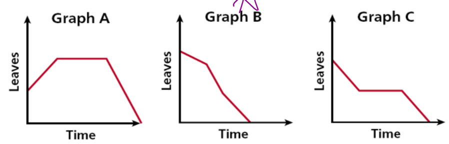
3.1 Graphing Relationships

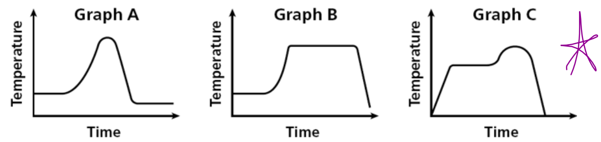
Objectives: 1. Match simple graphs with situations.
2. Graph relationships.

Relating Graphs to Situations

Each day several leaves fall from a tree. One day a gust of wind blows off many leaves. Eventually, there are no more leaves on the tree. Choose the graph that best represents the situation.

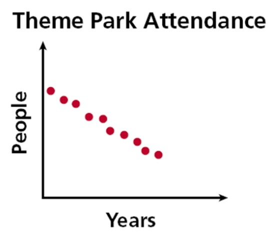


The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly before dropping sharply. Choose the graph that best represents this situation.



Some graphs are connected lines or curves called Continuous graphs. Some graphs are only distinct points. They are called discrete graphs.

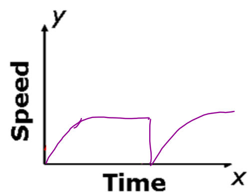
The graph on theme park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between whole numbers are not included, since they have no meaning for the situation.



Sketch a graph for the situation. Tell whether the graph is continuous or discrete.

A truck driver enters a street, drives at a constant speed, stops at a light, and then continues.

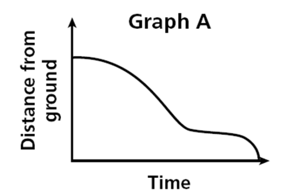
Continuous

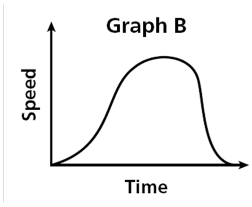


A small bookstore sold between 5 and 8 books each day for 7 days.

Discrete

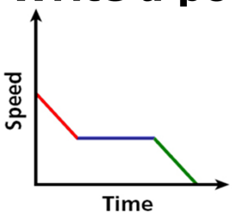
Both graphs show a relationship about a child going down a slide. Graph A represents the child's *distance from the ground* related to time. Graph B represents the child's *Speed* related to time.



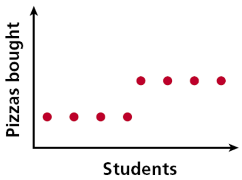


Continuous

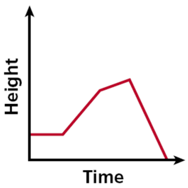
Write a possible situation for the given graph.



Car is slowing down to a constant speed. It continues at a constant speed then slows to a complete stop.



Pizza students bought for the occasion about 4 students bought 2 pizzas and 4 students bought 4 pizzas.



Growth of a plant. The sprout is the same height until it grows quickly and slows growth toward the end of life. It shrinks to nothing when it dies.

3.2 Relations and Functions

Objectives: 1. Identify functions.
2. Find the domain and range of relations and functions.

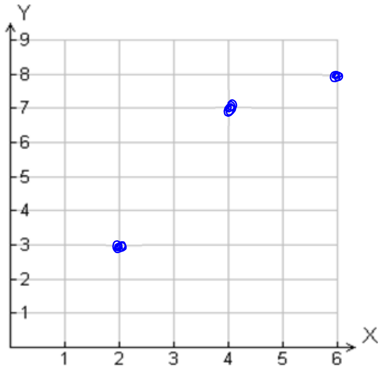
In Lesson 3-1 you saw relationships represented by graphs. Relationships can also be represented by a set of ordered pairs called a relation.

