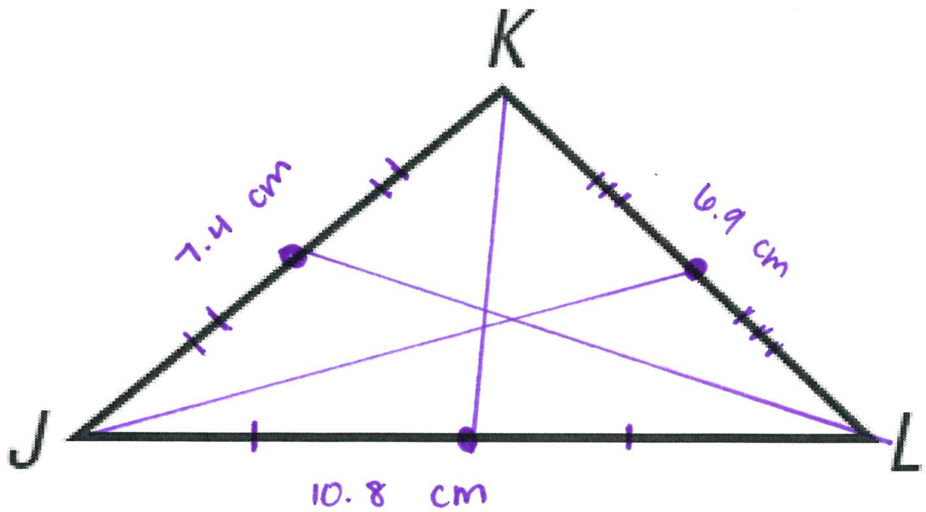


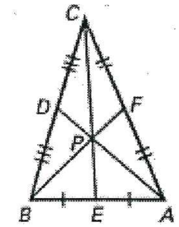
Use a ruler to draw all three medians in each triangle. Label the centroid of each triangle X.

↳ always use centimeters







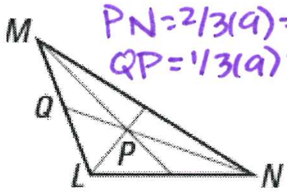
Intersections of Medians of a Triangle		
The medians of a triangle intersect at a point that is two- thirds of the distance from each vertex to the midpoint of the opposite side.	If P is the centroid of ΔABC, then $AP = \frac{2}{3}$AD and $BP = \frac{2}{3}$BF and $CP = \frac{2}{3}$CE	

*QN is whole segment

P is the centroid of ΔLMN. Find QP and PN.

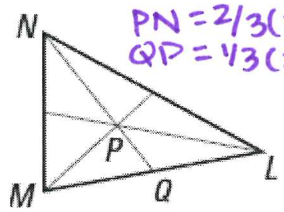
a) QP = 3 PN = 6 b) QP = 7 PN = 14 c) QP = 10 PN = 20

$QN = 9$



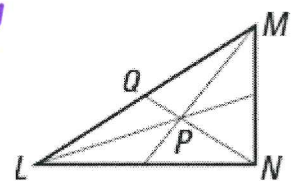
PN=2/3(9)=6
QP=1/3(9)=3

$QN = 21$



PN=2/3(21)=14
QP=1/3(21)=7

$QN = 30$



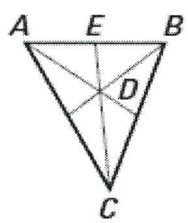
PN=2/3(30)=20
QP=1/3(30)=10

*DE is the 1/3 segment

D is the centroid of ΔABC. Find CD and CE.

a) CD = 10 CE = 15 b) CD = 22 CE = 33 c) CD = 18 CE = 27

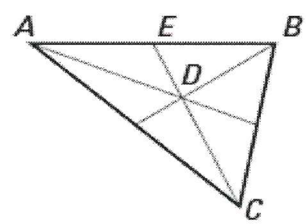
$DE = 5$



CD=2(DE)
CD = 2(5)=10

CE=10+5=15

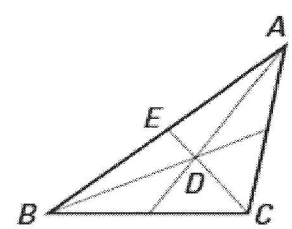
$DE = 11$



CD=2(DE)
CD=2(11)=22

CE=11+22=33

$DE = 9$



CD=2(DE)
CD=2(9)=18

CE=18+9=27

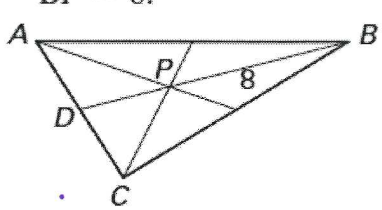
*All are the 2/3 segments

P is the centroid of each triangle. Use the information given to find the lengths.

a) BD = 12

BD=8+4

Find BD, given $BP = 8$.

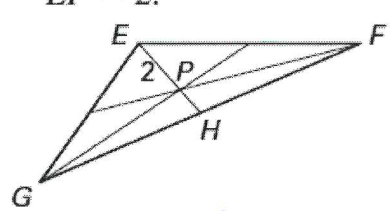


PD=1/2(BP)
PD=1/2(8)
PD=4

b) EH = 3

EH=2+1

Find EH, given $EP = 2$.

