

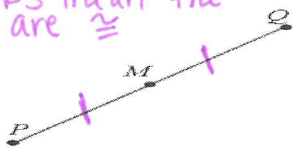
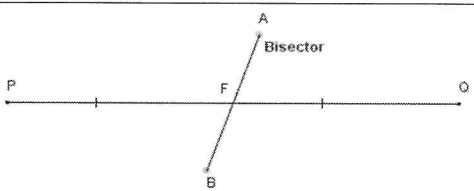
2.1 Segment Bisectors

Goal: Bisect a segment. Find the coordinates of the midpoint of a segment.

Midpoint: A point that divides a segment into two congruent segments

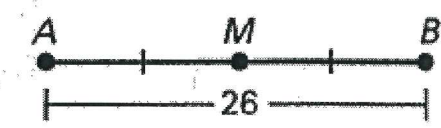
Segment Bisector: a segment, ray, line, or plane that intersects a segment at the midpoint

Bisect: to divide into two congruent pieces or to cut in half

<p style="color: purple; font-size: small;">tick marks mean the segments are \cong</p> 	<p style="color: purple; font-size: small;"><u>M</u> is the midpoint of <u>PQ</u></p> <p style="color: purple; font-size: small;">$\overline{PM} \cong \overline{MQ}$</p>
	<p style="color: purple; font-size: small;"><u>AB</u> is a bisector of <u>PQ</u></p> <p style="color: purple; font-size: small;">$\overline{PF} \cong \overline{FQ}$</p>

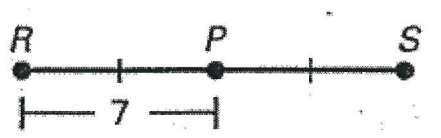
Example 1: M is the midpoint of \overline{AB} . Find AM and MB.

AM = 13 MB = 13 $\frac{26}{2} = 13$



Example 2: P is the midpoint of \overline{RS} . Find PS and RS.

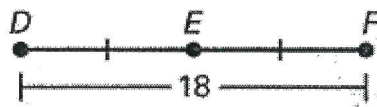
PS = 7 RS = 14 $7(2) = 14$



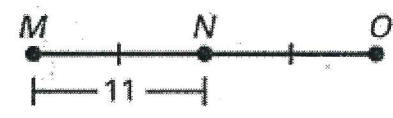
How is example 1 different from example 2?

Example 1 gives the length of the whole segment so we divide by 2. Example 2 gives the length of $\frac{1}{2}$ the segment so we multiply by 2 to get the whole segment.

a) DE = 9 EF = 9



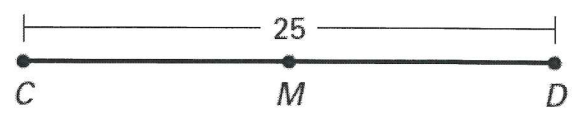
b) NO = 11 MO = 22



c) SM = 7 ST = 14



d) CM = 12.5 MD = 12.5



The Midpoint Formula

The coordinates of the midpoint are the average of the x-coordinates and the y-coordinates of the endpoints.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

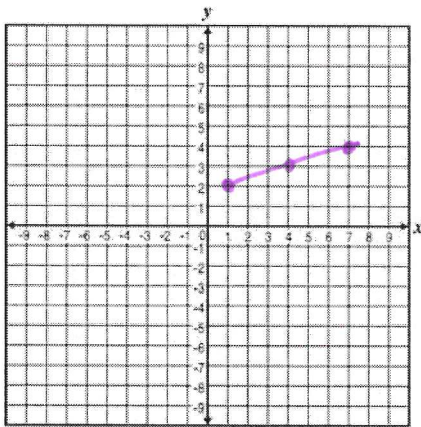
Plot the coordinates, then use the midpoint formula to find the coordinates of the midpoint.

a) (1, 2) and (7, 4)