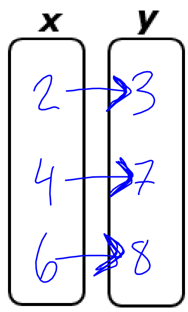
In the scoring systems of some track meets, for first place you get 5 points, for second place you get 3 points, for third place you get 2 points, and for fourth place you get 1 point. This scoring system is a relation, so it can be shown by ordered pairs. {(1, 5), (2, 3), (3, 2) (4, 1)}. You can also show relations in other ways, such as tables, graphs, or *mapping diagrams.*

x y

1. Express the relation {(2, 3), (4, 7), (6, 8)} as a table, as a graph, and as a mapping diagram.

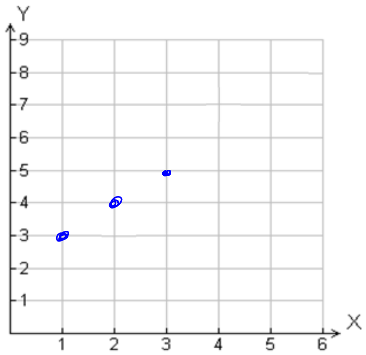
x	*y*
2	3
4	7
6	8

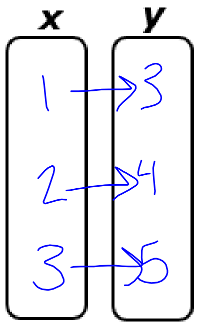




2. Express the relation {(1, 3), (2, 4), (3, 5)} as a table, as a graph, and as a mapping diagram.

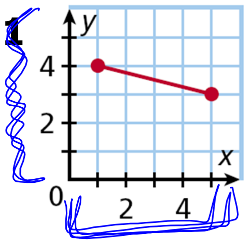
x	*y*
1	3
2	4
3	5





The Domain of a relation is the set of first coordinates (or *x*-values) of the ordered pairs.
The range of a relation is the set of second coordinates (or *y*-values) of the ordered pairs.

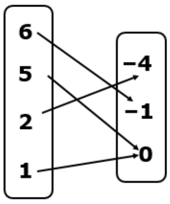
Give the domain and range of the relation.



Domain: $1 \le x \le 5$

Range: $3 \le y \le 4$

2.



Domain: {1, 2, 5, 6}

Range: {-4, -1, 0}

3.

x	*y*
1	**1**
4	**4**
8	**1**

Domain: {1, 4, 8}

Range: {1, 4}

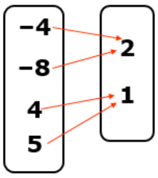
A function ________ is a special type of relation that pairs each domain value with exactly one range value.

Give the domain and range of the relation. Tell whether the relation is a function. Explain.

1. {(3, −2), (5, −1), (4, 0), (3, 1)} Not a function because the domain 3 is repeated.

D: {3, 4, 5} R: {-2, -1, 0, 1}

2.

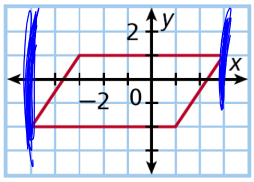


Yes a function because each domain has 1 range.

D: {-8, -4, 4, 5}

R: {1, 2}

3.



D: $-5 \le x \le 3$

Not a function

R: $-2 \le y \le 1$

Domain has more than one range value.

4. {(8, 2), (−4, 1), (−6, 2),(1, 9)}

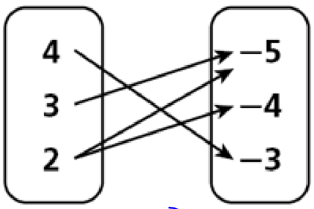
D: {8, -4, -6, 1}

R: {2, 1, 9}

Yes a function

Domain is with only 1 range.

5.

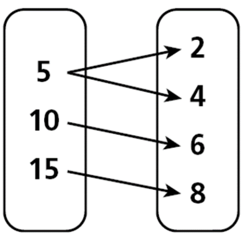


D: {4, 3, 2}

R: {-5, -4, -3}

Not a function Domain 2 repeats itself.

5.



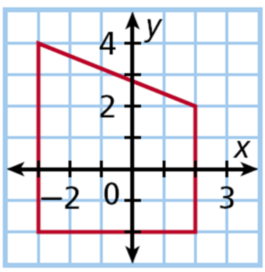
D: {5, 10, 15}

R: {2, 4, 6, 8}

Not a function

Domain 5 repeats.

6.



D: $-3 \le x \le 2$

R: $-2 \le y \le 4$

Not a function

Domain repeats.

3.3 Writing Functions

Obectives: 1. Identify independent and dependent variables.
2. Write an equation in function notation and evaluate a function for given input values.

