8.1 Relations and Functions

Objective: Use graphs to represent relations and functions.

A relation is a set of ordered pairs.
The domain of a relation is the set of all inputs, x-values.
The range of a relation is the set of all outputs, y-values.
Each number in a domain is an input.
Each number in a range is an output.
A function is a relation with the property that for each input their is exactly one output.
The vertical line test says that if you can find a vertical line passing through more than one point of a graph of a relation, then the relation is not a function. Otherwise, the relation is a function.

Example 1: Identifying the Domain and Range

Identify the domain and range of the relation.

1. The table below that shows one Norway Spruce tree's height at different ages.

Age (years), x	5	10	15	20	25
Height (ft), y	13	25	34	43	52

domain: 5, 10, 15, 20, 25 range: 13, 25, 34, 43, 52

2. (-5, 2), (-3, -1), (-1, 0), (2, 3), (5, 4)

domain: -5, -3, -1, 2, 5

range: 2, -1, 0, 3, 4

Domain: x
Range: y

Check It Out!

Identify the domain and range of the relation.

1. (-4, -3), (-3, 2), (0, 0), (1, -1), (2, 3), (3, 1), (3, -2)

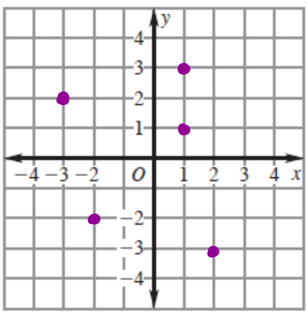
domain: -4, -3, 0, 1, 2, 3

range: -3, 2, 0, -1, 3, 1, -2

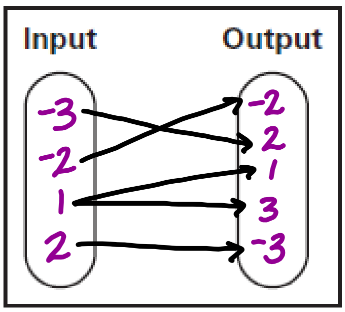
Example 2: Representing a Relation

Represent the relation (-3, 2), (-2, -2), (1, 1), (1, 3), (2, -3).

1. a graph

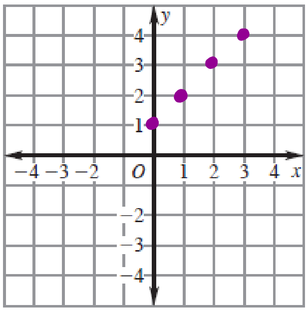


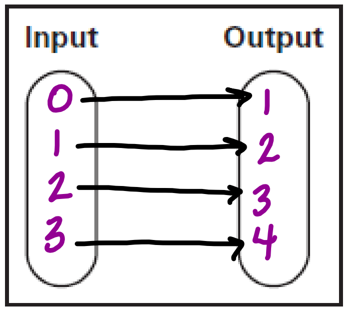
2. a mapping diagram



Check It Out!
Represent the relation (0, 1), (1, 2), (2, 3), (3, 4).

1. a graph
2. a mapping diagram





Example 3: Identifying Functions

Tell whether the relation is a function.

1. (-5, 2), (-3, -1), (-1, 0), (2, 3), (5, 4)

IN		OUT
-5	→	2
-3	→	-1
-1	→	0
2	→	3
5	→	4

YES it is a function

(one input per output)

2. (-4, -3), (-3, 2), (0, 0), (1, -1), (2, 3), (3, 1), (3, -2)

IN		OUT
-4	→	-3
-3	→	2
0	→	0
1	→	-1
2	→	3
3	→	1
3	→	-2

NO it is not a function

(for the '3' input there is two outputs)

Check It Out!
Tell whether the relation is a function.

