8.1 Relations and Functions

Objective: Use graphs to represent relations and functions.

A ___________________ is a set of ordered pairs.
The __________________ of a relation is the set of all inputs, x-values.
The _________________ of a relation is the set of all outputs, y-values.
Each number in a domain is an ______________.
Each number in a range is an ______________.
A ________________ is a relation with the property that for each input their is exactly one output.
The ______________________________ says that if you can find a vertical line passing through more than one point of a graph of a relation, then the relation is not a function. Otherwise, the relation is a function.

Example 1: Identifying the Domain and Range

Identify the domain and range of the relation.

1. The table below that shows one Norway Spruce tree's height at different ages.

Age (years), *x*	5	10	15	20	25
Height (ft), *y*	13	25	34	43	52

2. (-5, 2), (-3, -1), (-1, 0), (2, 3), (5, 4)

Check It Out!

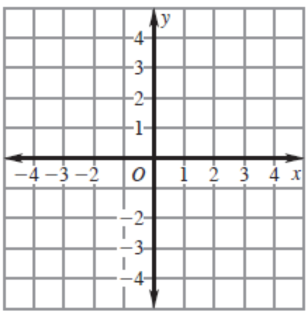
Identify the domain and range of the relation.

1. (-4, -3), (-3, 2), (0, 0), (1, -1), (2, 3), (3, 1), (3, -2)

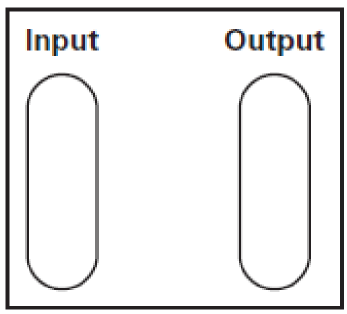
Example 2: Representing a Relation

Represent the relation (-3, 2), (-2, -2), (1, 1), (1, 3), (2, -3).

1. a graph



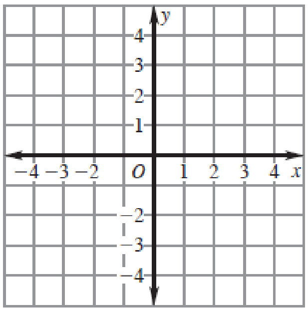
2. a mapping diagram



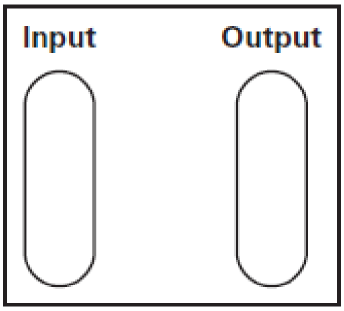
Check It Out!

Represent the relation (0, 1), (1, 2), (2, 3), (3, 4).

1. a graph



2. a mapping diagram



Example 3: Identifying Functions

Tell whether the relation is a function.

1. (-5, 2), (-3, -1), (-1, 0), (2, 3), (5, 4)

2. (-4, -3), (-3, 2), (0, 0), (1, -1), (2, 3), (3, 1), (3, -2)

Check It Out!

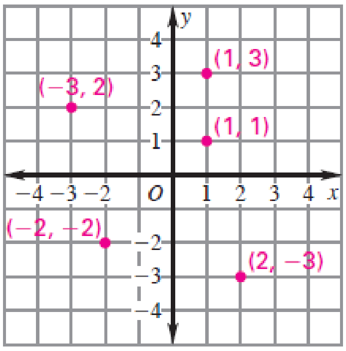
Tell whether the relation is a function.

1. (-3, 2), (-2, -2), (1, 1), (1, 3), (2, -3)

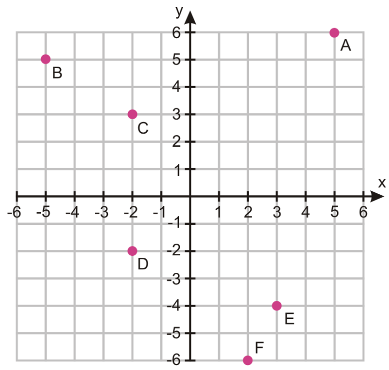
Example 4: Using the Vertical Line Test

Use the vertical line test to determine if the relation is a function.

1.



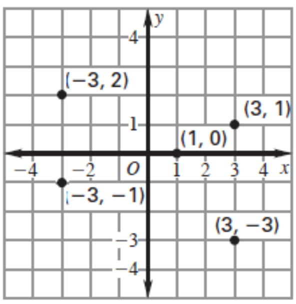
2.



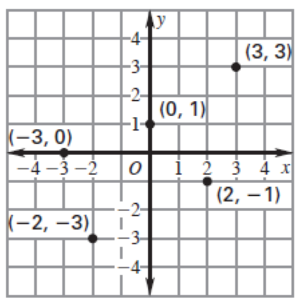
Check It Out!

Use the vertical line test to determine if the relation is a function.

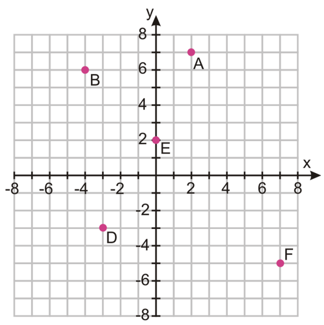
1.



2.



3.



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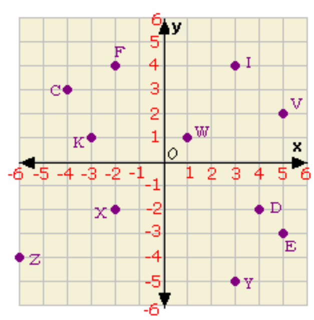


8.2 Linear Equations in Two Variables

Objective: Find solutions of equations in two variables.

An equation that contains two different variables is an _______________ __________________.

A ______________ of an equation in two variables in x and y is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the equation.

The _____________ of an equation in two variables is the set of points in a coordinate plane that represents all the solutions of the equation.

