1.1 Variables and Expressions

A __________ is a letter or a symbol used to represent a value that can change.

A ______________ is a value that does not change.

A ___________________ contains only constants and operations.

An ____________________ may contain variables, constants, and operations.

+	−
×	÷

Give two ways to write each algebraic expression in words.

A. $9 + r$ **B.** $q - 3$ **C.** $7m$ **D.** $j \div 6$

E. $\frac{t}{5}$ **F.** $4 - n$ **G.** $9 + q$ **H.** $3(h)$

Translating from Words to Algebra

John types 62 words per minute. Write an expression for the number of words he types in *m* minutes.

Roberto is 4 years older than Emily, who is *y* years old. Write an expression for Roberto's age

Joey earns $5 for each car he washes. Write an expression for the number of cars Joey must wash to earn *d* dollars.

To ____________ an expression is to find its value.

To evaluate an algebraic expression, substitute numbers for the variables in the expression and then simplify the expression.

Evaluating Algebraic Expressions

Evaluate each expression for $a = 4$, $b = 7$, and $c = 2$.

A. $b - c$ **B.** ac

Evaluate each expression for $m = 3$, $n = 2$, and $p = 9$.

a. mn **b.** $p - n$ **c.** $p \div m$

Approximately eighty-five 20-ounce plastic bottles must be recycled to produce the fiberfill for a sleeping bag.

a. Write an expression for the number of bottles needed to make *s* sleeping bags.

b. Find the number of bottles needed to make 20, 50, and 325 sleeping bags.

To make one sweater, 63 twenty ounce plastic drink bottles must be recycled.

a. Write an expression for the number of bottles needed to make *s* sweaters.

b. Find the number of bottles needed to make 12, 25 and 50 sweaters.

1.2 Solving Equations by Adding & Subtracting

An _______________ is a mathematical statement that two expressions are equal.

A ___________________________ is a value of the variable that makes the equation true.

To find solutions, ____________________________. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

Inverse Operations	
Operation	**Inverse Operation**
Addition	
Subtraction	

Solve the equation. Check your answer.

1. $y - 8 = 24$ **2.** $\frac{5}{16} = z - \frac{7}{16}$ **3.** $n - 3.2 = 5.6$

4. $-6 = k - 6$ **5.** $16 = m - 9$ **6.** $m + 17 = 33$

7. $4.2 = t + 1.8$ **8.** $d + \frac{1}{2} = 1$ **9.** $-5 = k + 5$

10. $6 + t = 14$ **11.** $\frac{5}{11} + p = -\frac{2}{11}$ **12.** $-2.3 + m = 7$

13. $-11 + x = 33$ **14.** $-\frac{3}{4} + z = \frac{5}{4}$

Over 20 years, the population of a town decreased by 275 people to a population of 850. Write and solve an equation to find the original population.

A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. One method to estimate maximum heart rate states that your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find a person's age if the person's maximum heart rate is 185 beats per minute.

1.3 Solving Equations by Multiplying & Dividing

Inverse Operations	
Operation	**Inverse Operation**
Multiplication	
Division	

Solve the equation.

1. $-8 = \frac{j}{3}$

2. $\frac{n}{6} = 2.8$

3. $\frac{p}{5} = 10$

4. $-13 = \frac{y}{3}$

5. $\frac{c}{8} = 7$

6. $9y = 108$

7. $-4.8 = -6v$

8. $16 = 4c$

9. $0.5y = -10$

Remember that dividing is the same as multiplying by the _______________. When solving equations, you will sometimes find it easier to multiply by a ___________ instead of dividing. This is often true when an equation contains fractions.

1. $\frac{5}{6}w = -20$

2. $\frac{3}{16} = \frac{1}{8}z$

3. $-\frac{1}{4} = \frac{1}{5}b$

4. $\frac{4j}{6} = \frac{2}{3}$

5. $\frac{1}{6}w = 102$

Ciro puts 1/4 of the money he earns from mowing lawns into a college education fund. This year Ciro added $285 to his college education fund. Write and solve an equation to find how much money Ciro earned mowing lawns this year.

The distance in miles from the airport that a plane should begin descending, divided by 3, equals the plane's height above the ground in thousands of feet. A plane began descending 45 miles from the airport. Use the equation to find how high the plane was flying when the descent began.

