2.1 Graphing & Writing Inequalities

Objectives: 1. Identify solutions of inequalities with one variable.
2. Write and graph inequalities with one variable.

An inequality is a statement that two quantities are not equal. The quantities are compared by using the following signs:

<	>	≤	≥	≠
$A < B$	$A > B$	$A \leq B$	$A \geq B$	$A \neq B$
A is less than B.	A is greater than B.	A is less than or equal to B.	A is greater than or equal to B.	A is not equal to B.

A solution **of an inequality** is any value of the variable that makes the inequality true.

1. Describe the solutions of $x - 6 \geq 4$ in words.

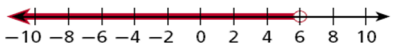
$+6 \quad +6$ $\quad x \geq 10$

Any value larger or equal to 10.

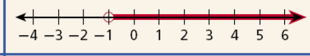
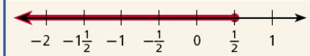
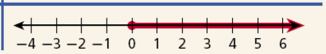
2. Describe the solutions of $2p > 8$ in words.

Any value larger than 4.

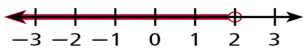
An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.



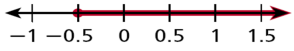
Graphing Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers less than 5	$x < 5$	
All real numbers greater than −1	$x > -1$	
All real numbers less than or equal to $\frac{1}{2}$	$x \leq \frac{1}{2}$	
All real numbers greater than or equal to 0	$x \geq 0$	

Write the inequality shown by each graph.

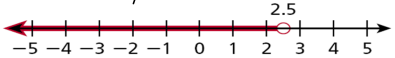
A. 

$2 > x$ $\quad x < 2$

B. 

$-.5 \leq x$

$x \geq -.5$

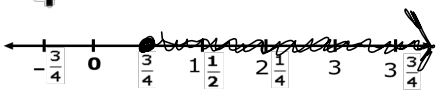
C. 

$x < 2.5$

$2.5 > x$

Graph each inequality.

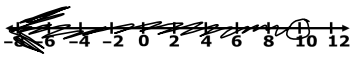
A. $m \geq \frac{3}{4}$



B. $t < 5(-1 + 3)$ $\quad -5 + 15$

$5(2)$ $\quad 10$

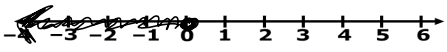
$t < 10$



C. $2^2 - 4 \geq w$

$4 - 4 \geq w$

$0 \geq w$

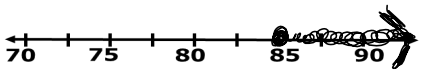


Ray's dad told him not to turn on the air conditioner unless the temperature is at least 85°F. Define a variable and write an inequality for the temperatures at which Ray can turn on the air conditioner. Graph the solutions.

x = temp when it can be turned on.

$x \geq 85$

$85 \leq x$

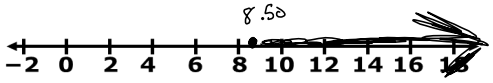


A store's employees earn at least $8.50 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

p = Payment per hour

$p \geq 8.50$

$8.50 \leq p$



2.2 Solving Inequalities by Adding & Subtracting

Objectives: 1. Solve one step inequalities by using addition.
2. Solve one step inequalities by using subtraction.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.

Properties of Inequality

Addition and Subtraction

WORDS	NUMBERS	ALGEBRA
Addition		
You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction		
You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$

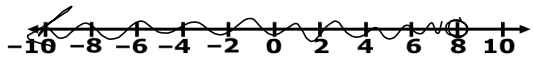
These properties are also true for inequalities that use the symbols >, ≥, and ≤.

Solve the inequality & graph the solutions.