1. (-3, 2), (-2, -2), (1, 1), (1, 3), (2, -3)

IN		OUT
-3	→	2
-2	→	-2
1	→	1
1	→	3
2	→	-3

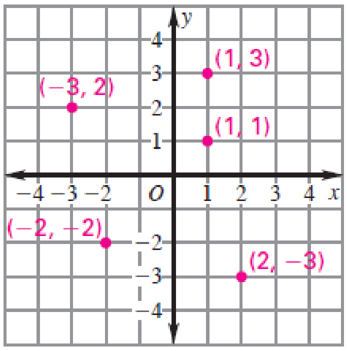
NO it is not a function

(for the '1' input there is two outputs)

Example 4: Using the Vertical Line Test

Use the vertical line test to determine if the relation is a function.

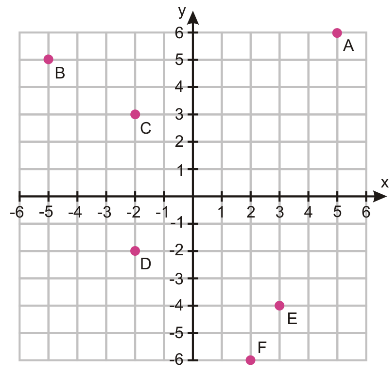
1.



Not a function

(fails at x=1)

2.



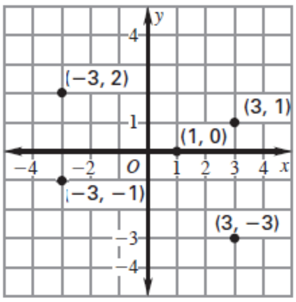
Not a function

(fails at x=-2)

Check It Out!

Use the vertical line test to determine if the relation is a function.

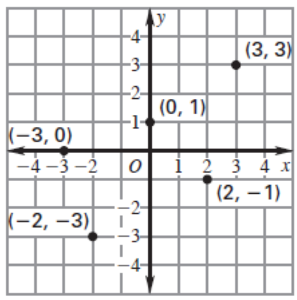
1.



Not a function

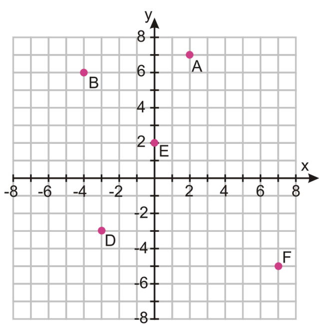
(fails at x=-3)

2.



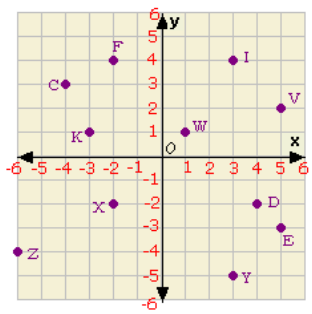
Function

3.



Function

4.



Not a Function

(fails at x=-2,3,5)

8.2 Linear Equations in Two Variables

Objective: Find solutions of equations in two variables.

An equation that contains two different variables is an equation in two variables.

A solution of an equation in two variables in x and y is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the equation.

The graph of an equation in two variables is the set of points in a coordinate plane that represents all the solutions of the equation.

An equation whose graph is a line is called a linear equation.

A function whose graph is a nonvertical line is called a linear function.

An equation solved for y is in function form.

Example 1: Checking Solutions

Tell whether the ordered pair is a solution of the equation.

1. (5, -1); x - 3y = 8

(x,y)

$5-3(-1)=8$

$5-(-3)=8$

$8=8$ ✓

$\boxed{(5,-1) \text{ is a solution}}$

Check It Out!

Tell whether the ordered pair is a solution of the equation.

1. (0, -5); 2x - y = 5

(x,y)

$2(0)-(-5)=5$

$0-(-5)=5$

$5=5$ ✓

$\boxed{(0,-5) \text{ is a solution}}$

2. (3, 2); 2x - y = 5

(x,y)

$2(3)-2=5$

$6-2=5$

$4\neq5$

$\boxed{(3,2) \text{ is NOT a solution}}$

Example 2: Graphing a Linear Equation

Graph the following equations by making an xy chart.