Midpoint: (4, 3)

$$\left(\frac{1+7}{2}, \frac{2+4}{2} \right)$$

$$\left(\frac{8}{2}, \frac{6}{2} \right)$$

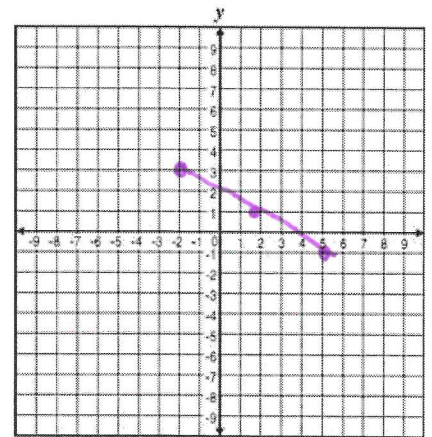


c) (-2, 3) and (5, -1)

Midpoint: (1.5, 1)

$$\left(\frac{-2+5}{2}, \frac{3+(-1)}{2} \right)$$

$$\left(\frac{3}{2}, \frac{2}{2} \right)$$

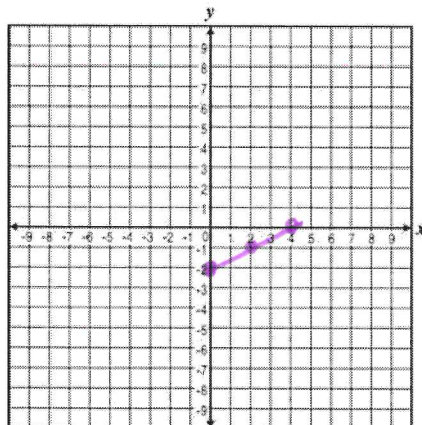


b) (0, -2) and (4, 0)

Midpoint: (2, -1)

$$\left(\frac{0+4}{2}, \frac{-2+0}{2} \right)$$

$$\left(\frac{4}{2}, \frac{-2}{2} \right)$$

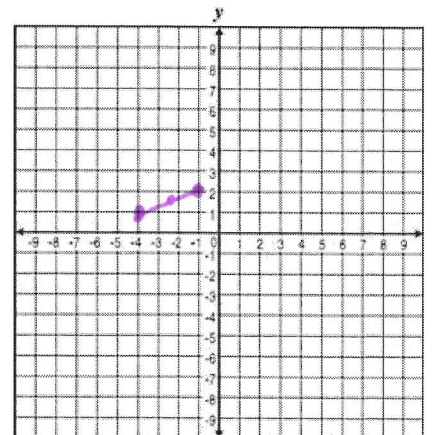


d) (-1, 2) and (-4, 1)

Midpoint: (-2.5, 1.5)

$$\left(\frac{-1+(-4)}{2}, \frac{2+1}{2} \right)$$

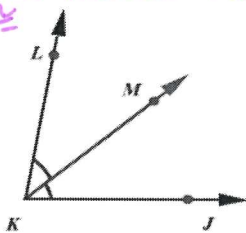
$$\left(\frac{-5}{2}, \frac{3}{2} \right)$$



2.2 Angle Bisectors

Goal: Use properties of angle bisectors to find missing measures.

Angle Bisector: is a ray that divides an angle into two angles that are congruent

<p>* the arc means that the \angle's are \cong</p> 	<p style="text-align: center;">\vec{KM} bisects $\angle LKJ$ so $\angle LKM \cong \angle MKJ$</p>
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\vec{HK} bisects $\angle GHJ$. Find $m\angle GHK$ and $m\angle KHJ$.

a) $m\angle GHK = 32^\circ$

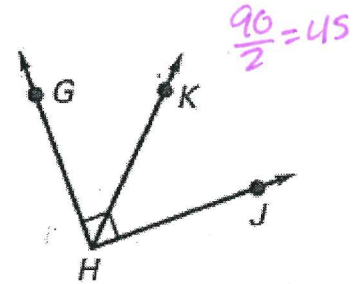
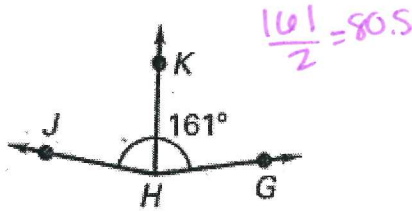
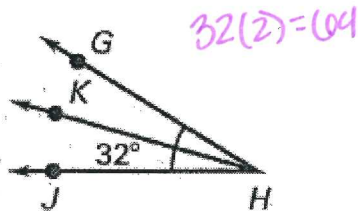
b) $m\angle GHK = 80.5^\circ$

c) $m\angle GHK = 45^\circ$

$m\angle KHJ = 64^\circ$

$m\angle KHJ = 80.5^\circ$

$m\angle KHJ = 45^\circ$



\vec{QS} bisects $\angle PQR$. Find $m\angle SQP$ and $m\angle PQR$. Then tell whether $\angle PQR$ is acute, right, obtuse, or straight.