1. $x + 12 < 20$

$-12 \quad -12$

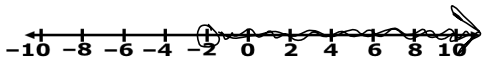
$x < 8$



2. $d - 5 > -7$

$+5 \quad +5$

$d > -2$



3. $0.9 \geq n - 0.3$

$+0.3 \quad +0.3$

$1.2 \geq n$

3. $s + 1 \leq 10$

$-1 \quad -1$

$s \leq 9$

4. $2\frac{1}{2} > -3 + t$

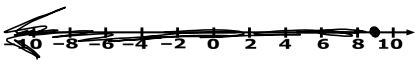
$+3 \quad +3$

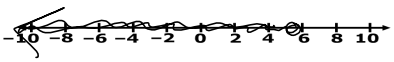
$5.5 > t$

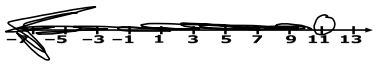
5. $q - 3.5 < 7.5$

$+3.5 \quad +3.5$

$q < 11$







Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

The solutions of $x + 9 < 15$ are given by $x < 6$.

Step 1 Check the endpoint.

Substitute 6 for x in the related equation $x + 9 = 15$. The endpoint should be a solution of the equation.

$x + 9 = 15$

$6 + 9$	15
15	15 ✓

Step 2 Check the inequality symbol.

Substitute a number less than 6 for x in the original inequality. The number you choose should be a solution of the inequality.

$x + 9 < 15$

$4 + 9$	$<$	15
13	$<$	15 ✓

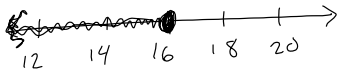
Sami has a gift card. She has already used $14 of the total value, which was $30. Write, solve, and graph an inequality to show how much more she can spend.

$14 + m \leq 30$

$30 \geq 14 + m$

$-14 \quad -14$

$16 \geq m$



The Recommended Daily Allowance (RDA) of iron for a female in Sarah's age group (14-18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding RDA.

$15 \geq 11 + g$

$-11 \quad -11$

$4 \geq g$

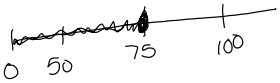
She can consume 0 - 4 mg

Mrs. Lawrence wants to buy an antique bracelet at an auction. She is willing to bid no more than $550. So far, the highest bid is $475. Write and solve an inequality to determine the amount Mrs. Lawrence can add to the bid. Check your answer.

$475 + B \leq 550$

$-475 \quad -475$

$B \leq \$75$



2.3 Solving Inequalities by Multiplication & Division

Objectives: 1. Solve one step inequalities by using multiplication.
2. Solve one step inequalities by using division.

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The following rules show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.

Properties of Inequality

Multiplication and Division by Positive Numbers

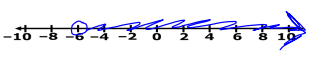
WORDS	NUMBERS	ALGEBRA
Multiplication		
You can multiply both sides of an inequality by the same *positive* number, and the statement will still be true.	$7 < 12$ $7(3) < 12(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division		
You can divide both sides of an inequality by the same *positive* number, and the statement will still be true.	$15 < 35$ $\frac{15}{5} < \frac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

Solve the inequality and graph the solutions.

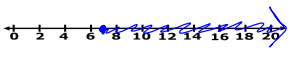
1. $\frac{7x}{7} > \frac{-42}{7}$

$x > -6$



2. $\frac{3}{1}(2.4) \leq \frac{m}{3} \cdot 3$

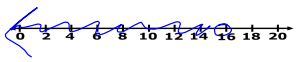
$7.2 \leq m$



3. $\frac{3}{4}r < 12\left(\frac{4}{3}\right)$

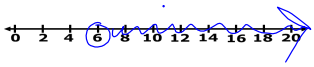
$\left(\frac{4}{3}\right)$

$r < 16$



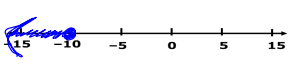
4. $\frac{4k}{4} > \frac{24}{4}$

$k > 6$



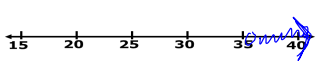
5. $\frac{-50}{5} \geq \frac{5q}{5}$

$-10 \geq q$



6. $\left(\frac{4}{3}\right)\frac{3}{4}g > 27\left(\frac{4}{3}\right)$

$g > 36$



If you multiply or divide both sides of an inequality by a negative number, the resulting inequality is not a true statement. You need to reverse the inequality symbol to make the statement true.