1. y = -x + 1

x	y	
0	1	(0,1)
1	0	(1,0)
2	-1	(2,-1)

$y=-0+1$

$\boxed{y=1}$

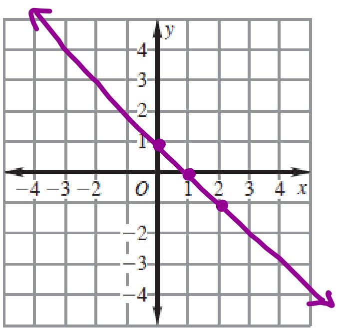
$y=-2+1$

$\boxed{y=-1}$

$0=-x+1$

$+x \quad +x$

$\boxed{x=1}$



Check It Out!

Graph the following equations by making an xy chart

1. y = x + 2

x	y	
0	2	(0,2)
-2	0	(-2,0)
1	3	(1,3)

$y = 0 + 2$

$\boxed{y=2}$

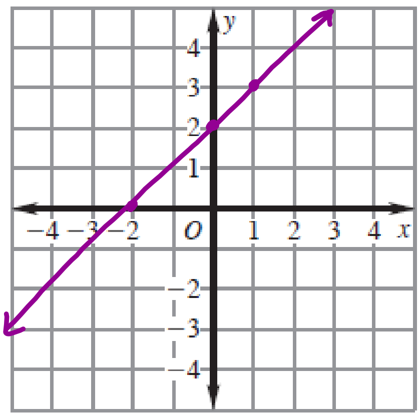
$y = 1 + 2$

$\boxed{y=3}$

$0 = x + 2$

$-2 \quad -2$

$\boxed{-2 = x}$



2. y = 3x - 3

x	y	
0	-3	(0,-3)
1	0	(1,0)
2	3	(2,3)

$y = 3(0) - 3$

$\boxed{y=-3}$

$y = 3(2) - 3$

$y = 6 - 3$

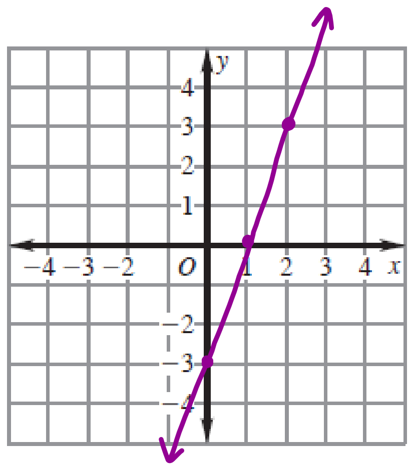
$\boxed{y=3}$

$0 = 3x - 3$

$+3 \quad +3$

$\frac{3}{3} = \frac{3x}{3}$

$\boxed{1 = x}$



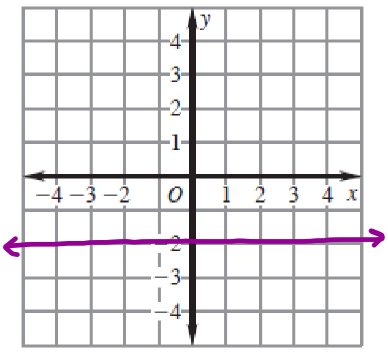
Example 3: Graphing Horizontal and Vertical Lines

Graph the following equations. Then tell whether the equation is a function.

1. y = -2

YES

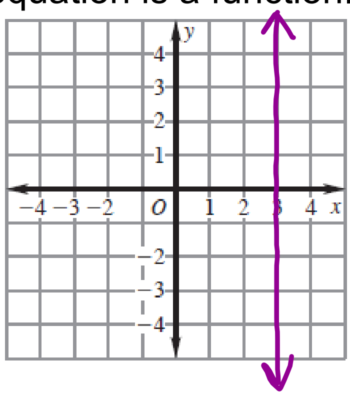
* passes the vertical line test



2. x = 3

NO

* Does NOT pass the vertical line test



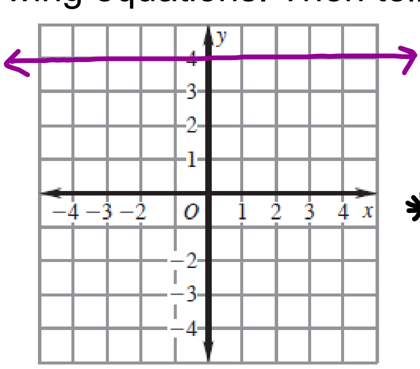
Check It Out!

