

# Chapter 2: Solving Equations

## 2.1 Properties and Operations

**Objective:** Use properties of addition and multiplication.

| <b>Commutative and Associative Properties</b>  |  |
|--|--|
| <b>Commutative Property of Addition</b><br><b>Words</b> In a sum, you can add the numbers in any order.<br><b>Numbers</b> $4 + (-7) = -7 + 4$<br><b>Algebra</b> $a + b = b + a$  | <b>Commutative Property of Multiplication</b><br><b>Words</b> In a product, you can multiply the numbers in any order.<br><b>Numbers</b> $8(-5) = -5(8)$<br><b>Algebra</b> $ab = ba$   |
| <b>Associative Property of Addition</b><br><b>Words</b> Changing the grouping of the numbers in a sum does not change the sum.<br><b>Numbers</b><br>$(9 + 6) + 2 = 9 + (6 + 2)$<br><b>Algebra</b><br>$(a + b) + c = a + (b + c)$ | <b>Associative Property of Multiplication</b><br><b>Words</b> Changing the grouping of the numbers in a product does not change the product.<br><b>Numbers</b><br>$(3 \cdot 10) \cdot 4 = 3 \cdot (10 \cdot 4)$<br><b>Algebra</b><br>$(a \cdot b) \cdot c = a \cdot (b \cdot c)$ |

### Example 1: Using Properties of Addition

This week, you rode in a car for 42 miles, rode a bike for 5 miles, and rode in a bus for 23 miles. Find the total distance.

Evaluate the expression when  $x = 7$  and  $y = 25$ .

1.  $(2x + y) + 46$

2.  $x + (2y + 7)$

3.  $(2y + 3x) + 12$

### Example 2: Using Properties of Multiplication

Evaluate the expression when  $x = -8$  and  $y = 15$ .

1.  $4xy$

2.  $4x^2y$

3.  $2xy$

### Example 3: Using Properties to Simplify Expressions

Simplify the expression.

1.  $x + 5 + 2$

2.  $3(9y)$

3.  $n + 6 + 7$

4.  $(4r)(-3)$

| Identity Properties   |   |
|---|---|
| <b>Identity Property of Addition</b><br><b>Words</b> The sum of a number and the <b>additive identity</b> , 0, is the number.<br><b>Numbers</b> $-6 + 0 = -6$<br><b>Algebra</b> $a + 0 = a$ | <b>Identity Property of Multiplication</b><br><b>Words</b> The product of a number and the <b>multiplicative identity</b> , 1, is the number.<br><b>Numbers</b> $4 \cdot 1 = 4$<br><b>Algebra</b> $a \cdot 1 = a$ |

### Example 4: Identify Properties

1.  $(3 + 2) + 4 = 3 + (2 + 4)$

4.  $cd = dc$

2.  $0 + b = b$

5.  $(2 \cdot 6) \cdot 3 = 2 \cdot (6 \cdot 3)$

3.  $1(-7) = -7$

6.  $q + (-r) = -r + q$

### Example 5: Use Conversion Factors

Use a conversion factor to perform the indicated conversions.

1. 180 mins to hrs

2. 10,560 miles to feet

3. 64 oz to lbs.

4. 255 mm to cm

## 2.2 The Distributive Property

**Objective:** Use the distributive property.

\_\_\_\_\_ are numerical expressions that have the same value.

\_\_\_\_\_ have the same value for all values of the variable(s).

| <b>The Distributive Property</b>    |  |
|-------------------------------------|--|
| <b>Algebra</b> $a(b + c) = ab + ac$ | <b>Numbers</b> $4(6 + 3) =$ <input type="text"/> |
| $(b + c)a = ba + ca$                | $(6 + 3)4 =$ <input type="text"/>                |
| $a(b - c) = ab - ac$                | $5(7 - 2) =$ <input type="text"/>                |
| $(b - c)a = ba - ca$                | $(7 - 2)5 =$ <input type="text"/>                |

### Example 1: Use the Distributive Property

You are buying beads for a craft project. You need gold, silver, and white beads. A bag of each type of bead costs \$3.99. Use the distributive property and mental math to find the total cost of the beads

Use the distributive property to evaluate the expression.

1.  $2(9 + 4)$

2.  $(12 - 3)3$

3.  $(4 - 11)(-4)$

4.  $-5(4 - 1)$

5.  $5(103)$

6.  $4(3.8)$

7.  $3(6.03)$

8.  $9(205)$

### Example 2: Writing Equivalent Variable Expressions

Use the distributive property to write an equivalent variable expression.