1.4 Solving Two-Step & Multi-Step Equations

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

Cost per CD → $3.95c$; **Total cost** → 63.40

$$3.95c + 19.95 = 63.40$$

Cost of discount card → 19.95

Operations in the Equation	**To Solve**
1. First *c* is **multiplied** by 3.95.	1. **Subtract** 19.95 from both sides of the equation.
2. Then 19.95 is **added**.	2. Then **divide** both sides by 3.95.

Solve two-step equations and check your answer.

1. $18 = 4a + 10$ **2.** $5t - 2 = -32$ **3.** $-4 + 7x = 3$

4. $1.5 = 1.2y - 5.7$ **5.** $\frac{n}{7} + 2 = 2$ **6.** $\frac{y}{8} - \frac{3}{4} = \frac{7}{12}$

7. $\frac{2}{3}r + \frac{3}{4} = \frac{7}{12}$

8. $\frac{2x}{5} - \frac{1}{2} = 5$

9. $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$

Simplifying Before Solving Equations

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

1. $8x - 21 + 5x = -15$

2. $10y - (4y + 8) = -20$

3. $2a + 3 - 8a = 8$

4. $-2(3 - d) = 4$

5. $4(x - 2) + 2x = 40$

Jan joined the dining club at the local café for a fee of $29.95. Being a member entitles her to save $2.50 every time she buys lunch. So far, Jan calculates that she has saved a total of $12.55 by joining the club. Write and solve an equation to find how many time Jan has eaten lunch at the café.

Sara paid $15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was $735.95. How much was the monthly fee?

1.5 Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, use inverse operations to "collect" variable terms on one side of the equation.

Solving Equations with Variables on Both Sides

1. $7n - 2 = 5n + 6$

2. $4b + 2 = 3b$

3. $0.5 + 0.3y = 0.7y - 0.3$

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

1. $4 - 6a + 4a = -1 - 5(7 - 2a)$

2. $\frac{1}{2}(b+6) = \frac{3}{2}b - 1$

3. $3x + 15 - 9 = 2(x + 2)$

An identity is an equation that is true for all values of the variable. An equation that is an identity has ________________________.

Some equations are always false. These equations have ________________.

1. $10 - 5x + 1 = 7x + 11 - 12x$

2. $12x - 3 + x = 5x - 4 + 8x$

3. $4y + 7 - y = 10 + 3y$

4. $2c + 7 + c = -14 + 3c + 21$

Jon and Sara are planting tulip bulbs. Jon has planted 60 bulbs and is planting at a rate of 44 bulbs per hour. Sara has planted 96 bulbs and is planting at a rate of 32 bulbs per hour. In how many hours will Jon and Sara have planted the same number of bulbs? How many bulbs will that be?

Person	Bulbs
Jon	60 bulbs plus 44 bulbs per hour
Sara	96 bulbs plus 32 bulbs per hour

Four times Greg's age, decreased by 3 is equal to 3 times Greg's age increased by 7. How old is Greg?

1.6 Solving for a Variable

A _______________ is an equation that states a rule for a relationship among quantities.

In the formula $d = rt$, d is isolated. You can "rearrange" a formula to isolate any variable by using inverse operations. This is called *solving for a variable*.

Solving for a Variable	
Step 1	Locate the variable you are asked to solve for in the equation.
Step 2	Identify the operations on this variable and the order in which they are applied.
Step 3	Use inverse operations to undo operations and isolate the variable.

The formula $C = \pi d$ gives the circumference of a circle C in terms of diameter d. The circumference of a bowl is 18 inches. What is the bowl's diameter? Leave the symbol π in your answer.

Solve the formula $d = rt$ for t. Find the time in hours that it would take Ernst Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour.

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base, and h is the height. Solve for h.

The formula for a person's typing speed is $s = \frac{w - 10e}{m}$, *where* s is speed in words per minute, w is number of words typed, e is number of errors, and m is number of minutes typing. Solve for e.

The formula for an object's final velocity is $f = i - gt$, where i is the object's initial velocity, g is acceleration due to gravity, and t is time. Solve for i.

A formula is a type of *literal equation*. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

1. Solve $x + y = 15$ for x.

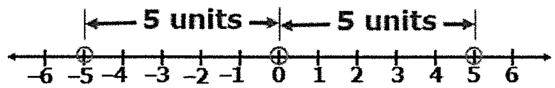
2. Solve $pq = x$ for q.