Graph the following equations. Then tell whether the equation is a function.

1. y = 4

YES

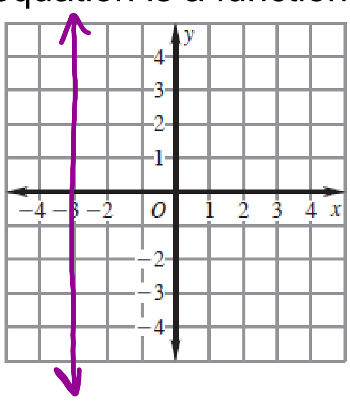
* passes the vertical line test



2. x = -3

NO

* Does NOT pass the vertical line test



Example 4: Writing an Equation in Function Form

Write the equation in function form. Then graph the following equations by making an xy chart.

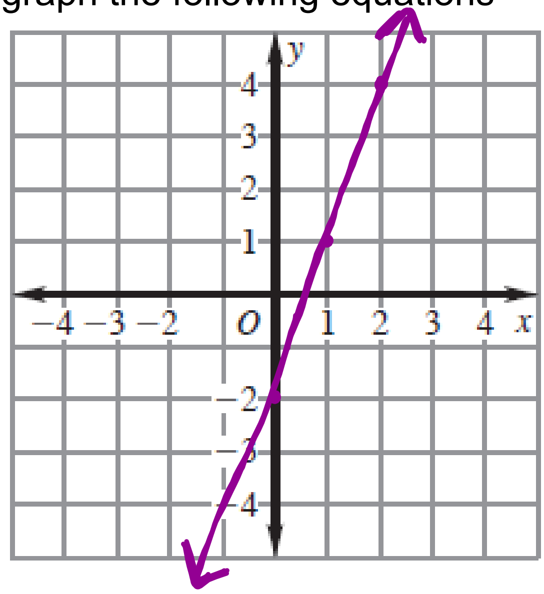
1. $3x - y = 2$

$$-3x \quad -3x$$

$$\frac{-y}{-1} = \frac{-3x+2}{-1}$$

$$\boxed{y = 3x - 2}$$

x	y	
0	-2	(0,-2)
1	1	(1,1)
2	4	(2,4)



$y = 3(0) - 2$
$\boxed{y = -2}$

$y = 3(2) - 2$
$y = 6 - 2$
$\boxed{y = 4}$

$y = 3(1) - 2$
$y = 3 - 2$
$\boxed{y = 1}$

Check It Out!

Write the equation in function form. Then graph the following equations by making an xy chart.

1. $x - 2y = 4$

$$-x \quad -x$$

$$\frac{-2y}{-2} = \frac{-x+4}{-2}$$

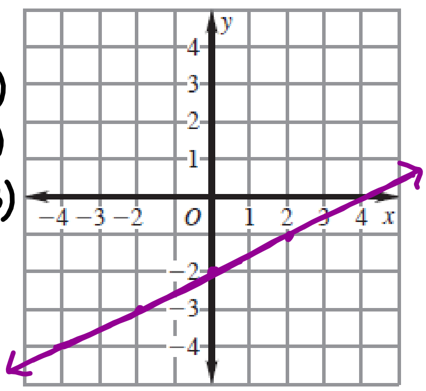
$$\boxed{y = \frac{1}{2}x - 2}$$

$y = \frac{1}{2}(0) - 2$
$\boxed{y = -2}$

$y = \frac{1}{2}(2) - 2$
$y = 1 - 2$
$\boxed{y = -1}$

$y = \frac{1}{2}(-2) - 2$
$y = -1 - 2$
$\boxed{y = -3}$

x	y	
0	-2	(0,-2)
2	-1	(2,-1)
-2	-3	(-2,-3)