Properties of Inequality

Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication		
If you multiply both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$8 > 4$ $8(-2) \square 4(-2)$ $-16 \square -8$ $-16 < -8$	If $a > b$ and $c < 0$, then $ac < bc$.
Division		
If you divide both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$12 > 4$ $\frac{12}{-4} \square \frac{4}{-4}$ $-3 \square -1$ $-3 < -1$	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

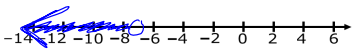
These properties are also true for inequalities that use the symbols $<$, $\geq$, and $\leq$.

Solve the inequality and graph the solutions.

1. $-12x > 84$

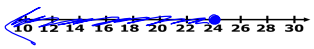
$\frac{-12x}{-12} > \frac{84}{-12}$

$x < -7$



2. $\frac{x}{-3} \geq -8$ · -3

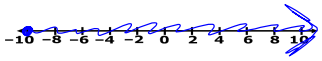
$x \leq 24$



3. $10 \geq -x$

$\frac{10}{-1} \geq \frac{-x}{-1}$

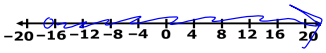
$-10 \leq x$



4. $4.25 > -0.25h$

$\frac{4.25}{-.25} > \frac{-0.25h}{-.25}$

$-17 < h$



$4\frac{1}{4}$ $\left(\frac{17}{4}\right)\left(\frac{1}{4}\right)\frac{4}{4}$

Jill has a $20 gift card to an art supply store where 4 oz tubes of paint are $4.30 each after tax. What are the possible numbers of tubes that Jill can buy?

$\frac{20}{4.3} \geq \frac{4.30p}{4.3}$

$4.65 \geq p$

$\{0, 1, 2, 3, 4\}$

A pitcher holds 128 ounces of juice. What are the possible numbers of 10-ounce servings that one pitcher can fill?

$\frac{128}{10} \geq \frac{10x}{10}$

$12.8 \geq x$

$\{12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0\}$

2.4 Solving Two-Step & Multi-Step Inequalties

Objectives: Solve inequalities that contain more then one operation.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.

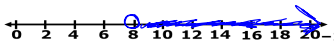
Solve the inequality and graph the solutions.

1. $45 + 2b > 61$

$-45 \quad -45$

$\frac{2b}{2} > \frac{16}{2}$

$b > 8$

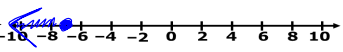


2. $8 - 3y \geq 29$

$-8 \quad -8$

$\frac{-3y}{-3} \geq \frac{21}{-3}$

$y \leq -7$

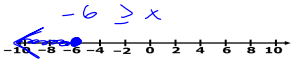


3. $-12 \geq 3x + 6$

$-6 \quad -6$

$\frac{-18}{3} \geq \frac{3x}{3}$

$-6 \geq x$

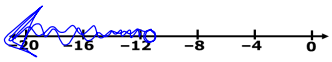


4. $\frac{x+5}{-2} > 3$ (-2)

$x + 5 < -6$

$-5 \quad -5$

$x < -11$



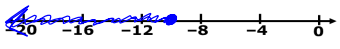
5. $\frac{1-2n}{3} \geq 7$ (3)

$1 - 2n \geq 21$

$-1 \quad -1$

$\frac{-2n}{-2} \geq \frac{20}{-2}$

$n \leq -10$



To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

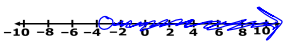
Solve the inequality and graph the solutions.