1.  $2(x + 10)$

2.  $(m + 3)(-4)$

3.  $-3(2y - 6)$

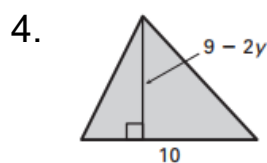
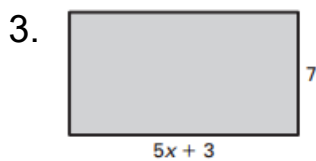
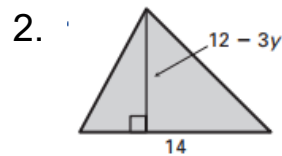
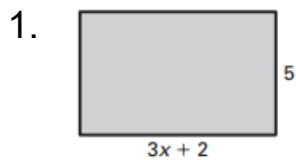
4.  $9(x - 2)$

5.  $(x + 7)4$

6.  $-3(4m - 7)$

### Example 3: Finding Areas of Geometric Figures

Find the area of the figure.



## 2.3 Simplifying Variable Expressions

**Objective:** Simplify variable expressions.

The parts of an expression that are added together are called \_\_\_\_\_.

The \_\_\_\_\_ with a variable is the number part of the term.

A \_\_\_\_\_ has a number but no variable.

\_\_\_\_\_ are terms that have identical variable parts.

### Example 1: Identify Parts of an Equation

Identify the terms, like terms, coefficients, and constant terms of the expressions.

1.  $5 - 2x - 3 + x$

terms: \_\_\_\_\_

like terms: \_\_\_\_\_

coefficients: \_\_\_\_\_

constant terms: \_\_\_\_\_

2.  $4y - 6 + 3y$

terms: \_\_\_\_\_

like terms: \_\_\_\_\_

coefficients: \_\_\_\_\_

constant terms: \_\_\_\_\_

3.  $9 + w - 5 - 8w$

terms: \_\_\_\_\_

like terms: \_\_\_\_\_

coefficients: \_\_\_\_\_

constant terms: \_\_\_\_\_

simplified expression: \_\_\_\_\_

2.  $3a - 2 + 5a + 9$

terms: \_\_\_\_\_

like terms: \_\_\_\_\_

coefficients: \_\_\_\_\_

constant terms: \_\_\_\_\_

simplified expression: \_\_\_\_\_

### Example 2: Simplifying an Expression

Simplify the expression.

1.  $5m + 8 - 3m - 7$

2.  $4y - 6 + 3y$

3.  $9 + w - 5 - 8w$

4.  $-6a^2 - 2 - 4a^2 - 10$

**Example 3: Simplifying an Expression with Parentheses**

Simplify the expression.

1.  $3(x + 2) - x + 9$

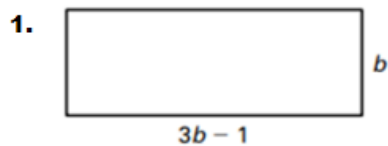
2.  $2k - 5(k + 4)$

3.  $5a - (5a - 7)$

4.  $4(x - 1) - 2x - 7$

**Example 4: Simplifying a Geometry Expression**

Write and simplify an expression for the perimeter of the triangle or rectangle.





### Example 3: Solving Equations Using Mental Math

Solve the equation using mental math. Check your answer.

1.  $x + 4 = 7$

2.  $12 - n = 5$

3.  $18 = 3t$

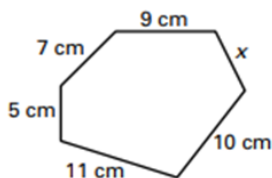
4.  $x - 8 = 10$

5.  $24 = 4m$

6.  $\frac{c}{3} = 9$

### Example 4: Finding the Side Length of Geometric Figures

The perimeter of the figure is 48 centimeters.



a. Write and simplify an equation that you can use to find  $x$ .

b. Solve your equation. What is the value of  $x$ ?



## 2.5 Solving Equations Using Addition or Subtraction

**Objective:** Solve equations using addition or subtraction.

Inverse \_\_\_\_\_ are two operations that undo each other, such as addition and subtraction.

### Subtraction Property of Equality

**Words** Subtracting the same number from each side of an equation produces an equivalent equation.

**Numbers** If  $x + 3 = 5$ , then  $x + 3 - \square = 5 - \square$ , or  $x = \square$ .

**Algebra** If  $x + a = b$ , then  $x + a - \square = b - \square$ , or  $x = \square$ .

### Example 1: Solving an Equation Using Subtraction

Solve the equation. Show your work. Check your answer.