a) $m\angle SQP = 29^\circ$

b) $m\angle SQP = 45^\circ$

c) $m\angle SQP = 60^\circ$

$m\angle PQR = 58^\circ$

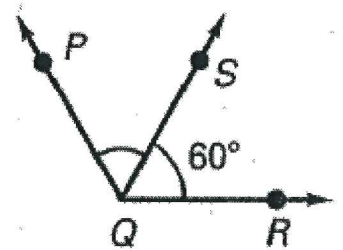
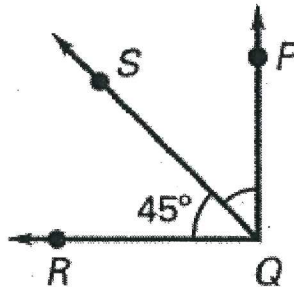
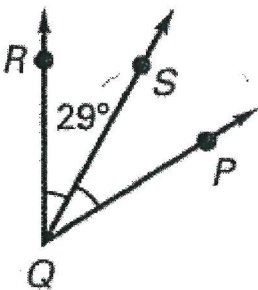
$m\angle PQR = 90^\circ$

$m\angle PQR = 120^\circ$

Classify: acute

Classify: right

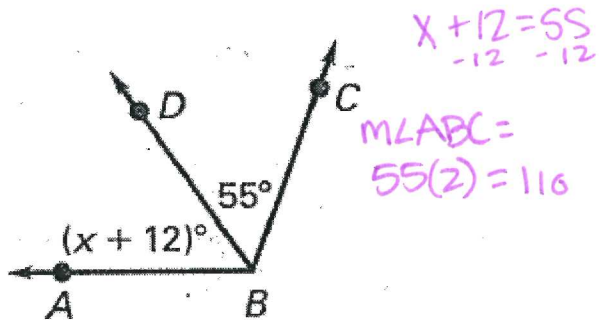
Classify: obtuse



\overline{BD} bisects $\angle ABC$. Find the value of x and then the measure of each missing angle.

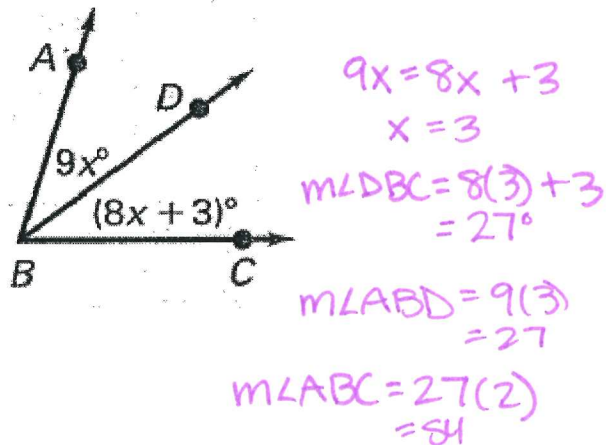
a) $x = 43$ $m\angle ABD = 55^\circ$

$m\angle ABC = 110^\circ$



b) $x = 3$ $m\angle DBC = 27^\circ$

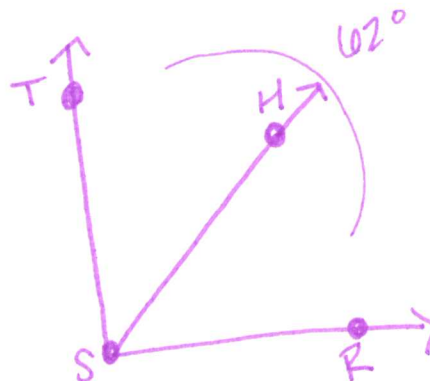
$m\angle ABD = 27^\circ$ $m\angle ABC = 54^\circ$



Draw a picture of the situation, then find the indicated information.

