4.1 Factors and Prime Factorization

Objective: Write the prime factorization of a number.

Example 1: Writing Factors

1. A rectangle has an area of 18 square feet. Find all possible whole number dimensions of the rectangle.

2. Write all of the factors of the following numbers.

   a. 28

   b. 48

   c. 49

   d. 15

   e. 24

   f. 39

   g. 40

A _____ number is a whole number that is greater than 1 and has exactly two whole number factors, 1 and itself.

A _____ number is a whole number that is greater than 1 and has more than two whole number factors.

When you write a number as a product of prime numbers, you are writing its _____.

Example 2: Writing a Prime Factorization

Tell whether the number is prime or composite. If it is composite, write its prime factorization by completing a factor tree.

a. 420 b. 97 c. 117 d. 50

A _____ is a number, a variable, or the product of a number and one or more variables raised to whole number powers.

Example 3: Factoring a Monomial

Factor the given monomial.

a. $24x^4y$ b. $21n^5$ c. $30xy^2$ d. $18x^2y^3$

4.2 Greatest Common Factor

Objective: Find the greatest common factor of two or more numbers.

The _____ is the greatest whole number that is a factor of two or more nonzero whole numbers.

Example 1: Finding the Greatest Common Factor

1. A high school asks for volunteers to help clean up local highways on one Saturday each month. The group of volunteers has 27 freshman, 18 sophmores, 36 juniors, and 45 seniors. What is the greatest number of groups that can be formed if each group is to have the same number of each type of student? How many freshman, sophomores, juniors, and seniors will be in each group?

2. Find the greatest common factor (GCF) of the numbers.

   a. 54, 81

   b. 12, 48, 66

   c. 35, 20

Two or more numbers are _____ if their greatest common factor is 1.

Example 2: Identifying Relatively Prime Numbers

1. Find the greatest common factor of the numbers. Then tell whether the numbers are relatively prime.

a. 28, 63

b. 42, 55

c. 30, 49

d. 52, 78

Example 3: Finding the GCF of Monomials

1. Find the greatest common factor of the monomials.

a. $16x^2y$, $26x^2y^3$

b. $27y$, $15y^5$

c. $12x^3$, $18x^2$

d. $40xy^3$, $24xy$

4.3 Equivalent Fractions

Objective: Write equivalent fractions.

Equivalent Fractions

Words To write equivalent fractions, multiply or divide the numerator and the denominator by the same nonzero number.

Algebra For all numbers a, b, and c, where $b \neq 0$ and $c \neq 0$, $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$.

Numbers $\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$ $\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$

Example 1: Write Equivalent Fractions

1. Write two fractions that are equivalent to the following numbers.

a. $\frac{6}{18}$

b. $\frac{7}{14}$

c. $\frac{4}{16}$

d. $\frac{10}{25}$

e. $\frac{1}{2}$

f. $\frac{2}{3}$

Example 2: Write Fractions in Simplest Form

1. Write the following fractions in simplest form.

a. $\frac{3}{18}$

b. $\frac{12}{32}$

c. $\frac{8}{36}$

d. $\frac{24}{42}$

e. $\frac{20}{30}$

f. $\frac{10}{25}$

Example 3: Simplifying a Variable Expression

1. Write the following expressions in simplest form.

a. $\frac{14x^2y}{35x^3}$

b. $\frac{9a}{15a^2}$

c. $\frac{16mn^2}{28n}$

d. $\frac{39st^2}{3s^2t}$

4.4 Least Common Multiple

Objective: Find the least common multiple of two numbers.

A _____ of a number is the product of the number and any nonzero whole number. A multiple that is shared by two or more numbers is a common multiple. The least of the common multiples of two or more numbers is the _____. The _____ of two or more fractions is the least common multiple of the denominators.

Example 1: Find the Least Common Multiple

1. Find the least common multiple of the following numbers.

   a. 6 and 14

   b. 8 and 18

   c. 4, 5, and 15

Example 2: Find the Least Common Multiple of Monomials

1. Find the least common multiple of the following monomials.

   a. $6xy$ and $16x^2$

   b. $12x$ and $18x^2$

   c. $4xy$ and $10xz^2$

Example 3: Comparing Fractions Using the LCD

1. Last year, a summer resort had 165,000 visitors, including 44,000 water skiers. This year, the resort had 180,000 visitors, including 63,000 water skiers. In which year was the fraction of water skiers greater?

Example 4: Ordering Fractions from Least to Greatest

1. Order the numbers from least to greatest.

   a. $4\frac{5}{12}$, $\frac{9}{2}$, and $\frac{33}{8}$

   b. $2\frac{7}{12}$, $\frac{24}{9}$, and $\frac{13}{6}$

4.5 Rules of Exponents

Objective: Multiply and divide powers.