1. $2 + (+10) > -4t$

$12 > -4t$

$\frac{12}{-4} > \frac{-4t}{-4}$

$-3 < t$



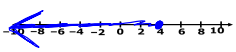
2. $-4(2 - x) \leq 8$

$-8 + 4x \leq 8$

$+8 \quad +8$

$\frac{4x}{4} \leq \frac{16}{4}$

$x \leq 4$



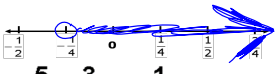
3. $\frac{2}{3}f + \frac{1}{2} > \frac{1}{3}$

$\frac{2}{6} - \frac{3}{6} = \frac{-1}{6}$

$-\frac{1}{2}$

$\left(\frac{3}{2}\right)\frac{2}{3}f > -\frac{1}{6}\left(\frac{3}{2}\right)$

$f > \frac{-3}{12} \quad f > -\frac{1}{4}$



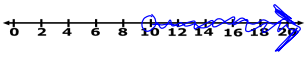
4. $2m + 5 > 5^2$

$2m + 5 > 25$

$-5 \quad -5$

$\frac{2m}{2} > \frac{20}{2}$

$m > 10$



5. $3 + 2(x + 4) > 3$

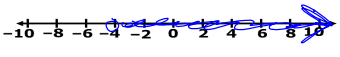
$3 + 2x + 8 > 3$

$11 + 2x > 3$

$-11 \quad -11$

$\frac{2x}{2} > \frac{-8}{2}$

$x > -4$



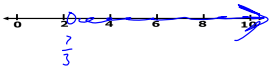
6. $\frac{5}{8} < \frac{3}{8}x - \frac{1}{4}$

$+\frac{2}{8} \quad +\frac{1}{4}$

$\left(\frac{8}{3}\right)\frac{7}{8} < \frac{3}{8}x\left(\frac{8}{3}\right)$

$2\frac{1}{3} < x$

$\frac{7}{3} < x$



To rent a certain vehicle, Rent-A-Ride charges $55.00 per day with unlimited miles. The cost of renting a similar vehicle at We Got Wheels is $38.00 per day plus $0.20 per mile. For what number of miles is the cost at Rent-A-Ride less than the cost at We Got Wheels?

$R < W$

More than 85 miles

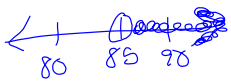
$85 < x$

$55 < 38 + .20x$

$-38 \quad -38$

$17 < .20x$

$\frac{17}{.2} < \frac{.20x}{.20}$



The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What grades can Jim get on his second test to make an A in the class?

$90 \leq \frac{2 \text{ scores}}{2}$

$(2)\,90 \leq \frac{95 + x}{2}(2)$

$180 \leq 95 + x$

$-95 \quad -95$

a grade 85% or higher

$85 \leq x$

2.5 Solving Inequalities with Variables on Both Sides

Objectives: Solve inequalities that contain variables on both sides.

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to "collect" all the variable terms on one side and all the constant terms on the other side.

Solve the inequality and graph the solutions.

1. $y \leq 4y + 18$

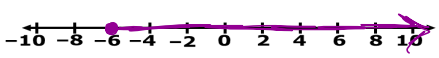
$-y \quad -y$

$0 \leq 3y + 18$

$-18 \quad -18$

$\frac{-18}{3} \leq \frac{3y}{3}$

$-6 \leq y$



2. $4m - 3 < 2m + 6$

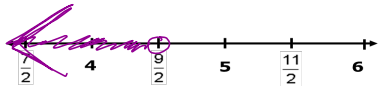
$-2m \quad -2m$

$2m - 3 < 6$

$+3 \quad +3$

$\frac{2m}{2} < \frac{9}{2}$

$m < \frac{9}{2}$

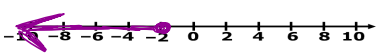


3. $4x \geq 7x + 6$

$-7x \quad -7x$

$\frac{-3x}{-3} \geq \frac{6}{-3}$

$x \leq -2$



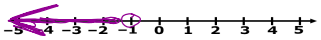
4. $5t + 1 < -2t - 6$

$+2t \quad +2t$

$7t + 1 < -6$

$-1 \quad -1$

$\frac{7t}{7} < \frac{-7}{7} \quad t < -1$



The Home Cleaning Company charges $312 to power-wash the siding of a house plus $12 for each window. Power Clean charges $36 per window, and the price includes power-washing the siding. How many windows must a house have to make the total cost from The Home Cleaning Company less expensive than Power Clean?