An equation whose graph is a line is called a _______________________.

A function whose graph is a nonvertical line is called a ____________ ______________.

An equation solved for y is in ________________________.

Example 1: Checking Solutions

Tell whether the ordered pair is a solution of the equation.

1. (5, -1); $x - 3y = 8$

Check It Out!

Tell whether the ordered pair is a solution of the equation.

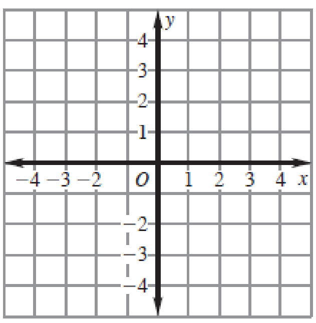
1. (0, -5); $2x - y = 5$

2. (3, 2); $2x - y = 5$

Example 2: Graphing a Linear Equation

Graph the following equations by making an xy chart.

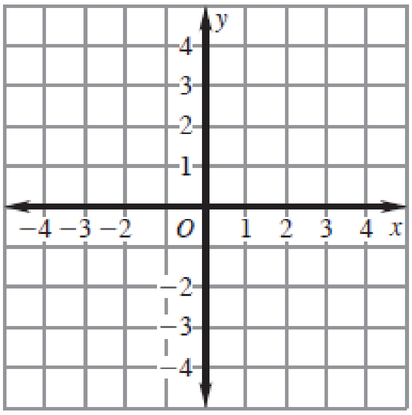
1. $y = -x + 1$



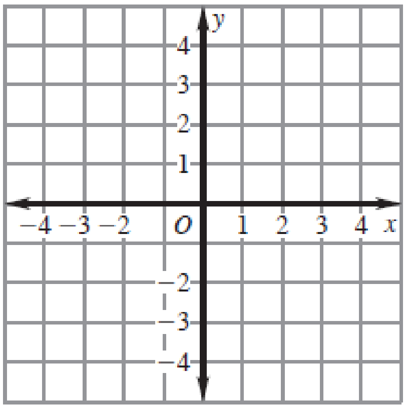
Check It Out!

Graph the following equations by making an xy chart

1. y = x + 2



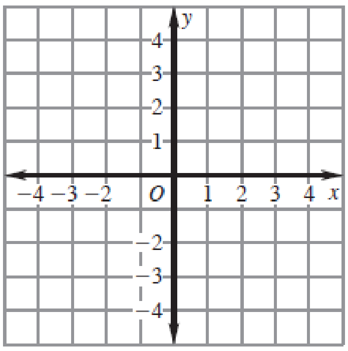
2. y = 3x - 3



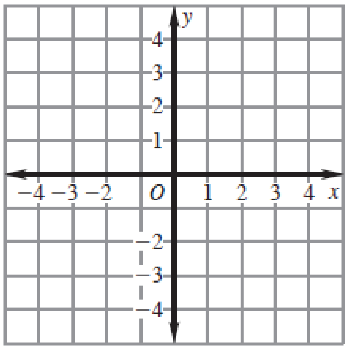
Example 3: Graphing Horizontal and Vertical Lines

Graph the following equations. Then tell whether the equation is a function.

1. y = -2



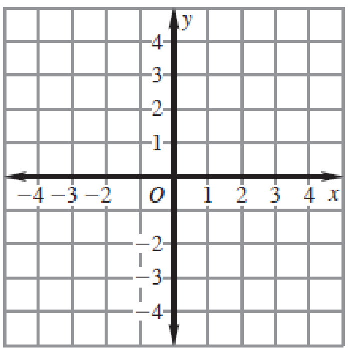
2. x = 3



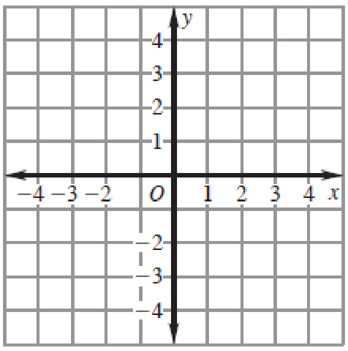
Check It Out!

Graph the following equations. Then tell whether the equation is a function.

1. y = 4



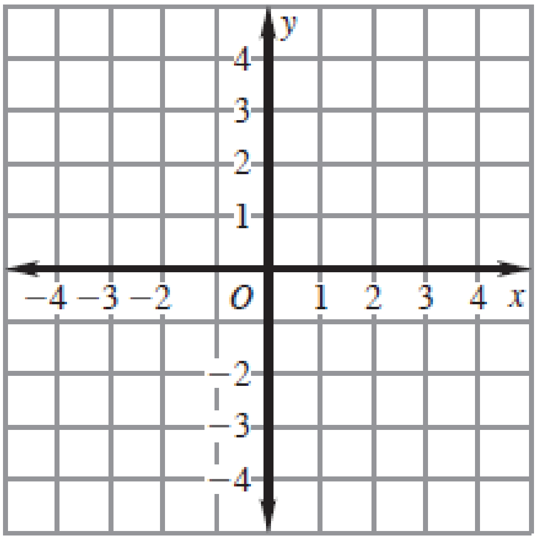
2. x = -3



Example 4: Writing an Equation in Function Form

Write the equation in function form. Then graph the following equations by making an xy chart.

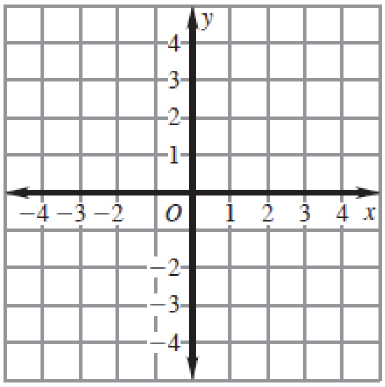
1. $3x - y = 2$



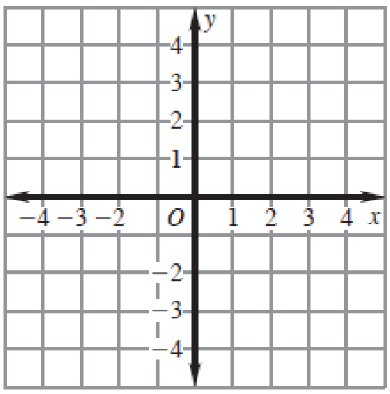
Check It Out!