Determine a relationship between the *x*- and *y*-values. Write an equation.

1.

x	5	10	15	20
y	1	2	3	4

+5 +5 +5

+1 +1 +1

$\frac{5(y)}{5} = \frac{x}{5}$

$y = \frac{x}{5}$

2.

x	1	2	3	4
y	3	6	9	12

x3 x3 x3 x3

$y = 3x$

The equation in Example 1 describes a function because for each *x*-value (input), there is only one *y*-value (output).

The input of a function is the independent **variable**. The output of a function is the dependent **variable.** The value of the dependent variable *depends* on, or is a function of, the value of the independent variable.

Identify the independent and dependent variables in the situation.

1. A painter must measure a room before deciding how much paint to buy.

Independent Variable: measure of room
Dependent Variable: amount of paint

2. The height of a candle decreases *d* centimeters for every hour it burns.

Independent Variable: time (hours)
Dependent Variable: height

3. A veterinarian must weigh an animal before determining the amount of medication.

Independent Variable: weight
Dependent Variable: amount of medication

4. A company charges $10 per hour to rent a jackhammer.

Independent Variable: time
Dependent Variable: fee

5. Apples cost $0.99 per pound.

Independent Variable: pound
Dependent Variable: cost

Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-value	y-value
Domain	Range
Input	Output
x	f(x)

An algebraic expression that defines a function is a function rule.
Suppose Tasha earns $5 for each hour she baby-sits. Then $5 \bullet x$ is a function rule that models her earnings.

If x is the independent variable and y is the dependent variable, then function notation for y is $f(x)$, read "f of x," where f names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable is a function of the independent variable.

y	**is**	**a function of**	***x*.**
y	**=**	***f***	**(*x*)**

$$y = f(x)$$

Identify the independent and dependent variables.
Write an equation in function notation for the situation.

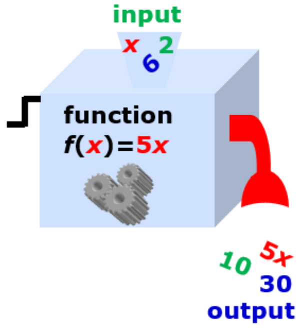
1. A math tutor charges $35 per hour.
Independent Variable: hour
Dependent Variable: cost
Equation: $f(x) = 35x$

2. A fitness center charges a $100 initiation fee plus $40 per month.
Independent Variable: months
Dependent Variable: cost
Equation: $f(x) = 100 + 40x$

3. Steven buys lettuce that costs $1.69/lb.
Independent Variable: Pound
Dependent Variable: cost
Equation: $f(x) = 1.69x$

4. An amusement park charges a $6.00 parking fee plus $29.99 per person.
Independent Variable: People
Dependent Variable: cost
Equation: $f(x) = 6 + 29.99x$

You can think of a function as an input-output machine. For Tasha's earnings, $f(x) = 5x$. If you input a value x, the output is $5x$.



Evaluate the function for the given input values.

1. For $f(x) = 3x + 2$, find $f(x)$ when $x = 7$ and when $x = -4$.

$f(7) = 3(7) + 2$
$f(7) = 21 + 2$
$f(7) = 23$

$f(-4) = 3(-4) + 2$
$f(-4) = -12 + 2$
$f(-4) = -10$

2. For $g(t) = 1.5t - 5$, find $g(t)$ when $t = 6$ and when $t = -2$.

$g(6) = 1.5(6) - 5$
$g(6) = 9 - 5$
$g(6) = 4$

$g(-2) = 1.5(-2) - 5$
$g(-2) = -3 - 5$
$g(-2) = -8$

3. For $h(r) = \frac{1}{3}r + 2$, find $h(r)$ when $r = 600$ and when $r = -12$.

$h(600) = \frac{1}{3}(600) + 2$
$h(600) = 200 + 2$
$h(600) = 202$

$h(-12) = \frac{1}{3}(-12) + 2$
$h(-12) = -4 + 2$
$h(-12) = -2$

4. For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and when $c = -3$.

$h(1) = 2(1) - 1$
$h(1) = 2 - 1$
$h(1) = 1$

$h(-3) = 2(-3) - 1$
$h(-3) = -6 - 1$
$h(-3) = -7$

5. For $g(t) = \frac{1}{4}t + 1$, find $g(t)$ when $t = -24$ and when $t = 400$.

$g(-24) = \frac{1}{4}(-24) + 1$
$g(-24) = -6 + 1$
$g(-24) = -5$

$g(400) = \frac{1}{4}(400) + 1$
$g(400) = 100 + 1$
$g(400) = 101$

When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

Write a function to describe the situation. Find the reasonable domain and range of the function.