2. $y - x = -1$

$$+x \quad +x$$

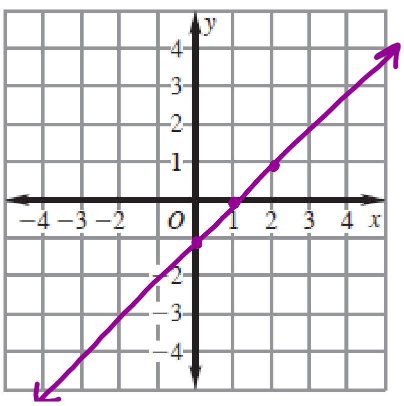
$$\boxed{y = x - 1}$$

$y = 2 - 1$
$\boxed{y = 1}$

$y = 0 - 1$
$\boxed{y = -1}$

$0 = x - 1$
$+1 \quad +1$
$\boxed{1 = x}$

x	y
0	-1
1	0
2	1



8.3 Using Intercepts

Objective: Use decimals to solve percent problems.

The x-coordinate of a point where a graph crosses the x-axis is an x-intercepts. The y-coordinate of a point where a graph crosses the y-axis is an y-intercepts.

Finding Intercepts

To find the x-intercept of a line, substitute $\boxed{0}$ for y in the line's equation and solve for $\boxed{x}$.

To find the y-intercept of a line, substitute $\boxed{0}$ for x in the line's equation and solve for $\boxed{y}$.

Example 1: Finding the Intercepts of a Graph

Find the intercepts of the graph.

1. $2x - 5y = -10$

x-int:
$2x-5(0)=-10$
$\frac{2x}{2}=\frac{-10}{2}$
$\boxed{x=-5}$

y-int:
$2(0)-5y=-10$
$\frac{-5y}{-5}=\frac{-10}{-5}$
$\boxed{y=2}$

2. $3x - 2y = 6$

x-int:
$3x-2(0)=6$
$\frac{3x}{3}=\frac{6}{3}$
$\boxed{x=2}$

y-int:
$3(0)-2y=6$
$\frac{-2y}{-2}=\frac{6}{-2}$
$\boxed{y=-3}$

Check It Out!

Find the intercepts of the graph.

1. $2x + 3y = 6$

x-int:
$2x+3(0)=6$
$\frac{2x}{2}=\frac{6}{2}$
$\boxed{x=3}$

y-int:
$2(0)+3y=6$
$\frac{3y}{3}=\frac{6}{3}$
$\boxed{y=2}$

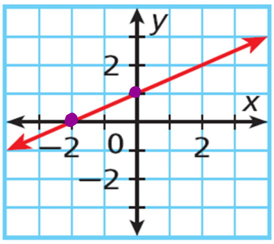
2. $3x - 6y = 12$

x-int:
$3x-6(0)=12$
$\frac{3x}{3}=\frac{12}{3}$
$\boxed{x=4}$

y-int:
$3(0)-6y=12$
$\frac{-6y}{-6}=\frac{12}{-6}$
$\boxed{y=-2}$

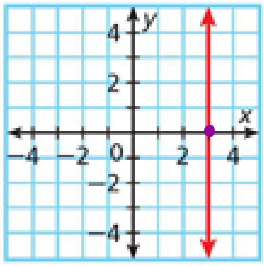
Example 2: Identifying Intercepts on a Graph

Identify the x-intercept and y-intercept of each graph.