Write the equation in function form. Then graph the following equations by making an xy chart.

1. $x - 2y = 4$



2. $y - x = -1$



8.3 Using Intercepts

Objective: Use decimals to solve percent problems.

The x-coordinate of a point where a graph crosses the x-axis is an __________________. The y-coordinate of a point where a graph crosses the y-axis is an __________________.

Finding Intercepts To find the *x*-intercept of a line, substitute ☐ for *y* in the line's equation and solve for ☐. To find the *y*-intercept of a line, substitute ☐ for *x* in the line's equation and solve for ☐.

Example 1: Finding the Intercepts of a Graph

Find the intercepts of the graph.

1. $2x - 5y = -10$

2. $3x - 2y = 6$

Check It Out!

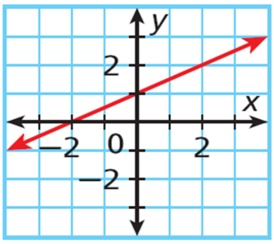
Find the intercepts of the graph.

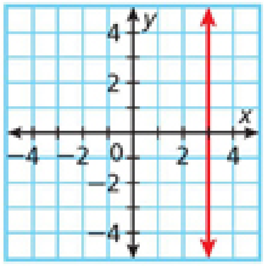
1. $2x + 3y = 6$

2. $3x - 6y = 12$

Example 2: Identifying Intercepts on a Graph

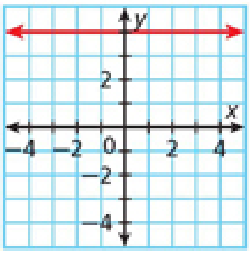
Identify the x-intercept and y-intercept of each graph.

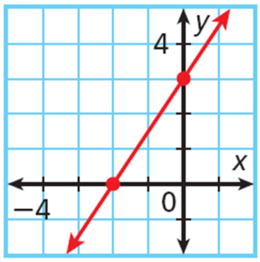
1. 

2. 

Check It Out!

Identify the x-intercept and y-intercept of each graph.

1. 

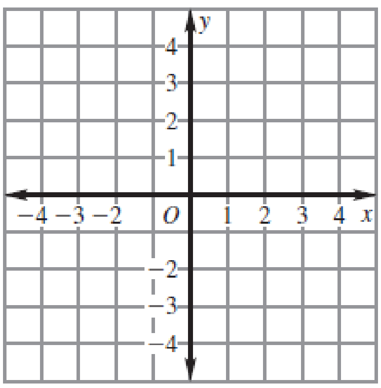
2. 

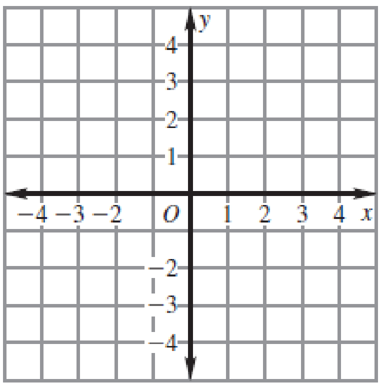
Example 3: Using Intercepts to Graph a linear Equation

Find the intercepts of the graph. Graph the equations using the intercepts.

1. $x - 2y = -2$

2. $4x + 3y = 12$



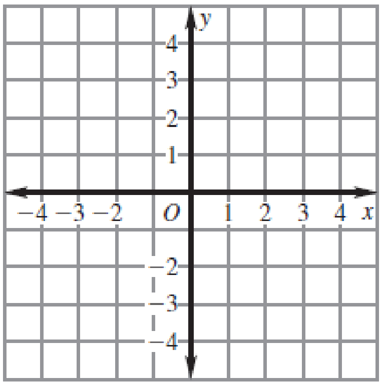


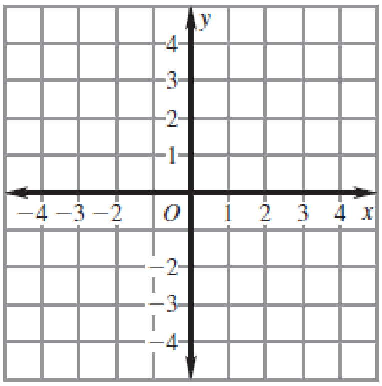
Check It Out!

Find the intercepts of the graph. Graph the equations using the intercepts.

1. $x - 2y = 4$

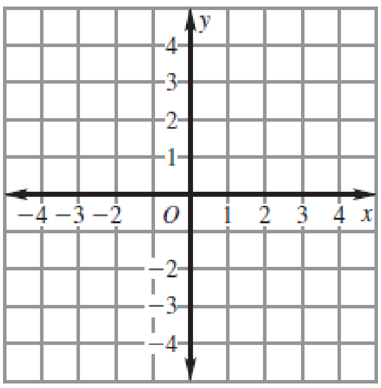
2. $2x + y = 2$





Example 4: Writing and Graphing an Equation

1. You run and walk on a fitness trail that is 12 miles long. You can run 6 miles per hour and walk 3 miles per hour. Write and graph an equation describing your possible running and walking times on the fitness trail. Give three possible combinations of running and walking times.



8.4 The Slope of a Line

Objective: Find and interpret slopes of lines.

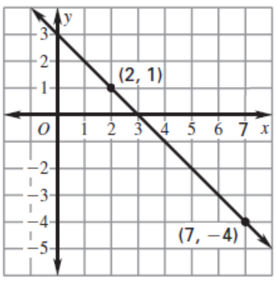
The _______________ of a line is the ratio of the line's vertical change to its horizontal change. The line's vertical change between two points is called its _____________. A line's horizontal change between two points is called its ____________. The slope of a line is the rise over the run.