1. Joe has enough money to buy 1, 2, or 3 DVDs at $15.00 each, if he buys any at all.

$f(x) = 15x$

$h(0) = 15(0) = 0$
$h(1) = 15(1) = 15$
$h(2) = 15(2) = 30$
$h(3) = 15(3) = 45$

x D: $\{0, 1, 2, 3\}$
$f(x)$ R: $\{0, 15, 30, 45\}$

2. The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function to describe the number of watts used for each setting. Find the reasonable domain and range for the function.

$f(x) = 500x$

$f(0) = 500(0) = 0$
$f(1) = 500(1) = 500$
$f(2) = 500(2) = 1000$
$f(3) = 500(3) = 1500$

x D: $\{0, 1, 2, 3\}$
$f(x)$ R: $\{0, 500, 1,000, 1500\}$

3.4 Graphing Functions

Objectives: 1. Graph functions given a limited domain.
2. Graph functions given a domain of all real numbers.

Graph the function for the given domain.

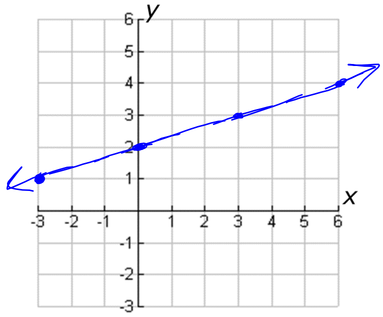
1. $x - 3y = -6$; D: {−3, 0, 3, 6}

$-x \qquad -x$

$\frac{-3y}{-3} = \frac{-x}{-3} \frac{-6}{-3}$

$y = \frac{x}{3} + 2$

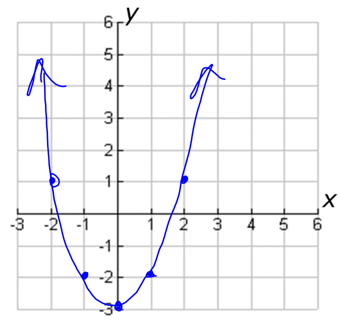
x	$y = x/3 + 2$	(x, y)
−3	$-3/3 + 2 = -1 + 2 = 1 = y$	(−3, 1)
0	$0/3 + 2 = 0 + 2 = 2 = y$	(0, 2)
3	$3/3 + 2 = 1 + 2 = 3 = y$	(3, 3)
6	$6/3 + 2 = 2 + 2 = 4 = y$	(6, 4)



2. $f(x) = x^2 - 3$; D: {−2, −1, 0, 1, 2}

Parabola

x	$y = x^2 - 3$	(x, f(x))
−2	$(-2)^2 - 3 = 4 - 3 = 1$	(−2, 1)
−1	$(-1)^2 - 3 = 1 - 3 = -2$	(−1, −2)
0	$0^2 - 3 = 0 - 3 = -3$	(0, −3)
1	$1^2 - 3 = 1 - 3 = -2$	(1, −2)
2	$2^2 - 3 = 4 - 3 = 1$	(2, 1)

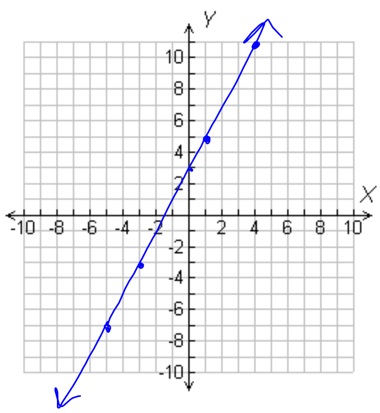


3. $-2x + y = 3$; D: {−5, −3, 1, 4}

$+2x \qquad +2x$

$y = 2x + 3$

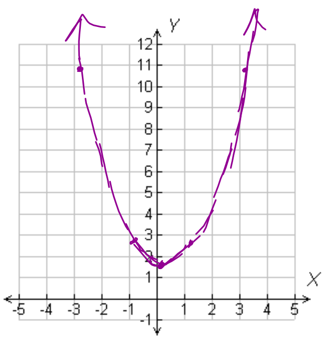
x	$y = 2x + 3$	(x, y)
−5	$2(-5) + 3 = -10 + 3 = -7$	(−5, −7)
−3	$2(-3) + 3 = -6 + 3 = -3$	(−3, −3)
1	$2(1) + 3 = 2 + 3 = 5$	(1, 5)
4	$2(4) + 3 = 8 + 3 = 11$	(4, 11)