1. 

x-int: $\boxed{x=-2}$

y-int: $\boxed{y=1}$

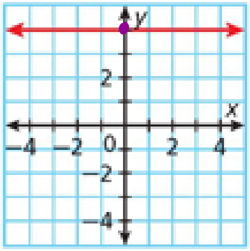
2. 

x-int: $\boxed{x=3}$

y-int: $\boxed{\text{NONE}}$

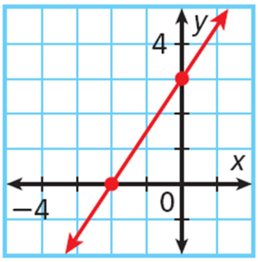
Check It Out!

Identify the x-intercept and y-intercept of each graph.

1. 

x-int: $\boxed{\text{NONE}}$

y-int: $\boxed{y=4}$

2. 

x-int: $\boxed{x=-2}$

y-int: $\boxed{y=3}$

Example 3: Using Intercepts to Graph a linear Equation

Find the intercepts of the graph. Graph the equations using the intercepts.

1. x - 2y = -2

x-int:

$x-2(0)=-2$

$\boxed{x=-2}$

y-int:

$0-2y=-2$

$\frac{-2y}{-2}=\frac{-2}{-2}$

$\boxed{y=1}$

2. 4x + 3y = 12

x-int:

$4x+3(0)=12$

$\frac{4x}{4}=\frac{12}{4}$

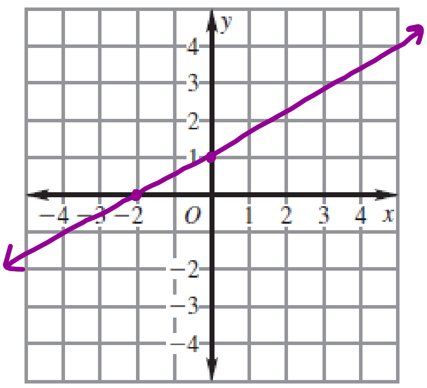
$\boxed{x=3}$

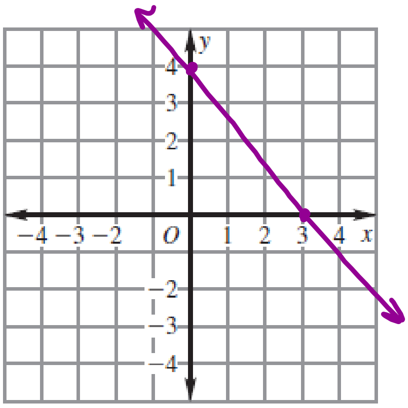
y-int:

$4(0)+3y=12$

$\frac{3y}{3}=\frac{12}{3}$

$\boxed{y=4}$





Check It Out!

Find the intercepts of the graph. Graph the equations using the intercepts.

1. $x - 2y = 4$

x-int:

$x - 2(0) = 4$

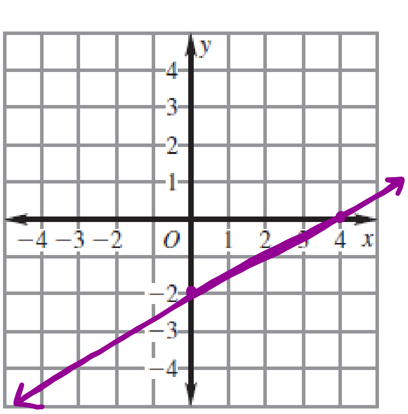
$\boxed{x = 4}$

y-int:

$0 - 2y = 4$

$\frac{-2y}{-2} = \frac{4}{-2}$

$\boxed{y = -2}$



2. $2x + y = 2$

x-int:

$2x + (0) = 2$

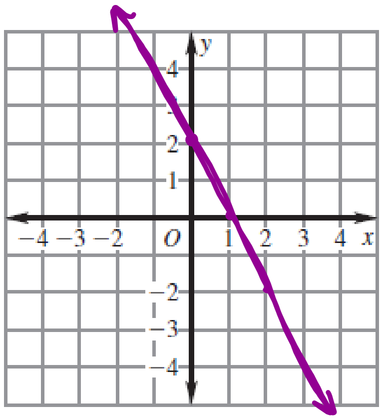
$\frac{2x}{2} = \frac{2}{2}$

$\boxed{x = 1}$

y-int:

$2(0) + y = 2$

$\boxed{y = 2}$



Example 4: Writing and Graphing an Equation

1. You run and walk on a fitness trail that is 12 miles long. You can run 6 miles per hour and walk 3 miles per hour. Write and graph an equation describing your possible running and walking times on the fitness trail. Give three possible combinations of running and walking times.