Example 1: Finding Slope

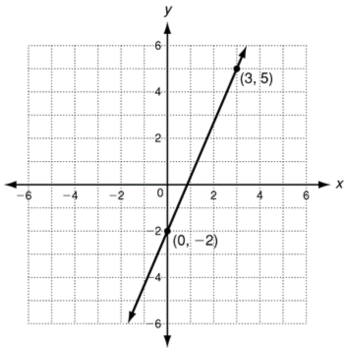
1. A building's access ramp has a rise of 2 feet and a run of 24 feet. Find its slope.

Find the slope of each line:

2.



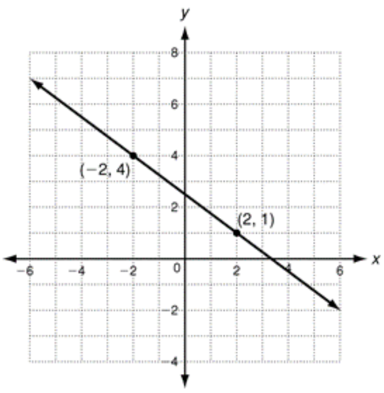
3.



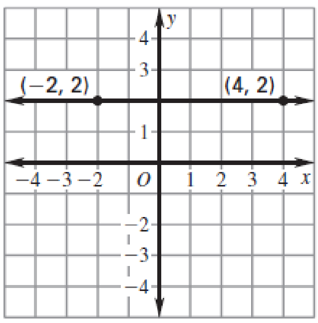
Check It Out!

Find the slope of each line:

1.



2.



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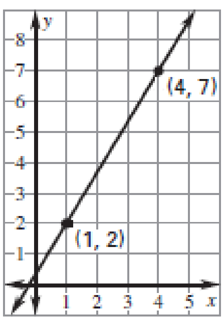

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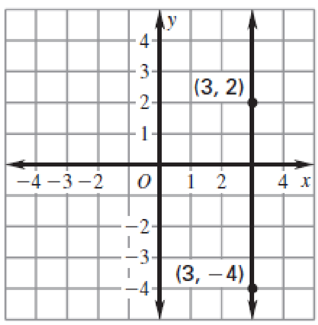
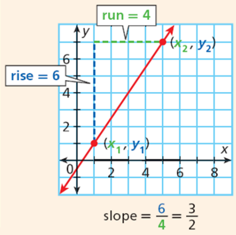


Slope Formula:

Slope of a Line

DEFINITION	EXAMPLE
The **rise** is the difference in the *y*-values of two points on a line.	
The **run** is the difference in the *x*-values of two points on a line.	
The **slope** of a line is the ratio of the rise to run. If (x_1, y_1) and (x_2, y_2) are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	

Example 2: Finding Positive and Negative Slope

Find the slope of the line containing these two points.

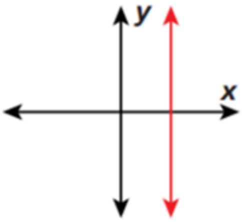
1. (0, 2) and (-2, 3)
2. (2, -2), (0, 4)

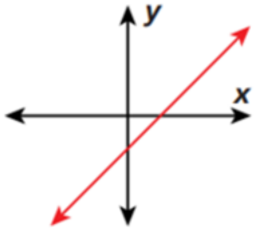
Check It Out!

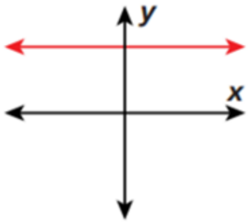
Find the slope of the line containing these two points.

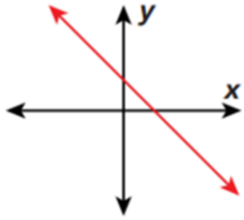
1. (1, 6) and (3, 10)
2. (7, 5), (3, 2)
3. (-2, 10), (-4, -4)
4. (-2, 4), (6, 2)

Tell whether the slope is *positive*, *negative*, *zero* or *undefined*.
Remember a fraction with zero in the denominator is undefined because it is impossible to divide by zero.









______________ ______________ ______________ ______________

Example 3: Zero and Undefined Slope

Find the slope of the containing these two points. Tell whether the slope is positive, negative, zero, or undefined.

1. (-2, 7) (3, 7)

2. (3, -1), (3, 5)

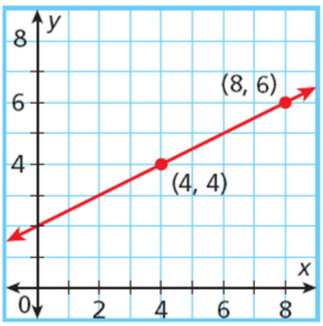
Check It Out!

Find the slope of the line shown or through the given points. Tell whether the slope is positive, negative, zero, or undefined.

1. (-10, 2) (-10, -2)

2. (1, -1), (7, -1)

3.



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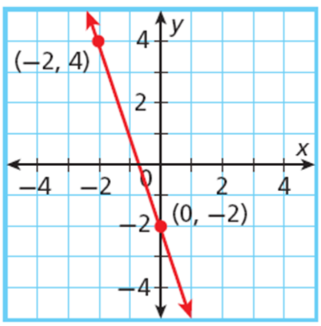


8.5 Slope-Intercept Form

Objective: Graph linear equations in slope-intercept form.

Slope-Intercept Form:

Slope-Intercept Form

Words A linear equation of the form $y = mx + b$ is said to be in **slope-intercept form**. The ______ is m and the ______ is b.

Algebra $y = mx + b$ **Numbers** $y = 2x + 3$

Example 1: Identifying the Slope and y-intercept

Identify the slope and y-intercept in the equation of the line.

1. $y = 2x - 3$
2. $4x + 3y = 9$
3. $y = -3x - 4$
4. $x - 2y = 10$

Check It Out!

Identify the slope and y-intercept in the equation of the line.

1. $y = x - 5$
2. $4x - 2y = -16$
3. $y = -6x$
4. $x - 2y = 6$

Example 2: Writing an equation

Write the equation that describes the line in slope-intercept form.