- a) If \overrightarrow{SH} is the bisector of $\angle TSR$ and $m\angle TSR = 62^\circ$, then what is $m\angle TSH$?

$m\angle TSH = \frac{62}{2} = 31^\circ$



- b) \overrightarrow{RT} is the bisector of $\angle ARC$

If $m\angle ART = (\frac{1}{2}x + 24)^\circ$,

and $m\angle TRC = (3x - 46)^\circ$,

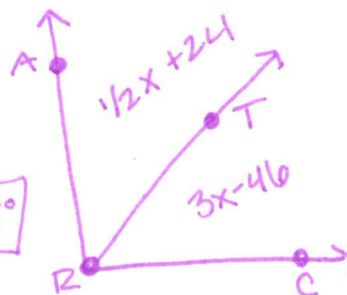
then find x and $m\angle ART$.

$\frac{1}{2}x + 24 = 3x - 46$

$70 = 2.5x$

$x = 28$

$m\angle ART = \frac{1}{2}(28) + 24 = 38^\circ$



- c) \overrightarrow{EF} is the bisector of $\angle AEC$.

If $m\angle AEF = (5x - 17)^\circ$,

and $m\angle FEC = (3x + 13)^\circ$,

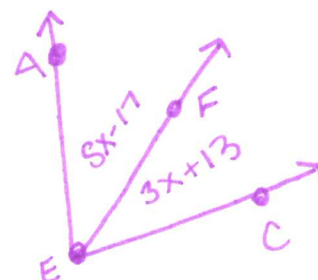
then find x and $m\angle FEC$.

$5x - 17 = 3x + 13$

$2x = 30$


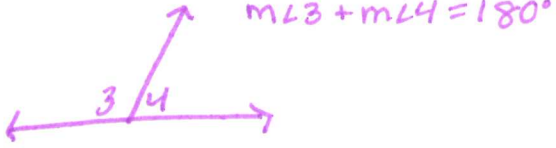
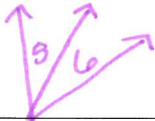
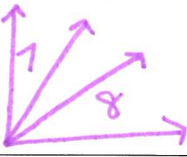
$x = 15$

$m\angle FEC = 3(15) + 13 = 43^\circ$



2.3 Complementary and Supplementary Angles

Goal: Find measures of complementary and supplementary angles.

Angle Pairs	
Complementary Angles: two angles whose sum of their measures is <u>90°</u>	
Supplementary Angles: two angles whose sum of their measures is <u>180°</u>	
Adjacent Angles: two angles that share a common vertex and <u>side</u> , but have no common interior points.	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <u>Adjacent</u>  </div> <div style="text-align: center;"> <u>Nonadjacent</u>  </div> </div>

Think of a way to help you remember the difference between complementary and supplementary!

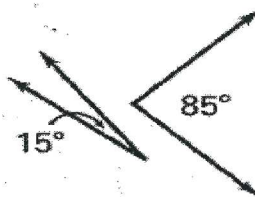
*MS. B's tip: If you get a compliment, you did something right. Right = 90° .

State whether the angles are complement, supplementary, or neither.

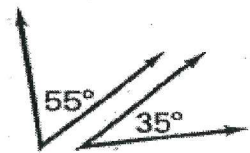
a) supplementary



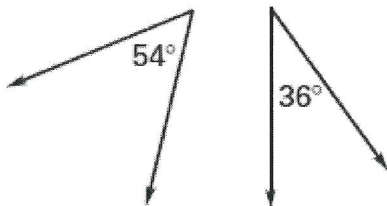
b) neither



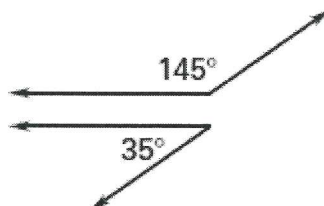
c) complementary



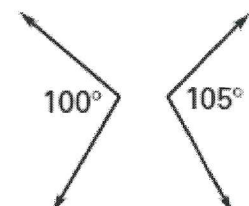
d) complementary



e) supplementary



f) neither

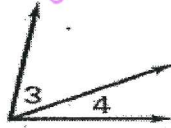


Tell whether the numbered angles are adjacent or nonadjacent.