4. $f(x) = x^2 + 2$; D: {−3, −1, 0, 1, 3}

x^2: Parabola

x	$f(x) = x^2 + 2$	(x, f(x))
−3	$(-3)^2 + 2 = 9 + 2$	(−3, 11)
−1	$(-1)^2 + 2 = 1 + 2$	(−1, 3)
0	$(0)^2 + 2 = 0 + 2$	(0, 2)
1	$(1)^2 + 2 = 1 + 2$	(1, 3)
3	$(3)^2 + 2 = 9 + 2$	(3, 11)



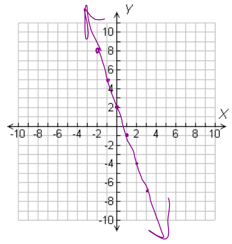
If the domain of a function is all real numbers, any number can be used as an input value. This process will produce an infinite number of ordered pairs that satisfy the function. Therefore, arrowheads are drawn at both "ends" of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not given, assume that the domain is all real numbers.

Graphing Functions Using a Domain of All Real Numbers	
Step 1	use the function to generate ordered pairs with specific domain
Step 2	Plot enough points to find a pattern on the graph
Step 3	Connect points with a line or a smooth curve.

(you pick domain values)

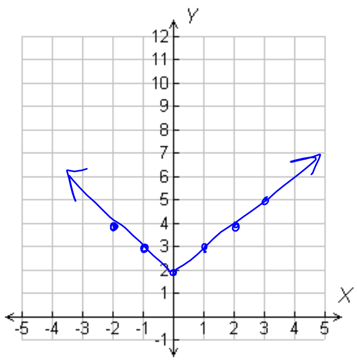
1. Graph the function $-3x + 2 = y$.

x	$-3x + 2 = y$	(x, y)
-2	$-3(-2)+2 = 6+2$	$(-2, 8)$
-1	$-3(-1)+2 = 3+2$	$(-1, 5)$
0	$-3(0)+2 = 0+2$	$(0, 2)$
1	$-3(1)+2 = -3+2$	$(1, -1)$
2	$-3(2)+2 = -6+2$	$(2, -4)$
3	$-3(3)+2 = -9+2$	$(3, -7)$



2. Graph the function $g(x) = |x| + 2$.

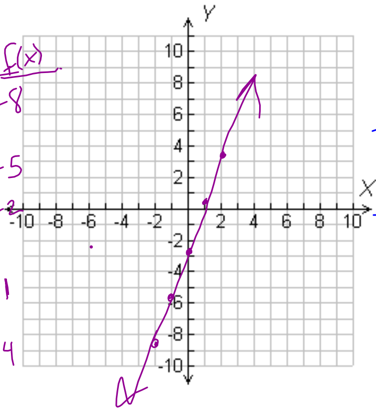
x	$g(x) = \|x\| + 2$	$(x, g(x))$
-2	$\|-2\|+2 = 2+2 = 4$	$(-2, 4)$
-1	$\|-1\|+2 = 1+2 = 3$	$(-1, 3)$
0	$\|0\|+2 = 0+2 = 2$	$(0, 2)$
1	$\|1\|+2 = 1+2 = 3$	$(1, 3)$
2	$\|2\|+2 = 2+2 = 4$	$(2, 4)$
3	$\|3\|+2 = 3+2 = 5$	$(3, 5)$



3. Graph the following functions

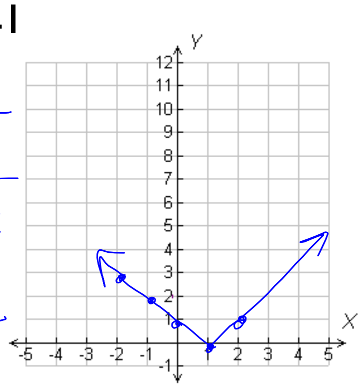
$f(x) = 3x - 2$

X	$3x-2=f(x)$	$f(x)$
-2	$3(-2)-2=-6-2$	-8
-1	$3(-1)-2=-3-2$	-5
0	$3(0)-2=0-2$	-2
1	$3(1)-2=3-2$	1
2	$3(2)-2=6-2$	4