1. slope = $\frac{1}{4}$; y-intercept = 4
2. slope = -9 ; y-intercept = $-\frac{5}{4}$
3. slope = 1 ; y-intercept = 0
4. slope = 2 ; (3, 4) is on the line

Check It Out!

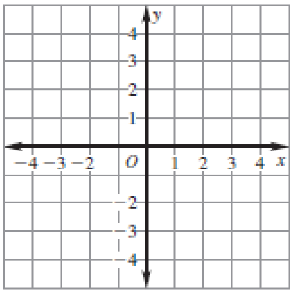
Write the equation that describes the line in slope-intercept form.

1. slope = 0 ; y-intercept = 1
2. slope = 3 ; y-intercept = -1
3. slope = -12 ; y-intercept = $-\frac{1}{2}$
4. slope = 8 ; (-3, 1) is on the line

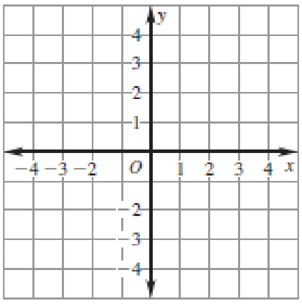
Example 3: Graphing an Equation in Slope-Intercept Form

Graph the equation.

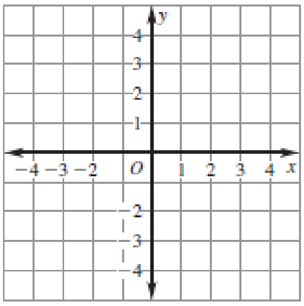
1. $y = -\frac{3}{4}x + 2$



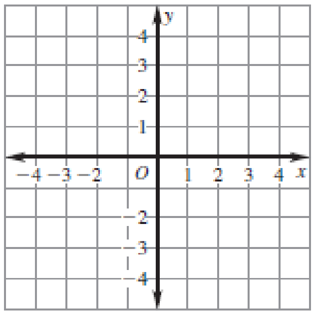
2. $y = -5x - 4$



3. $2x - 3y = 6$



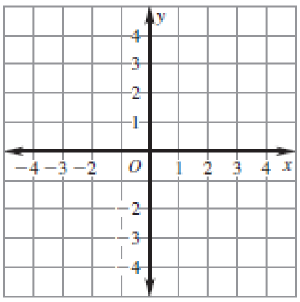
4. $3x - 2y = 6$



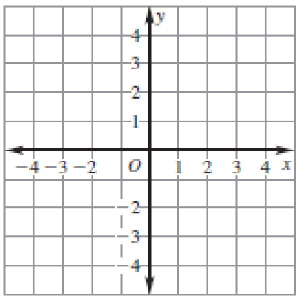
Check It Out!

Graph the equation.

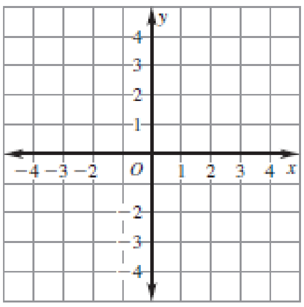
1. $y = -\frac{2}{3}x + 4$



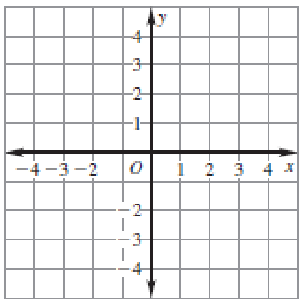
2. $y = -x + 1$



3. $y = 4x$



4. $3x + y = -1$

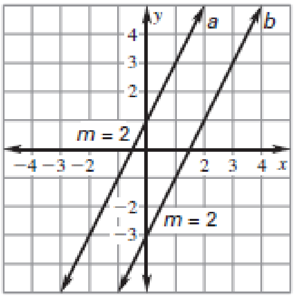


If m is any nonzero number, then the negative reciprocal of m is $-\frac{1}{m}$. Note that the product of a number and its negative reciprocal is -1:

$$m\left(-\frac{1}{m}\right) = -1$$

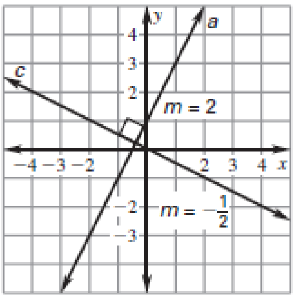
Slopes of Parallel and Perpendicular Lines

Two nonvertical parallel lines have ______. For example, the parallel lines a and b below ______.



$a \parallel b$

Two nonvertical perpendicular lines, such as lines a and c below, have slopes that are ______.



$a \perp c$

Example 4: Finding Slopes of Parallel or Perpendicular Lines

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

1. $y = -\frac{3}{4}x + 2$
2. $2x - 3y = 6$

Check It Out!

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

1. $y = -5x - 4$
2. $10x + 5y = 15$

8.5.5 Point-Slope Form

Objective: Graph linear equations in point-slope form.

Point-Slope Form:

Point-Slope Form of a Linear Equation

The line with slope m that contains the point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

Example 1: Identifying the Slope and Point

Identify the slope and point in equation of the line.

1. $y - 2 = -7(x + 1)$
2. $y + 3 = -(x + 1)$

Check It Out!

Identify the slope and point in equation of the line.

1. $y - 10 = \frac{1}{4}(x - 6)$
2. $y + 5 = 2x$

Example 2: Writing an equation

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. Slope = $\frac{1}{6}$; (5, 1)
2. Slope = -4 ; (0, 3)

Check It Out!

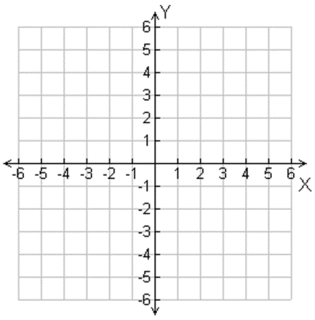
Write an equation in point-slope form for the line with the given slope that contains the given point.