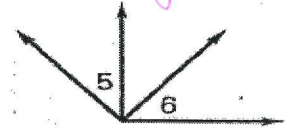
a) nonadjacent



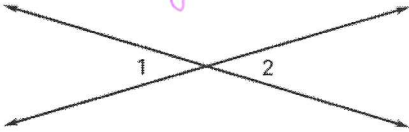
b) adjacent



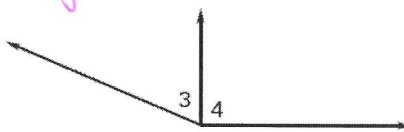
c) nonadjacent



d) nonadjacent



e) adjacent



Find the complement or supplement of each angle.

a) $m\angle A = 47^\circ$ 90-47
Complement of $\angle A =$ 43°

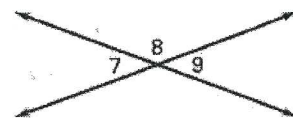
c) $m\angle C = 133^\circ$ 180-133
Supplement of $\angle C =$ 47°

b) $m\angle B = 68^\circ$ 90-68
Complement of $\angle B =$ 22°

d) $m\angle D = 13^\circ$ 180-13
Supplement of $\angle D =$ 167°

<p>Congruent Complements Theorem: If two angles are complementary to the same angle, then they are <u>Congruent</u>.</p> <p>Symbols If $m\angle 1 + m\angle 2 = 90^\circ$ and $m\angle 2 + m\angle 3 = 90^\circ$, then $\angle 1 \cong \angle 3$.</p>	
<p>Congruent Supplements Theorem: If two angles are supplementary to the same angle, then they are <u>Congruent</u>.</p> <p>Symbols If $m\angle 4 + m\angle 5 = 180^\circ$ and $m\angle 5 + m\angle 6 = 180^\circ$, then $\angle 4 \cong \angle 6$.</p>	

$\angle 7$ and $\angle 8$ are supplementary, and $\angle 8$ and $\angle 9$ are supplementary. Name a pair of congruent angles. Explain your reasoning.

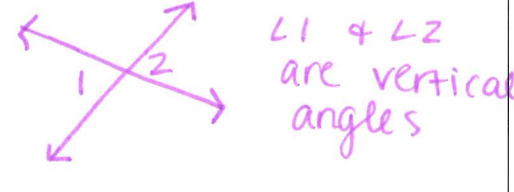
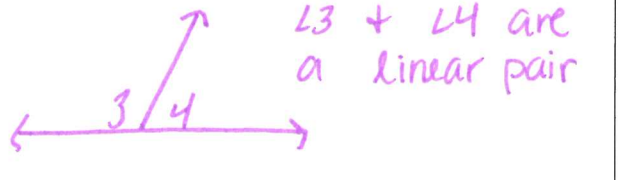


Solution

$\angle 7$ and $\angle 9$ are both supplementary to $\angle 8$. So, from the Congruent Supplements Theorem, it is true that $\angle 7 \cong \angle 9$.

2.4 Vertical Angles

Goal: Find measures of angles formed by intersecting lines.

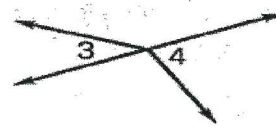
<p>Vertical Angles: two angles that are not <u>adjacent</u> and their sides are formed by two <u>intersecting</u> lines</p>	
<p>Linear Pair: two <u>adjacent</u> angles whose noncommon sides are on the same <u>line</u></p>	

Determine whether the labeled angles are vertical angles, a linear pair, or neither.

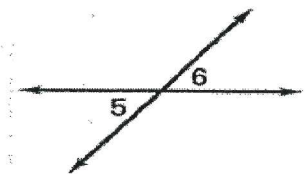
a) linear pair



b) neither



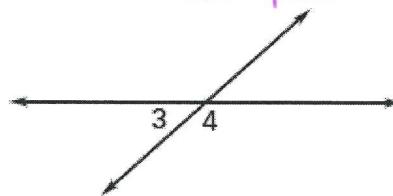
c) vertical L's



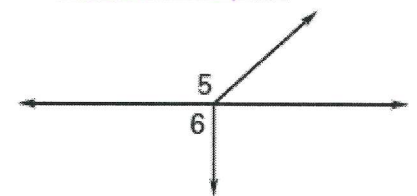
d) vertical L's

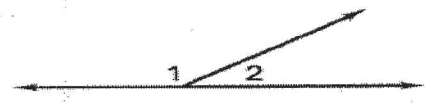
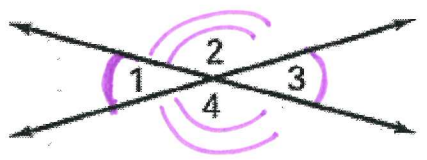