$y = |x - 1|$

X	$\|x-1\|$	y
-2	$\|-2-1\| = \|-3\| = 3$	3
-1	$\|-1-1\| = \|-2\|$	2
0	$\|0-1\| = \|-1\|$	1
1	$\|1-1\| = \|0\|$	0
2	$\|2-1\| = \|1\|$	1



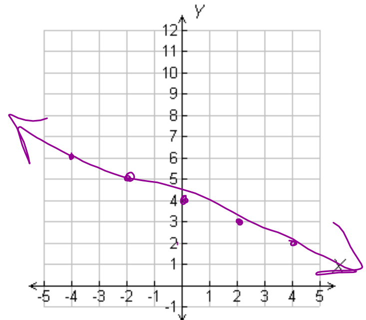
1. Use a graph of the function $f(x) = -\frac{1}{2}x + 4$ to find the value of $f(x)$ when $x = -4$. Check your answer.

x	$f(x) = -\frac{1}{2}x + 4$	$(x, f(x))$
-2	$-\frac{1}{2}(-2)+4 = 1+4$	$(-2, 5)$
0	$-\frac{1}{2}(0)+4 = 0+4$	$(0, 4)$
2	$-\frac{1}{2}(2)+4 = -1+4$	$(2, 3)$
4	$-\frac{1}{2}(4) = -2+4$	$(4, 2)$

$-\frac{1}{2}(4)+4$

$2+4=6$

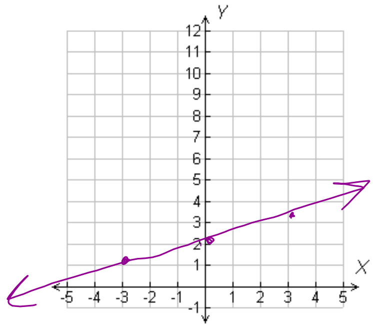
$\boxed{x=-4 \quad f(x)=6}$



2. Use the graph of $f(x) = \frac{1}{3}x + 2$ to find the value of x when $f(x) = 3$. Check your answer.

x	$f(x) = \frac{1}{3}x + 2$	$(x, f(x))$
-6	$\frac{1}{3}(-6)+2 = -2+2$	$(-6, 0)$
-3	$\frac{1}{3}(-3)+2 = -1+2$	$(-3, 1)$
0	$\frac{1}{3}(0)+2 = 0+2$	$(0, 2)$
3	$\frac{1}{3}(3)+2 = 1+2$	$(3, 3)$
6	$\frac{1}{3}(6)+2 = 2+2$	$(6, 4)$

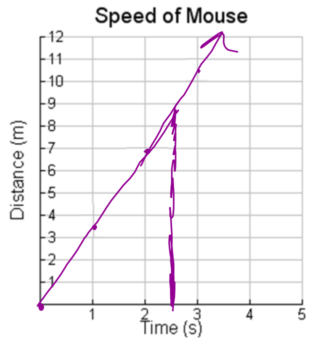
$\boxed{x=3 \quad f(x)=3}$



A mouse can run 3.5 meters per second. The function *y* = 3.5*x* describes the distance in meters the mouse can run in *x* seconds. Graph the function. Use the graph to estimate how many meters a mouse can run in 2.5 seconds.

x	$3.5x = y$	**(x, y)**
0	$3.5(0) = 0$	$(0, 0)$
1	$3.5(1) = 3.5$	$(1, 3.5)$
2	$3.5(2) = 7$	$(2, 7)$
3	$3.5(3) = 10.5$	$(3, 10.5)$

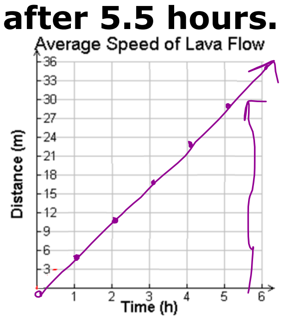
about 9 m



The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function *y* = 6*x* describes the distance *y* the lava moved on average in *x* hours. Graph the function. Use the graph to estimate how many miles the lava moved after 5.5 hours.