1.  $x + 5 = -2$

2.  $x + 6 = 19$

3.  $-5 = y + 12$

4.  $7 + k = 42$

5.  $j + 13 = -2$

6.  $-1 = x + 5$

### Addition Property of Equality

**Words** Adding the same number to each side of an equation produces an equivalent equation.

**Numbers** If  $x - 3 = 5$ , then  $x - 3 + \square = 5 + \square$ , or  $x = \square$ .

**Algebra** If  $x - a = b$ , then  $x - a + \square = b + \square$ , or  $x = \square$ .

### Example 2: Solving Equations Using Addition

Solve the equation. Show your work. Check your answer.

1.  $12 = y - 7$

2.  $m - 3 = -11$

3.  $a - 5 = 5$

4.  $f - 11 = 2$

5.  $y - 12 = -8$

6.  $-7 = z - 5$

### Check It Out!

Solve the equation. Show your work. Check your answer.

1.  $m + 9 = 12$

2.  $k - 2 = -15$

3.  $3 + a = 5$

4.  $b - 10 = 2$

5.  $x + 12 = -2$

6.  $7 = x - 4$

7.  $n + 3 + 8 = 20$

8.  $x - 1 - 7 = -16$

### Example 3: Solving Story Problems

You spend \$12.75 at the movie theater. You purchase a ticket, a container of popcorn of \$3.25, and a bottle of water for \$2. How much does the movie ticket cost?

## 2.6 Solving Equations Using Multiplication or Division

**Objective:** Solve equations using multiplication or division.

**Division Property of Equality**

**Words** Dividing each side of an equation by the same nonzero number produces an equivalent equation.

**Numbers** If  $3x = 12$ , then  $\frac{3x}{\square} = \frac{12}{\square}$ , or  $x = \square$ .

**Algebra** If  $ax = b$  and  $a \neq 0$ , then  $\frac{ax}{\square} = \frac{b}{\square}$ , or  $x = \square$ .

### Example 1: Solving Equations Using Division

Solve the equation. Show your work. Check your answer.

1.  $-7x = 42$

2.  $5x = 45$

3.  $-56 = -8y$

**Multiplication Property of Equality**

**Words** Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

**Numbers** If  $\frac{x}{3} = 12$ , then  $\square \cdot \frac{x}{3} = \square \cdot 12$ , or  $x = \square$ .

**Algebra** If  $\frac{x}{a} = b$  and  $a \neq 0$ , then  $\square \cdot \frac{x}{a} = \square \cdot b$ , or  $x = \square$ .

### Example 2: Solving Equations Using Multiplication

Solve the equation. Show your work. Check your answer.

1.  $5 = \frac{w}{11}$

2.  $\frac{m}{4} = 11$

3.  $-9 = \frac{c}{6}$

### Check It Out!

Solve the equation. Show your work. Check your answer.

1.  $3x = -3$

2.  $8a = 32$

3.  $\frac{1}{2}x = 3$

4.  $8 = \frac{n}{5}$

5.  $-\frac{1}{3}x = 4$

6.  $\frac{x}{-2} = 5$

### Example 3: Solving Story Problems

1. Sixty-four people show up for a volleyball tournament. Write and solve an equation to find how many 4-person teams can be formed.

2. A high-speed rail service connects London, England, Paris, France, and Brussels, Belgium. One of the high-speed trains travels about 233 miles from London to Brussels at a speed of about 87 miles per hour. How long does the trip take?

## 2.7 Decimal Operations and Equations with Decimals

**Objective:** Solve equations with decimals.

### Example 1: Adding and Subtracting Decimals

Find the sum.

1.  $-1.7 + (-3.4)$

2.  $-2.8 + (-5.9)$

3.  $2.1 + (-1.1)$

Find the difference.

1.  $-21.29 - (-34.62)$

2.  $7.12 - (-3.46)$

3.  $-4.3 - (1.2)$

### Example 2: Multiplying and Dividing Decimals

Find the product or quotient.

1.  $-0.4(13.7)$

2.  $-2.5(-6.75)$

3.  $18.05 \div (-1.9)$

4.  $7.093 \div (-3.46)$

### Example 3: Solve Equations using Addition or Subtraction

Solve the equation.

1.  $x + 6.3 = 4.8$

2.  $y - 5.74 = -3.51$

3.  $x + 5.6 = 9.4$

4.  $-3.5 = y + 1.27$

### Example 3: Solve Equations using Multiplication or Division

Solve the equation.

1.  $0.8m = 4.8$

2.  $\frac{n}{5} = -2.15$

3.  $6x = -43.2$

4.  $\frac{y}{-3.1} = -8.4$