3. Solve $5 - b = 2t$ for t.

4. Solve $D = \frac{m}{V}$ for V

1.7 Solving Absolute-Value Equations

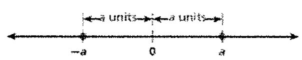
Recall that the absolute-value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.



For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and –5 have an absolute value of 5.

To write this statement using algebra, you would write $|x| = 5$. This equation asks, "What values of x have an absolute value of 5?" The solutions are 5 and –5. Notice this equation has two solutions.

Absolute-Value Equations

WORDS	NUMBERS
The equation $\|x\| = a$ asks, "What values of x have an absolute value of a?" The solutions are a and the opposite of a.	$\|x\| = 5$ $x = 5$ or $x = -5$
GRAPH	**ALGEBRA**
	$\|x\| = a$ $x = a$ or $x = -a$ $(a \geq 0)$

Solving an Absolute-Value Equation
1. Use inverse operations to isolate the absolute-value expression.
2. Rewrite the resulting equation as two cases that do not involve absolute values.
3. Solve the equation in each of the two cases.

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

1. $|x| = 12$ **2.** $3|x + 7| = 24$ **3.** $|x| - 3 = 4$ **4.** $8 = |x - 2.5|$

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute-value is negative, there are ________________.

5. $-8 = |x + 2| - 8$ **6.** $3 + |x + 4| = 0$

7. $2 - |2x - 5| = 7$ **8.** $-6 + |x - 4| = -6$

A support beam for a building must be 3.5 meters long. It is acceptable for the beam to differ from the ideal length by 3 millimeters. Write and solve an absolute-value equation to find the minimum and maximum acceptable lengths for the beam.

Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

1.8 Rates, Ratios, & Proportions

A ________ is a comparison of two quantities by division. The ratio of *a* to *b* can be written *a*:*b* or $\frac{a}{b}$, where b ≠ 0. ________ that name the same comparison are said to be equivalent.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.

1. The ratio of the number of bones in a human's ears to the number of bones in the skull is 3:11. There are 22 bones in the skull. How many bones are in the ears?

2. The ratio of games won to games lost for a baseball team is 3:2. The team has won 18 games. How many games did the team lose?

A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi.}}{2 \text{ gal.}}$ Rates are usually written as *unit rates.* A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi.}}{1 \text{ gal.}}$ or 17 mi/gal. You can convert any rate to a unit rate.

1. Raulf Laue of Germany flipped a pancake 416 times in 120 seconds to set the world record. Find the unit rate. Round your answer to the nearest hundredth.

2. Cory earns $52.50 in 7 hours. Find the unit rate.

Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft.}}$ in which the two quantities are equal but use different units, is called a **conversion factor**. To convert a rate from one set of units to another, multiply by a conversion factor.

1. A fast sprinter can run 100 yards in approximately 10 seconds. Use dimensional analysis to convert 100 yards to miles. Round to the nearest hundredth. (*Hint:* There are 1760 yards in a mile.)

2. A cheetah can run at a rate of 60 miles per hour in short bursts. What is this speed in feet per minute?

3. A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second? Round your answer to the nearest tenth, and show that your answer is reasonable.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \bullet d$ *and* $b \bullet c$ are called ________ ________________. You can solve a proportion for a missing value by using the Cross Products property.

Cross Products Property

WORDS	NUMBERS	ALGEBRA
In a proportion, cross products are equal.	$\frac{2}{3} = \frac{4}{6}$ $2 \bullet 6 = 3 \bullet 4$	If $\frac{a}{b} = \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$ then $ad = bc$.

Solve each proportion.

1. $\frac{3}{9} = \frac{5}{m}$ **2.** $\frac{6}{y-3} = \frac{2}{7}$ **3.** $\frac{-5}{2} = \frac{y}{8}$ **4.** $\frac{g+3}{5} = \frac{7}{4}$

A **scale** is a ratio between two sets of measurements, such as 1 in:5 mi. A **scale drawing** or **scale model** uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

1. A contractor has a blueprint for a house drawn to the scale 1 in: 3 ft.

a. A wall on the blueprint is 6.5 inches long. How long is the actual wall?

b. One wall of the house will be 12 feet long when it is built. How long is the wall on the blueprint?

3. A scale model of a human heart is 16 ft. long. The scale is 32:1. How many inches long is the actual heart it represents?