e) linear pair



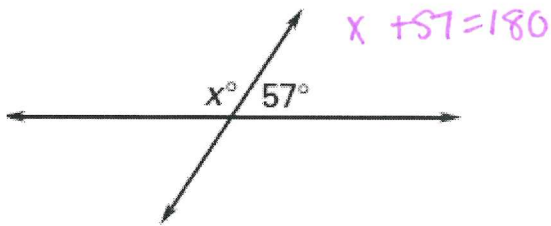
f) neither



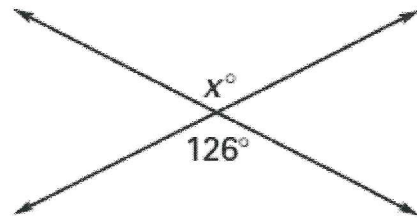
<p>Linear Pair Postulate: If two angles form a linear pair, then they are <u>supplementary</u>. $m\angle 1 + m\angle 2 = 180^\circ$</p>	
<p>Vertical Angles Theorem: Vertical angles are <u>congruent</u>. $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$</p>	

Use the linear pair postulate and the vertical angles theorem to find the value of the variable.

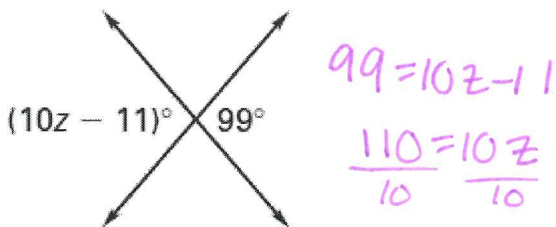
a) Type of Angles: linear pair $x = 123$