1. Slope = 1 ; (-1, -4)
2. Slope = 0 ; (3, -4)

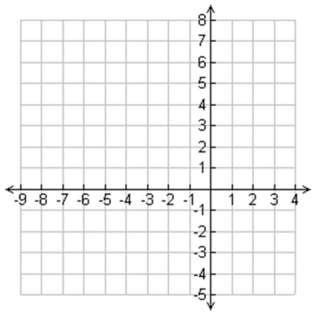
Example 3: Graphing an Equation in Point-Slope Form

Graph the line described by the equation.

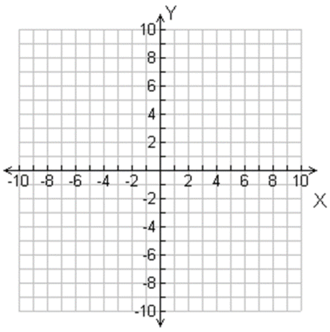
1. $y - 1 = 2(x - 3)$



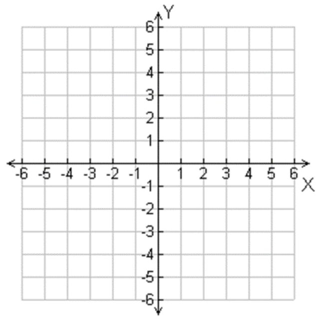
2. $y - 4 = \frac{3}{4}(x + 2)$



3. $y + 3 = 0(x - 4)$



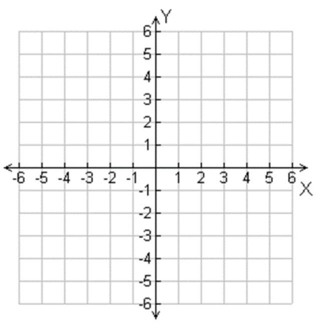
4. $y - 3 = -2(x + 4)$



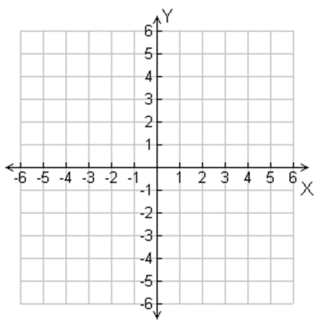
Check It Out!

Graph the line described by the equation.

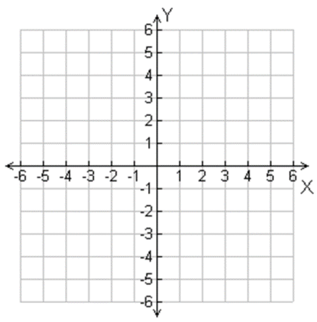
1. $y - 1 = -\frac{2}{3}(x + 2)$



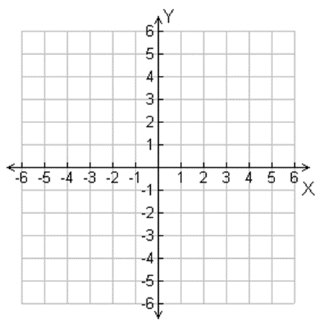
2. $y + 4 = 3(x + 4)$



3. $y = \frac{1}{2}(x - 3)$



4. $y - 1 = -(x + 1)$



8.6 Writing Linear Equations

Objective: Write linear equations.

The ________________________ is the line that lies as close as possible to the points in a data set.

Example 1: Writing an Equation

Write an equation of the line with the given information in the given form.

1. slope of -2 ; y-intercept of -5
   slope-intercept form

2. slope of $\frac{1}{2}$; through (-3, -3)
   point-slope form

3. slope of 1 ; through (-1, 2)
   point-slope form

4. slope of 4 ; y-intercept of -3
   slope-intercept form

Check It Out!

Write an equation of the line with the given information in the given form.

1. slope of 1 ; y-intercept of -2
   slope-intercept form

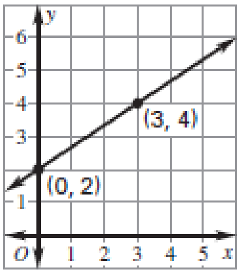
2. slope of 0 ; through (2, 4)
   point-slope form

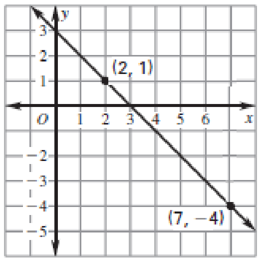
3. slope of -10 ; through (0, 7)
   point-slope form

4. slope of -6 ; y-intercept of 3
   slope-intercept form

Example 2: Writing an Equation of a Graph

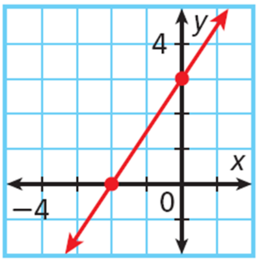
Write an equation of the line shown.

1. 

2. 

Check It Out!

Write an equation of the line shown.

1. 

Example 3: Writing an Equation in Slope-Intercept Form

Write the equation in slope-intercept form. Then identify the slope and y-intercept.

1. $y - 3 = -2(x + 4)$

2. $6x + 2y = 10$

Check It Out!

Write the equation in slope-intercept form. Then identify the slope and y-intercept.

1. $y - 10 = 2(x - 1)$

2. $2y - 4x = 16$

8.7 Function Notation

Objective: Use Function Notation.

An equation written in ___________________________ uses f(x) to represent the output of the function *f* for an input of *x*.

Example 1: Working with Function Notation

Let f(x) = 2x - 5. Find the missing value with the given information.

1. x = -3 ; Find f(x)
2. f(x) = 13 ; Find x

Check It Out!