x	$y = 6x$	**(x, y)**
1	$6(1) = 6$	$(1, 6)$
2	$6(2) = 12$	$(2, 12)$
3	$6(3) = 18$	$(3, 18)$
4	$6(4) = 24$	$(4, 24)$

about 33 m



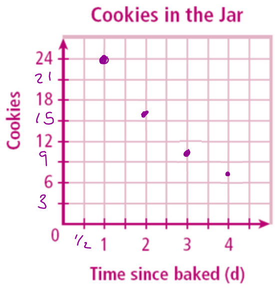
3.5 Scatter Plots and Trend Lines

Objectives: 1. Create and interpret scatter plots.
2. Use rend lines to make predictions.

A Scatterplots is a graph with points plotted to show a possible relationship between two sets of data. A scatter plot is an effective way to display some types of data.

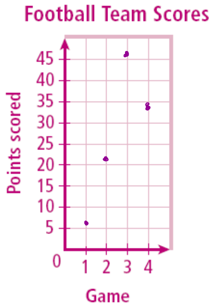
1. The table shows the number of cookies in a jar from the time since they were baked. Graph a scatter plot using the given data.

Cookies in the Jar				
Time Since Baked (d)	1	2	3	4
Cookies	24	16	10	7

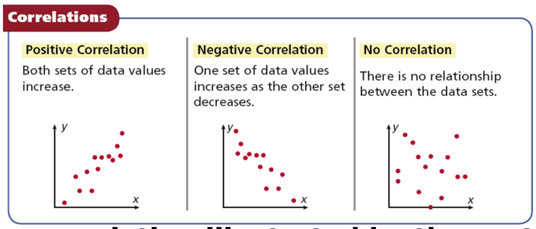


2. The table shows the number of points scored by a high school football team in the first four games of a season. Graph a scatter plot using the given data.

Game	1	2	3	4
Score	6	21	46	34

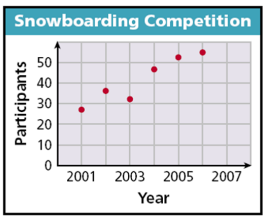


A correlation describes a relationship between two data sets. A graph may show the correlation between data. The correlation can help you analyze trends and make predictions. There are three types of correlations between data.



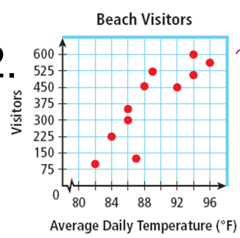
Describe the correlation illustrated by the scatter plot.

1.



Positive

2.



Positive

Identify the correlation you would expect to see between the pair of data sets. Explain.

1. the average temperature in a city and the number of speeding tickets given in the city

No Correlation

2. the number of people in an audience and ticket sales

Positive

3. a runner's time and the distance to the finish line

Negative

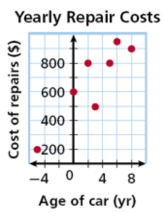
4. the temperature in Houston and the number of cars sold in Boston

No Correlation

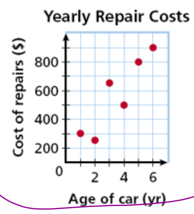
Choose the scatter plot that best represents the relationship between the age of a car and the amount of money spent each year on repairs. Explain.

The older the car the more likely you need to make repairs.

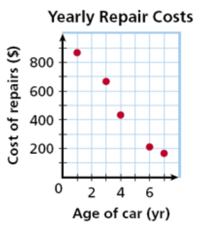
Graph A



Graph B



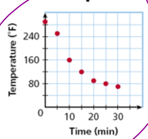
Graph C



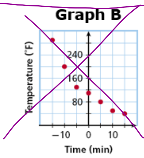
Choose the scatter plot that best represents the relationship between the number of minutes since a pie has been taken out of the oven and the temperature of the pie. Explain.

Starts hot and the air starts to cool it down.

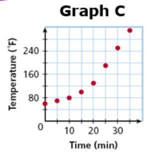
Graph A



Graph B



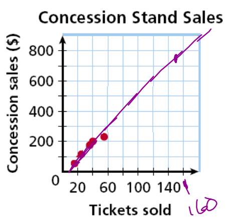
Graph C



You can graph a function on a scatter plot to help show a relationship in the data. Sometimes the function is a straight line. This line, called a trend line, helps show the correlation between data sets more clearly. It can also be helpful when making predictions based on the data.