$6x + 3y = 12$

x = running time
y = walking time

x-int:

$6x + 3(0) = 12$

$\frac{6x}{6} = \frac{12}{6}$

$\boxed{x = 2}$

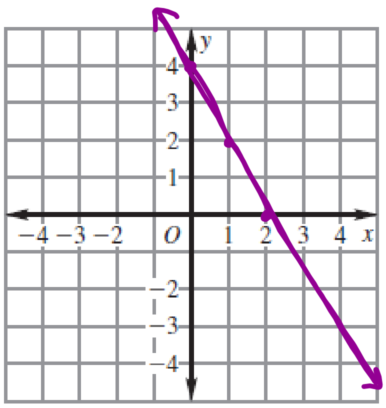
y-int:

$6(0) + 3y = 12$

$\frac{3y}{3} = \frac{12}{3}$

$\boxed{y = 4}$

x	y
1	2



(2,0): run 2 hours, no walk
(0,4): no run, walk 4 hours
(1,2): run 1 hour, walk 2 hours

8.4 The Slope of a Line

Objective: Find and interpret slopes of lines.

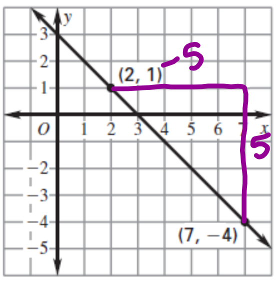
The __slope__ of a line is the ratio of the line's vertical change to its horizontal change. The line's vertical change between two points is called its __rise__. A line's horizontal change between two points is called its __run__. The slope of a line is the rise over the run.

Example 1: Finding Slope

1. A building's access ramp has a rise of 2 feet and a run of 24 feet. Find its slope.

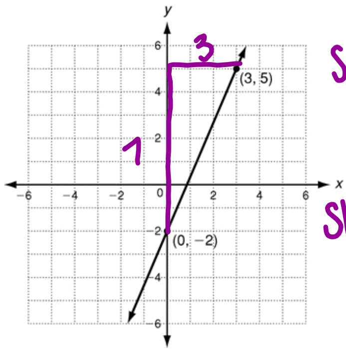
$$\text{slope} = \frac{\text{rise}}{\text{run}} \qquad \text{slope} = \frac{2 \div 2}{24 \div 2} = \boxed{\frac{1}{12}}$$

Find the slope of each line:

2. 

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{5 \div 5}{-5 \div 5} = \boxed{-1}$$

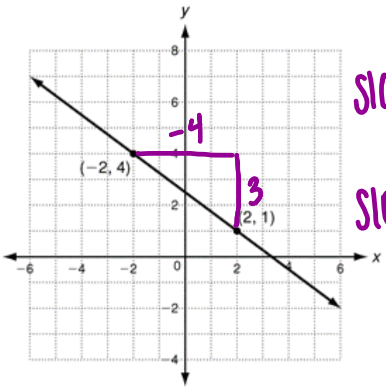
3. 

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \boxed{\frac{7}{3}}$$

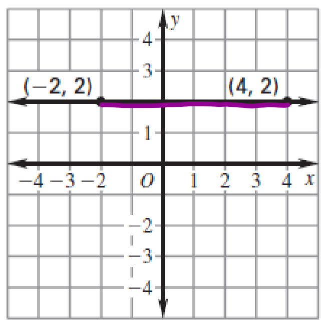
Check It Out!

Find the slope of each line:

1. 