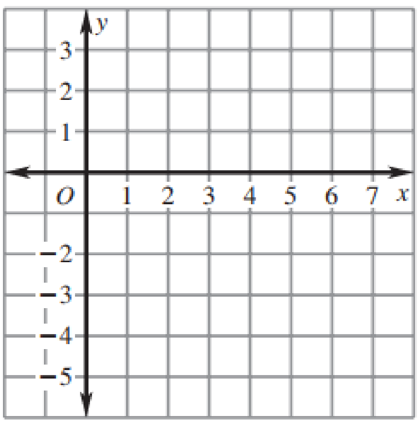
Let f(x) = -x + 7. Find the missing value with the given information.

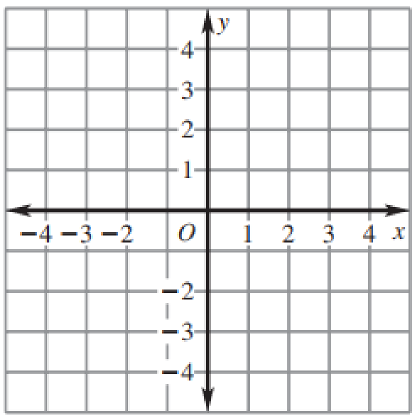
1. x = 4 ; Find f(x)
2. f(x) = 9 ; Find x

Example 2: Graphing a function

Graph the function.

1. $f(x) = \frac{5}{6}x - 3$
2. $g(x) = -\frac{2}{3}x + 2$

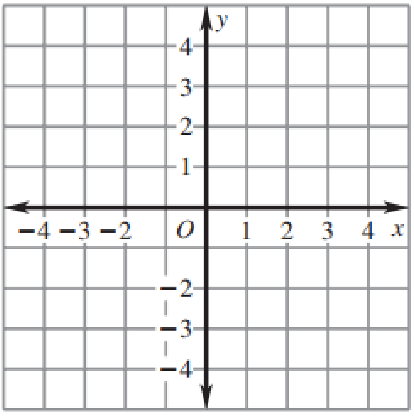




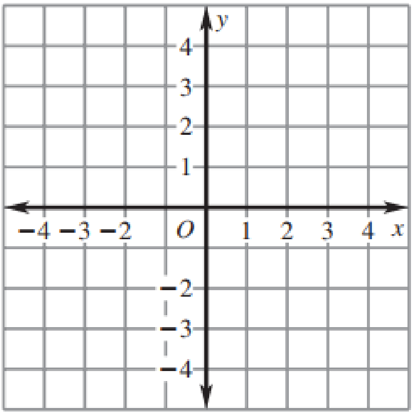
Check It Out!

Graph the function.

1. $h(x) = \frac{3}{2}x - 1$



2. $f(x) = 4x - 5$



Example 3: Using Function Notation in real life

You ride your bike at a speed of 12 miles per hour. Use function notation to write an equation giving the distance traveled as a function of time. How long will it take you to travel 30 miles ?

8.8 Systems of Linear Equations

Objective: Graph and solve systems of linear equations.

A _______________ of linear equations consists of two or more linear equations with the same variables.

A _________________ of a linear system in two variables is an ordered pair that is a solution of each equation in the system.

Example 1: Solutions of systems of equations

Determine whether the given ordered pair is a solution to the systems of equations.

1. (0, 3); $\begin{cases} y = 3x + 3 \\ y = -3x + 3 \end{cases}$

2. (3, -2); $\begin{cases} y = -4x + 10 \\ y = \frac{1}{3}x + 3 \end{cases}$

Check It Out!

Determine whether the given ordered pair is a solution to the systems of equations.

1. (2, 4); $\begin{cases} y = 4x - 4 \\ y = \frac{1}{2}x + \frac{3}{2} \end{cases}$

2. (-2, 1); $\begin{cases} y = 2x + 5 \\ y = -x - 1 \end{cases}$

Example 2: Solving a System of Linear Equations

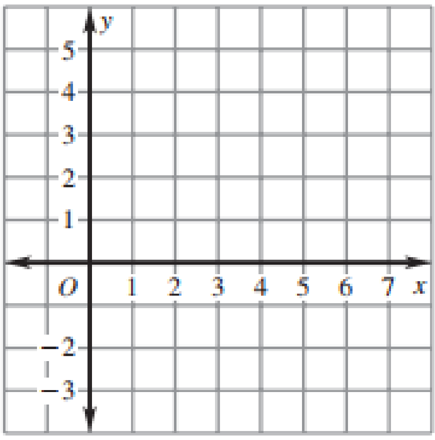
Solve the linear equations by graphing.

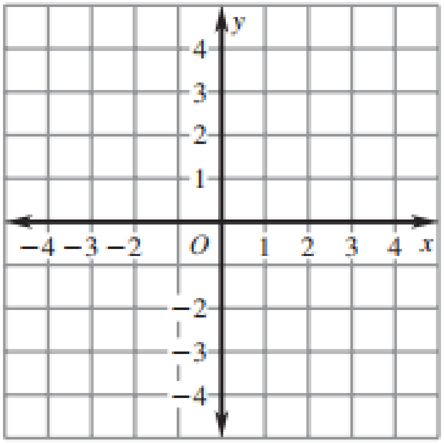
1. $y = x - 3$

   $y = -\frac{1}{5}x + 3$

2. $y = -x + 3$

   $y = \frac{1}{3}x - 1$





Check It Out!

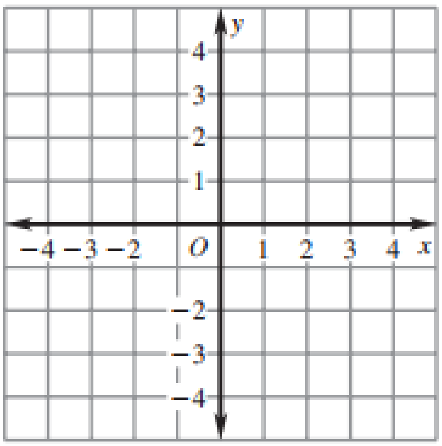
Solve the linear equations by graphing.