The scatter plot shows a relationship between the total amount of money collected at the concession stand and the total number of tickets sold at a movie theater. Based on this relationship, predict how much money will be collected at the concession stand when 150 tickets have been sold.

around $750



3.6 Arithmetic Sequences

Objectives: 1. Recognize and extend an arithmetic sequence.
2. Find a given term of an arithmetic sequence.

When you list the times and distances in order, each list forms a sequence. A <u>sequence</u> is a list of numbers that often forms a pattern. Each number in a sequence is a <u>term</u>.

Time (s)	1	2	3	4	5	6	7	8
Distance (mi)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6

+0.2 +0.2 +0.2 +0.2 +0.2 +0.2 +0.2

In the distance sequence, each distance is 0.2 mi greater than the previous distance. When the terms of a sequence differ by the same nonzero number d, the sequence is an <u>arithmetic sequence</u> and d is the <u>common difference</u>. The distances in the table form an arithmetic sequence with $d = 0.2$.

Finding a Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with common difference d is

$$a_n = a_{n-1} + d.$$

Determine whether the sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

1. 9, 13, 17, 21,...

+4 +4 +4

$d = 4$

<u>25</u>, <u>29</u>, <u>33</u>

2. 10, 8, 5, 1,...

−2 −3 −4

Not arithmetic sequence

3. $-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \ldots$

$+\frac{2}{4}$ $+\frac{2}{4}$ $+\frac{2}{4}$

$d = \frac{2}{4} = \frac{1}{2}$

<u>$\frac{5}{4}$</u>, <u>$\frac{7}{4}$</u>, <u>$\frac{9}{4}$</u>

4. −4, −2, 1, 5,...

+2 +3 +4

Not arithmetic

Finding the *n*th Term of an Arithmetic Sequence

The nth term of an arithmetic sequence with common difference d and first term a_1 is

1st term

$$a_n = a_1 + (n-1)d.$$

\# term ← common diff.

Find the indicated term of the arithmetic sequence.

1. 16th term: 4, 8, 12, 16, ...

+4 +4 +4

$a_1 = 4$

$n = 16$

$d = 4$

$a_{16} = 4 + (16-1)(4)$

$a_{16} = 4 + (15)(4)$

$a_{16} = 4 + 60$

$a_{16} = 64$

2. The 25th term: $a_1 = -5$; $d = -2$

$a_1 = -5$

$n = 25$

$d = -2$

$a_{25} = -5 + (25-1)(-2)$

$-5 + (24)(-2)$

a_{25} $-5 + -48$

$a_{25} = -53$

3. 60th term: 11, 5, −1, −7, ...

−6 −6 −6

$a_1 = 11$

$n = 60$

$d = -6$

$a_{60} = 11 + (60-1)(-6)$

$a_{60} = 11 + (59)(-6)$

$a_{60} = 11 + -354$

$a_{60} = -343$

4. 12th term: $a_1 = 4.2$; $d = 1.4$

$a_1 = 4.2$

$n = 12$

$d = 1.4$

$a_{12} = 4.2 + (12-1)(1.4)$

$a_{12} = 4.2 + (11)(1.4)$

$a_{12} = 4.2 + 15.4$

$a_{12} = 19.6$

A bag of cat food weighs 18 pounds at the beginning of day 1. Each day, the cats are fed 0.5 pound of food. How much does the bag of cat food weigh at the beginning of day 30?

$a_n = a_1 + (n-1)d$

$a_1 = 18$

$n = 30$

$d = -.5$

$a_{30} = 18 + (30-1)(-.5)$

$a_{30} = 18 + 29(-.5)$

$a_{30} = 18 + {}^{-}14.5$

$a_{30} = 3.5$ lbs

Each time a truck stops, it drops off 250 pounds of cargo. After stop 1, its cargo weighed 2000 pounds. How much does the load weigh after stop 6?

$a_1 = 2000$

$n = 6$

$d = -250$

$a_6 = 2000 + (6-1)(-250)$

$a_6 = 2000 + 5(-250)$

$a_6 = 2000 - 1250$

$a_6 = 750$ lbs