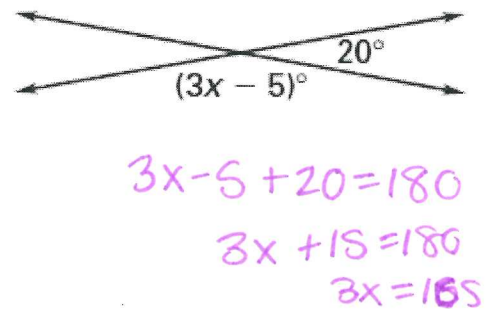
b) Type of Angles: Vertical $x = 126$



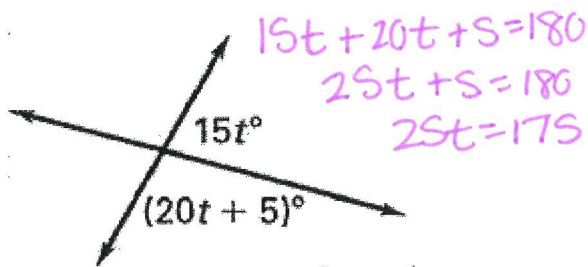
c) Type of Angles: vertical $z = 11$



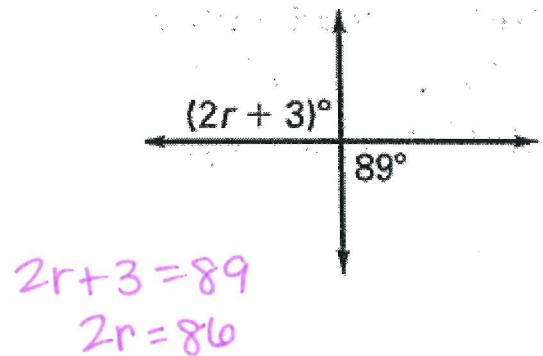
d) Type of Angles: linear pair $x = 55$



e) Type of Angles: linear pair $t = 7$

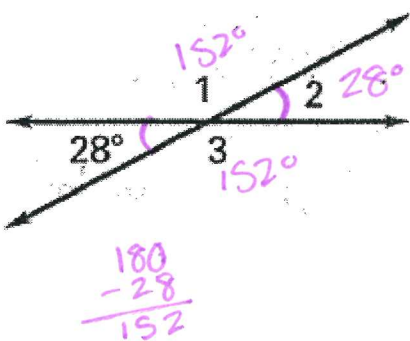


f) Type of Angles: Vertical $r = 43$

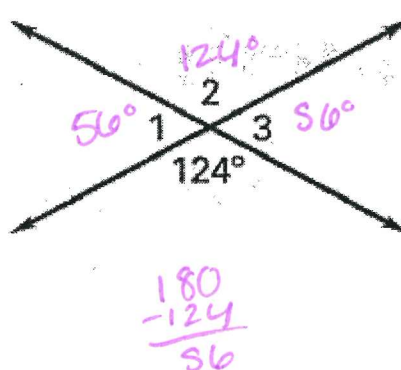


Find the measure of each missing angle.

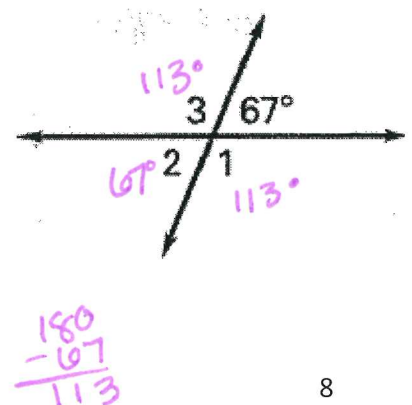
a)



b)



c)



2.5 If-Then Statements and Deductive Reasoning

Goal: Use if-then statements and apply laws of logic.

If-then statement: a statement with two parts: an "if" part that contains the hypothesis and a "then" part that contains the conclusion.

Hypothesis: the "if" part of an if-then statement

Conclusion: the "then" part of an if then statement

For each statement, underline the hypothesis and circle the conclusion.

a) If you attend T. F. Riggs High School, then your mascot is the Governors.

b) If it is raining outside, then there are clouds in the sky.

c) If you are in Basic Geometry, then Ms. Blaseg and Ms. Vockrodt are your teachers.

Rewrite each statement as an if-then statement.

a) I will buy the CD if it costs less than \$15.

If a CD costs less than \$15, then I will buy it.

b) A right angle measures 90 degrees.

If an angle is a right angle, then its measure is 90°.

c) All games involving zombies are fun to play.

If a game involves zombies, then it is fun to play.

d) I will give my dog a treat if she behaves.

If my dog behaves, then I will give her a treat.

Follow up: In a sentence that contains a hypothesis and a conclusion, is the conclusion always stated at the

end of the sentence? Explain.

No, it examples a + d, the conclusion was at the beginning.

<p>Law of Detachment: If the hypothesis of a true if-then statement is true, then the conclusion is <u>true</u></p>	<p>Law of Syllogism: If the following two statements are true, then the third statement is <u>true</u></p> <p>If statement p, then statement q. \searrow If statement q, then statement r. \swarrow If these statements are true, If statement p, then statement r. \longleftarrow then this statement is true.</p>
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What can you conclude from the following statements?

a) If you wash the cotton t-shirt in hot water, then it will shrink. You wash the cotton t-shirt in hot water.

Conclusion: The cotton t-shirt will shrink.

b) If x has a value of 7, then $2x-3$ has a value of 11. The value of x is 7.

Conclusion: $2x-3$ has a value of 11.

c) If you study at least 2 hours for the test, then you will pass the test. You study 3 hours for the test.

Conclusion: You will pass the test.

d) If you participate in class every day, Ms. Blaseg will be happy. You participate in class.

Conclusion: Ms. Blaseg is happy.

Use the Law of Syllogism to write a statement that follows the pair of true statements.

a) If I throw the stick, then my dog will go fetch it.
If my dog fetches the stick, then my dog will bring it back to me.

Conclusion: If I throw the stick, then my dog will bring it back to me.

b) If the juice is knocked over, then it will spill on the carpet.
If the juice spills on the carpet, then it will stain the carpet.

Conclusion: If the juice is knocked over, then it will stain the carpet.

c) If you give a mouse a cookie, he's going to ask for a glass of milk.
If you give him the milk, he'll probably ask for a straw.