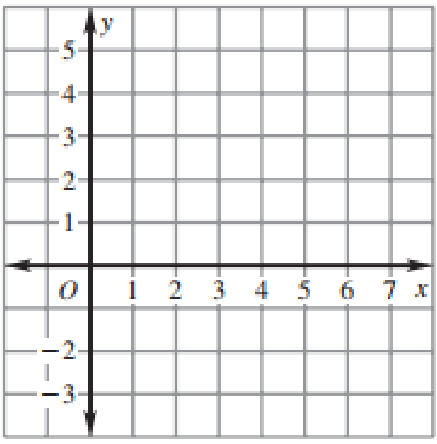
1. $y = -4x + 1$

   $y = 2x - 5$

2. $y = 2x - 4$

   $y = -3x + 6$





8.9 Graphs of Linear Inequalities

Objective: Graph inequalities in two variables.

A ____________________________ in two variables is the result of replacing the equal sign in a linear equation with $<, >, \leq, or \ \geq$.

The _______________ of a linear inequality is an ordered pair (x, y) that makes the inequality true when the values of x and y are substituted into the inequality.

The ____________ of a linear inequality in two variables is the set of points in a coordinate plane that represent the inequality's solutions.

Example 1: Checking Solutions of a Linear Inequality

Tell whether the ordered pair is a solution of 3x - y$>$ 2.

1. (3, 0)

2. (-1, 5)

Check It Out!

Tell whether the ordered pair is a solution of -x + 2y$>$ 4.

1. (1, 6)

2. (-7, -2)

3. (2, 3)

4. (0, 5)

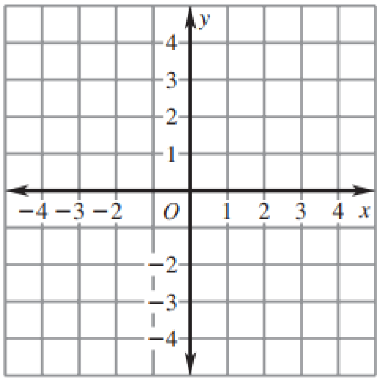
Graphing Linear Inequalities

1. Find the equation of the boundary line by replacing the inequality symbol with =. Graph this equation. Use a dashed line for $<$ or $>$. Use a solid line for $\le$ or $\ge$.
2. Test a point in one of the half-planes to determine whether it is a solution of the inequality.
3. If the test point is a solution, shade the half-plane that contains the point. If not, shade the other half-plane.

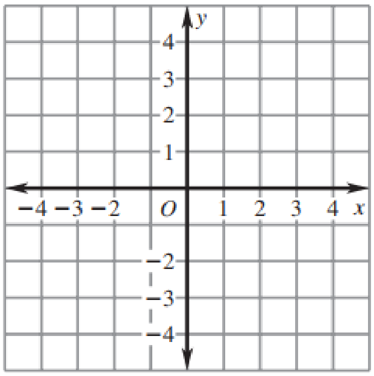
Example 2: Graphing a Linear Inequality

Graph the inequalities in the coordinate plane.

1. $y \ge -x + 1$



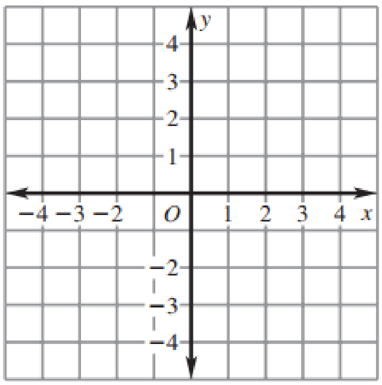
2. $y > 2x - 3$



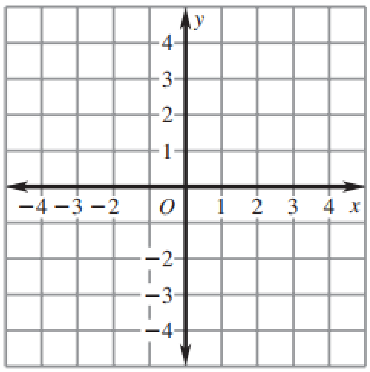
Check It Out!

Graph the inequalities in the coordinate plane.

1. $y \ge 2x + 4$



2. $y \le \frac{3}{4}x - 5$



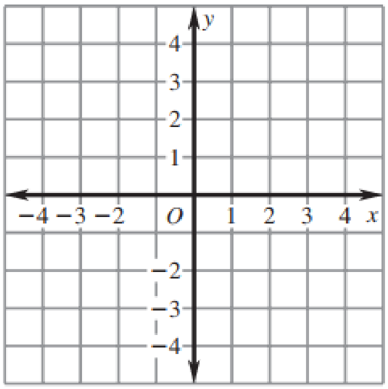
x = # Vertical Line

y = # Horizonal Line

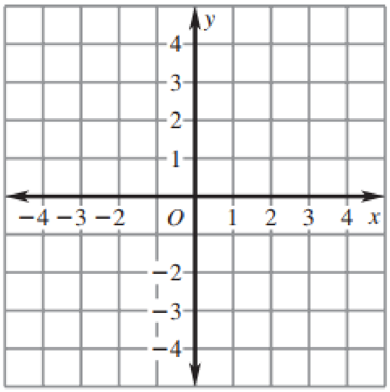
Example 3: Graphing Inequalities with One Variable

Graph the inequalities in the coordinate plane.

1. $x > -2$



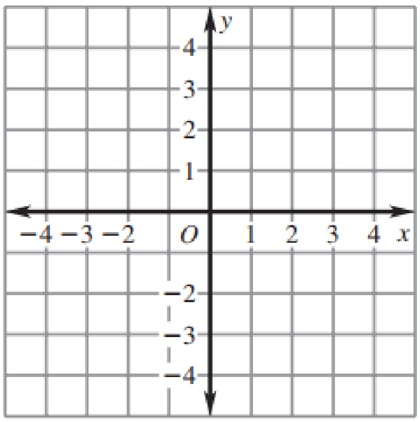
2. $y \leq 3$



Check It Out!

Graph the inequalities in the coordinate plane.

1. $x \leq 4$



2. $y < 1$