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Algebra $a^m \cdot a^n = a^{m+n}$

Numbers $4^3 \cdot 4^2 = 4^{\square} = 4^{\square}$

Example 1: Using the Product of Powers Property

1. Find the product. Write your answer using exponents.

a. $2^5 \cdot 2^{12}$ b. $4^7 \cdot 4^{11}$ c. $(0.4)^6 \cdot (0.4)^2 \cdot (0.4)^3$

d. $2x^2 \cdot 7x^6$ e. $4y \cdot 2y^4$ f. $10a^{11} \cdot 20a^7$

g. $x^6 \cdot x^{13}$ h. $b^2 \cdot b^4 \cdot b$ i. $\left(\frac{1}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^4$

j. $3x^2 \cdot 7x^6$ k. $s^3 \cdot 9s^3$ l. $y \cdot 2y$

Quotient of Powers Property

Words To divide powers with the same base, subtract the exponent of the denominator from the exponent of the numerator.

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Numbers $\frac{5^7}{5^4} = 5^{\square} = 5^{\square}$

Example 2: Using the Quotient of Powers Property

1. Find the quotient. Write your answer using exponents.

a. $\frac{(0.6)^8}{(0.6)^3}$

b. $\frac{5^9}{5^2}$

c. $\frac{(1.4)^7}{(1.4)^4}$

d. $\frac{4x^{13}}{24x^9}$

e. $\frac{14x^{16}}{6x^{11}}$

f. $\frac{2a^8}{12a^4}$

h. $\frac{(2.2)^4}{(2.2)^2}$

i. $\frac{10^2}{10}$

j. $\frac{99^{10}}{99^3}$

Example 3: Using Both Properties of Powers

1. Simplify.

a. $\frac{4m^3m^4}{12m^2}$

b. $\frac{6m^5m}{15m^3}$

c. $\frac{10n^2n^6}{5n^3}$

4.6 Negative and Zeros Exponents

Objective: Work with negative and zero exponents.

Negative and Zero Exponents

For any nonzero number a, $a^0 = 1$.

For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

Example 1: Powers with Negative and Zero Exponents

1. Write the expression using only positive exponents.

a. 4^{-3}

b. $m^{-5}n^0$

c. $13xy^{-8}$

d. $33{,}333^0$

e. 7^{-3}

f. $2z^{-2}$

g. $3x^{-4}y$

h. $5x^0y^{-1}$

Example 2: Rewriting Fractions

1. Write the expression without using a fraction bar.

a. $\frac{1}{15}$

b. $\frac{a^3}{c^5}$

c. $\frac{1}{18}$

d. $\frac{1}{100}$

e. $\frac{3}{c^2}$

f. $\frac{x^5}{y^7}$

g. $\frac{x^0}{4}$

g. $\frac{3}{x^0y^2}$

Example 3: Using Powers Properties with Negative Exponents

1. Find the product or quotient. Write your answer using only positive exponents.

a. $6^{12} \cdot 6^{-4}$

b. $\frac{0.7n^{-4}}{n}$

c. $(0.3)^{10} \cdot (0.3)^{-7}$

d. $\frac{7d^{-4}}{d^2}$

e. $10^1 \ 10^{-3}$

f. $(1.4)^5 \ (1.4)^{-7}$

4.7 Scientific Notation

Objective: Write numbers using scientific notation.

Using Scientific Notation

A number is written in **scientific notation** if it has the form $c \times 10^n$ where $1 \leq c < 10$ and n is an integer.

Standard form	Product form	Scientific notation
725,000	$7.25 \times 100{,}000$	7.25×10^5
0.006	6×0.001	6×10^{-3}

Example 1: Writing Numbers in Scientific Notation

1. Write the numbers in scientific notation.

   a. The average distance Mars is from the sun is 141,600,000 miles.

   b. The diameter of a quarter (American Eagle coin) is 0.022 meters.

2. Write the numbers in scientific notation.

   a. 3,050,000,000

   b. 0.000082

Example 2: Writing Numbers in Standard From

1. Write the numbers in standard form.

   a. 4.1×10^4

   b. 7.23×10^{-6}

   c. 6.53×10^7

   d. 9.2×10^{-4}

Example 3: Ordering Numbers Using Scientific Notation

1. Order 5.3×10^5, 520,000, and 7.5×10^4 from least to greatest.

2. Order the numbers from least to greatest.

   a. 23,000, 3.4×10^3, 2.2×10^4

   b. 4.5×10^{-4}, 0.000047, 4.8×10^{-5}

Example 4: Multiplying Numbers in Scientific Notation

1. The volume of one mole of oxygen atoms is about 1.736×10^{-5} cubic meters. Find the volume of 1.5×10^4 of oxygen atoms.

2. Find the product. Write your answer in scientific notation.

   A. $(2.5 \times 10^3)(2 \times 10^5)$

   B. $(1.5 \times 10^{-2})(4 \times 10^{-4})$