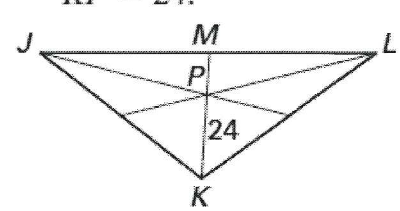


PH=1/2(EP)
PH=1/2(2)
PH=1

c) KM = 36

KM=12+24

Find KM, given $KP = 24$.



PM=1/2(PK)
PM=1/2(24)
PM=12

4.7 Investigation

Determine if a triangle can be made with the following colors as sides. List the side length of each color and whether or not it makes a triangle.

Orange: 5 cm Purple: 7 cm Green: 9 cm

Yellow: 10 cm Blue: 12 cm Red: 14 cm

Colors of Sides	Length of side 1	Length of side 2	Length of side 3	Did it make a triangle?
1. Purple, purple, orange	7	7	5	Yes
2. Orange, green, yellow	5	9	10	Yes
3. Green, orange, red	9	5	14	No
4. Orange, blue, red	5	12	14	Yes
5. Orange, purple, orange	5	7	5	Yes
6. Blue, orange, purple	12	5	7	No
7. Orange, orange, blue	5	5	12	No
8. Purple, orange, yellow	7	5	10	Yes
9. Orange, purple, red	5	7	14	No
10. Green, yellow, blue	9	10	12	Yes

Go back and observe the side lengths, find a rule for when the side lengths make a triangle.

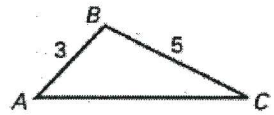
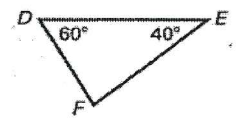
The sum of the shortest two sides must be greater than the longest side

Find a rule for when the side lengths do not make a triangle.

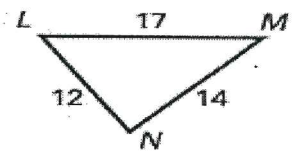
The side lengths do not make a triangle when the sum of the shorter two sides is less than the length of the longest side.

4.7 Triangle Inequalities

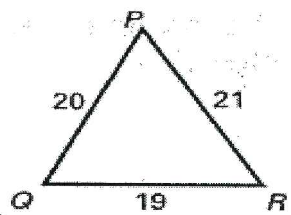
Goal: Use triangle measures to decide which side is the longest and which angle is the largest.

If one side of a triangle is longer than another side, then the angle opposite the longer side is <u>larger</u> than the angle opposite the shorter side.	**If $BC > AB$, then** $m\angle A$ <u>></u> $m\angle C$ 
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is <u>Shorter</u> than the side opposite the smaller angle.	**If $m\angle D > m\angle E$, then** EF <u>></u> DF 

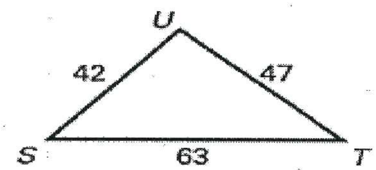
Name the angles from largest to smallest.



∠N, ∠L, ∠M

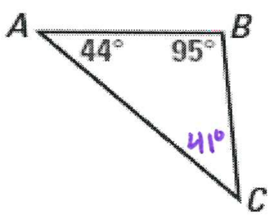


∠Q, ∠R, ∠P

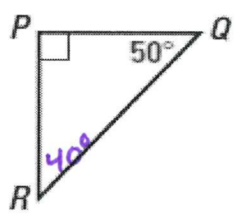


∠U, ∠S, ∠T

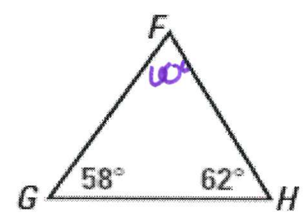
Name the sides from longest to shortest.



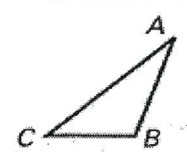
$\overline{AC}$, $\overline{BC}$, $\overline{AB}$



$\overline{RQ}$, $\overline{PR}$, $\overline{PQ}$



$\overline{FG}$, $\overline{GH}$, $\overline{FH}$

Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is <u>greater</u> than the length of the third side.	$CA + AB$ <u>></u> BC $AB + BC$ <u>></u> CA $BC + CA$ <u>></u> AB 

Determine whether the side lengths can form a triangle.

a) 3, 5, 9 NO
3+5 ? 9
8 < 9

b) 7, 5, 12 NO
7+5 ? 12
12 = 12

c) 6, 9, 4 Yes
6+4 ? 9
10 > 9

d) 18, 9, 11 Yes
9+11 ? 18
20 > 18

e) 20, 36, 13 NO
20+13 ? 36
33 < 36

f) 8.5, 7.2, 3.4 Yes
3.4+7.2 ? 8.5
10.6 > 8.5