More than 13 windows

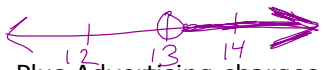
$CC < PC$

$312 + 12w < 36w$

$-12w \quad -12w$

$\frac{312}{24} < \frac{24w}{24}$

$13 < w$



A-Plus Advertising charges a fee of $24 plus $0.10 per flyer to print and deliver flyers. Print and More charges $0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost of Print and More?

$APA < PM$

$24 + .10f < .25f$

$-.10f \quad -.10f$

$\frac{24}{.15} < \frac{.15f}{.15}$

$160 < f$

More than 160 flyers

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use the Distributive Property.

Solve the inequality and graph the solutions.

1. $2(k - 3) > 6 + 3k - 3$

$2k - 6 > 6 + 3k - 3$

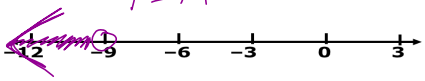
$2k - 6 > 3 + 3k$

$-2k \quad -2k$

$-6 > 3 + k$

$-3 \quad -3$

$-9 > k$

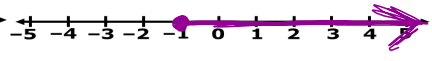


2. $0.9y \geq 0.4y - 0.5$

$-.4y \quad -.4y$

$\frac{.5y}{.5} \geq \frac{-.5}{.5}$

$y \geq -1$



3. $5(2 - r) \geq 3(r - 2)$

$10 - 5r \geq 3r - 6$

$+5r \quad +5r$

$10 \geq 8r - 6$

$+6 \quad +6$

$\frac{16}{8} \geq \frac{8r}{8} \quad 2 \geq r$

Some inequalities are true no matter what value is substituted for the variable. For these inequalities, all real numbers are solutions.

Some inequalities are false no matter what value is substituted for the variable. These inequalities have no solution.

Solve the inequality.

1. $2x - 7 \le 5 + 2x$

$+7 \quad +7$

$2x \le 12 + 2x$

$-2x \quad -2x$

$0 \le 12$

All Real Numbers

2. $2(3y - 2) - 4 \ge 3(2y + 7)$

$6y - 4 - 4 \ge 6y + 21$

$6y - 8 \ge 6y + 21$

$-6y \quad -6y$

$-8 \ge 21$

No Solution

3. $4(y - 1) \ge 4y + 2$

$4y - 4 \ge 4y + 2$

$-4y \quad -4y$

$-4 \ge 2$

No Solution

4. $x - 2 < x + 1$

$-x \quad -x$

$-2 < 1$

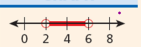
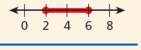
All Real Numbers

2.6 Solving Compound Inequalities

Objectives: 1. Solve compound inequalities with one variable.
2. Graph solution sets of compound inequalities with one variable.

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a compound inequality.

Compound Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $2 < x < 6$	
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \ge 2$ AND $x \le 6$ $2 \le x \le 6$	
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$	
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \le 2$ OR $x \ge 6$	

no less than — No more than

The pH level of a popular shampoo is between 6.0 and 6.5 inclusive. Write a compound inequality to show the pH levels of this shampoo. Graph the solutions.