Conclusion: If you give a mouse a cookie, he'll probably ask for a straw.

2.6 Properties of Equality and Congruence

Goal: Use properties of equality and congruence.

Properties of Equality and Congruence		
Reflexive Property	Equality $AB = AB$ $m\angle A = \underline{m\angle A}$	Congruence $\overline{AB} \cong \overline{AB}$ $\angle A \cong \underline{\angle A}$
Symmetric Property	Equality If $AB = CD$, then $CD = AB$. If $m\angle A = m\angle B$, then $\underline{m\angle B = m\angle A}$.	Congruence If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{AB}$. If $\angle A \cong \angle B$, then $\underline{\angle B \cong \angle A}$.
Transitive Property <i>*List of 3 things!</i>	Equality If $AB = CD$ and $CD = EF$, then $AB = EF$. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $\underline{m\angle A = m\angle C}$.	Congruence If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\underline{\angle A \cong \angle C}$.

Name the property that each statement illustrates.

- a) Reflexive Prop. of = $DE = DE$
- b) Transitive Prop. of \cong If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then $\angle P \cong \angle R$.
- c) Reflexive Prop. of \cong $\angle P \cong \angle P$
- d) Symmetric Prop. of = If $m\angle S = m\angle T$, then $m\angle T = m\angle S$.
- e) Transitive Prop. of = If $DF = FG$ and $FG = GH$, then $DF = GH$.
- f) Symmetric Prop. of \cong If $\angle G \cong \angle Z$, then $\angle Z \cong \angle G$.

Use the property to complete the statement.

Reflexive Property of Equality: $m\angle A = \underline{m\angle A}$

Symmetric Property of Equality: If $EF = GH$, then $\underline{GH} = \underline{EF}$.

Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{m\angle 1} = \underline{m\angle 3}$

Reflexive Property of Congruence: $\underline{\overline{KL}} \cong \underline{\overline{KL}}$

Symmetric Property of Congruence: If $\angle 5 \cong \angle 6$, then $\underline{\angle 6} \cong \underline{\angle 5}$.

Transitive Property of Congruence: If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\underline{\overline{AB}} \cong \underline{\overline{EF}}$.

Properties of Equality		
Addition Property	Adding a number to each side of an equation produces an equivalent equation	<u>Example:</u> If $x - 3 = 7$, then $+3 \quad +3$ $x = 10$
Subtraction Property	Subtracting a number to each side of an equation produces an equivalent equation	<u>Example:</u> If $y + 5 = 11$, then $-5 \quad -5$ $y = 6$
Multiplication Property	Multiplying a number to each side of an equation by the same nonzero number produces an equivalent equation	<u>Example:</u> If $1/4x = 6$, then $\times 4 \quad \times 4$ $x = 24$
Division Property	Dividing a number to each side of an equation by the same nonzero number produces an equivalent equation	<u>Example:</u> If $8x = 16$, then $\div 8 \quad \div 8$ $x = 2$
Substitution Property	Substituting a number to each side of an equation produces an equivalent equation	<u>Example:</u> If $x = 7$, then $2x + 4 =$ $2(7) + 4 = 18$

Name the property that each statement illustrates.

- a) Subtraction Prop — If $m\angle 1 = m\angle 4$, then $m\angle 1 - 30^\circ = m\angle 4 - 30^\circ$.
- b) Multiplication Prop — If $LM = NP$, then $2 \cdot LM = 2 \cdot NP$.
- c) Addition Prop — If $XY = EF$, then $XY + 7 = EF + 7$.
- d) Division Prop — If $m\angle A = m\angle B$, then $\frac{m\angle A}{3} = \frac{m\angle B}{3}$.
- e) Substitution Prop. — If $CD = 4$, then $CD + 12 = 4 + 12$.
- f) Addition Prop — If $m\angle S = 45^\circ$, then $m\angle S + 35^\circ = 80^\circ$.
- g) Multiplication Prop — If $m\angle K = 9^\circ$, then $3(m\angle K) = 27^\circ \rightarrow 9(3)$
- h) Substitution Prop — If $AB = 12$, then $2 \cdot AB + 3 = 2(12) + 3$.