1.9 Applications of Proportions

_______________ figures have exactly the same shape but not necessarily the same size.

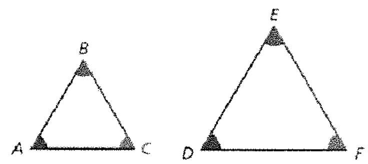
________________________ of two figures are in the same relative position, and ________________________ are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$m\angle A = m\angle D$

$m\angle B = m\angle E$

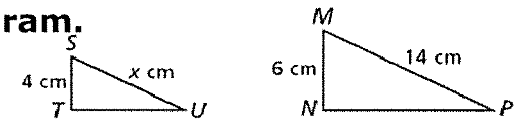
$m\angle C = m\angle F$



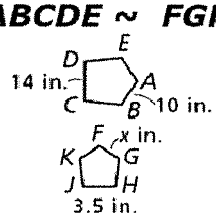
When stating that two figures are similar, use the symbol ~. For the triangles above, you can write $\Delta ABC \sim \Delta DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\Delta ABC \sim \Delta EFD$.

Find the value of *x* the diagram.

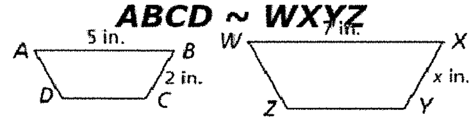
ΔMNP ~ ΔSTU



ABCDE ~ FGHJK



ABCD ~ WXYZ



You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement.** If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

1. A flagpole casts a shadow that is 75 ft long at the same time a 6-foot-tall man casts a shadow that is 9 ft long. Write and solve a proportion to find the height of the flag pole.

2. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.

If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a _______________.

1. The radius of a circle with radius 8 in. is multiplied by 1.75 to get a circle with radius 14 in. How is the ratio of the circumferences related to the ratio of the radii? How is the ratio of the areas related to the ratio of the radii?

	Circle A	Circle B
$C = 2\pi r$		
$A = \pi r^2$		

2. Every dimension of a rectangular prism with length 12 cm, width 3 cm, and height 9 cm is multiplied by 1/3 to get a similar rectangular prism. How is the ratio of the volumes related to the ratio of the corresponding dimensions?

	Prism A	Prism B
$V = lwh$		

3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides?

	Rectangle A	Rectangle B
$P = 2l + 2w$		

1.10 Precision & Accuracy

A ______________ is the level of detail in a measurement and is determined by the smallest unit or fraction of a unit that you can reasonably measure.

The _______________ of a measurement is the closeness of a measured value to the actual or true value.

________________ describes the amount by which a measurement is permitted to vary from a specified value.

Choose the more precise measurement in each pair.

A. 0.8 km; 830.2 m **B.** 2.45 in.; 2.5 in. **C.** 100 cm; 1 m

D. 2 lb; 17 oz. **E.** 7.85 m; 7.8 m. **F.** 6 kg; 6000 g.

Ida works in a deli. She is testing the scales at the deli to make sure they are accurate. She uses a weight that is exactly 1 pound and gets the following results:

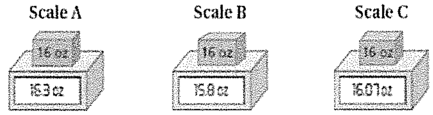
Scale 1: 1.019 lb
Scale 2: 1.01 lb
Scale 3: 0.98 lb

a. Which scale is the most precise?

b. Which scale is the most accurate?

A standard mass of 16 ounces is used to test three postal scales. The results are shown below.

a. Which scale is the most precise?



b. Which scale is the most accurate?

Bright Days Blinds makes window shades. The width of a 30-inch shade should be within 0.18 in. of 30 in. A batch of shades has the widths shown in the table.

Do all of the shades measure within the specified tolerance? If not, which shade(s) are not within the specified tolerance?

Shade	Width (in.)
A	30.06
B	29.75
C	29.84
D	30.12
E	29.93

A lacrosse ball must weigh 5.25 oz ± 0.25 oz. The weights of the lacrosse balls in one box are given in the table. Do all of the lacrosse balls weigh within the specified tolerance? If not, which lacrosse ball(s) are not within the specified tolerance?

Ball	Weight (oz)
A	5.41
B	5.23
C	5.54
D	5.33
E	5.21

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

A. 12 lb ± 3%   **B.** 15 oz ± 1.5%   **C.** 3 m ± 0.2%

D. 4.1 in. ± 5%   **E.** 475 m ± 2.5%   **F.** 85 mg ± 0.5%