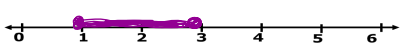
$6.0 \le x$ AND $x \le 6.5$

The free chlorine in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

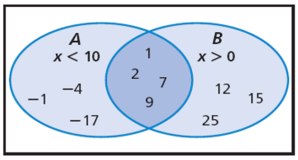
no less than 1 — no more than 3

$1 \le x \le 3$

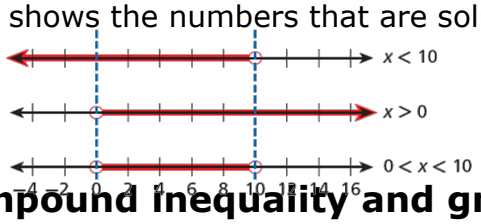
$1 \le x$ AND $x \le 3$



In this diagram, oval *A* represents some integer solutions of $x < 10$ and oval *B* represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of *both* $x < 10$ *and* $x > 0$.



You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.

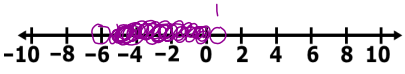


Solve the compound inequality and graph the solutions.

1. $-5 < x + 1 < 2$

$-1 \quad -1 \quad -1$

$-6 < x < 1$

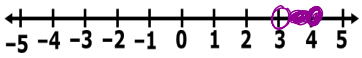


2. $8 < 3x - 1 \leq 11$

$+1 \quad +1 \quad +1$

$\frac{9}{3} < \frac{3x}{3} \leq \frac{12}{3}$

$3 < x \leq 4$



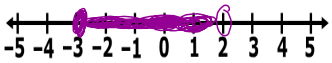
3. $-4 \leq 3n + 5 < 11$

$-5 \quad -5 \quad -5$

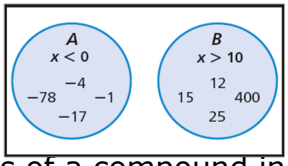
$-9 \leq 3n < 6$

$\frac{-9}{3} \leq \frac{3n}{3} < \frac{6}{3}$

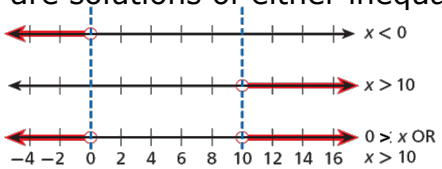
$-3 \leq n < 2$



In this diagram, circle *A* represents some integer solutions of $x < 0$, and circle *B* represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of *either* $x < 0$ *or* $x > 10$.



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combine regions are called the **union** and show the numbers that are solutions of either inequality.

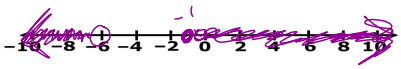


Solve the inequality and graph the solutions.

1. $8 + t \geq 7$ OR $8 + t < 2$

$-8 \quad -8 \quad -8 \quad -8$

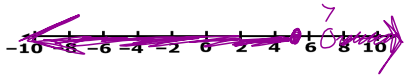
$t \geq -1$ OR $t < -6$



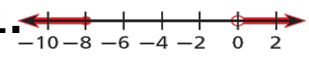
2. $4x \leq 20$ OR $3x > 21$

$\frac{4x}{4} \leq \frac{20}{4}$ OR $\frac{3x}{3} > \frac{21}{3}$

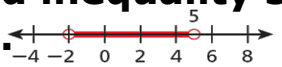
$x \leq 5$ OR $x > 7$



Write the compound inequality shown by the graph.

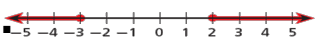
1. 

$-8 \geq x$ OR $0 < x$

2. 

$-2 < x < 5$

$-2 < x$ AND $5 > x$

3. 

$-3 \geq x$ OR $2 \leq x$

$x \leq -3$ OR $x \geq 2$

The target heart rate during exercise for a 15 year-old is between 154 and 174 beats per minute ~~inclusive.~~ Write a compound inequality to show the heart rates that are within the target range. Graph the solutions.

$154 \leq x \leq 174$ $\quad 174 \geq x \geq 154$