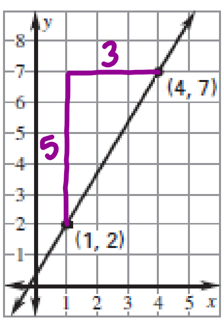
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \boxed{\frac{3}{-4}}$$

2. 

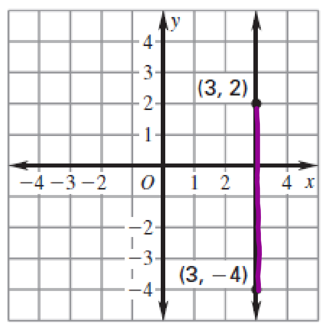
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{0}{6} = \boxed{0}$$

3. 

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \boxed{\frac{5}{3}}$$

4. 

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{6}{0}$$

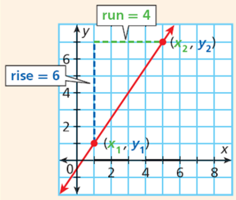
$\boxed{\text{undefined}}$

Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of a Line

DEFINITION	EXAMPLE
The **rise** is the difference in the *y*-values of two points on a line.	
The **run** is the difference in the *x*-values of two points on a line.	
The **slope** of a line is the ratio of the rise to run. If (x_1, y_1) and (x_2, y_2) are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	



slope $= \frac{6}{4} = \frac{3}{2}$

Example 2: Finding Positive and Negative Slope

Find the slope of the line containing these two points.

1. (0, 2) and (-2, 3)

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-2}{-2-0} = \boxed{\frac{1}{-2}}$$

2. (2, -2), (0, 4)

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-(-2)}{0-2} = \frac{6 \div 2}{-2 \div 2} = \boxed{-3}$$

Check It Out!

Find the slope of the line containing these two points.

1. (1, 6) and (3, 10)

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{10-6}{3-1} = \frac{4 \div 2}{2 \div 2} = \boxed{2}$$

2. (7, 5), (3, 2)

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-5}{3-7} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$$

3. (-2, 10), (-4, -4)

x_1 y_1 x_2 y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4-10}{-4-(-2)} = \frac{-14 \div -2}{-2 \div -2} = \boxed{7}$$

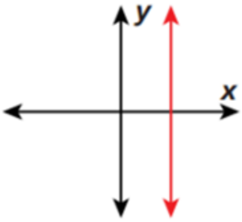
4. (-2, 4), (6, 2)

x_1 y_1 x_2 y_2

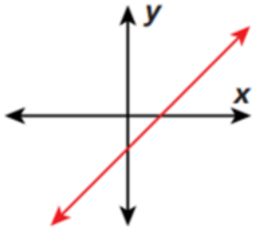
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-4}{6-(-2)} = \frac{-2 \div 2}{8 \div 2} = \boxed{\frac{-1}{4}}$$

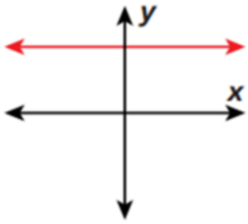
Tell whether the slope is *positive*, *negative*, *zero* or *undefined*.
Remember a fraction with zero in the denominator is undefined because it is impossible to divide by zero.



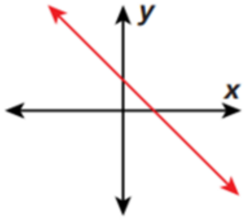
undefined



positive



zero



negative

Example 3: Zero and Undefined Slope

Find the slope of the containing these two points. Tell whether the slope is positive, negative, zero, or undefined.

1. (-2, 7) (3, 7)

$x_1 \quad y_1 \quad x_2 \quad y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{7-7}{3-(-2)} = \frac{0}{5} = \boxed{0}$

Zero slope

2. (3, -1), (3, 5)

$x_1 \quad y_1 \quad x_2 \quad y_2$

*cannot divide by zero

$m = \frac{5-(-1)}{3-3} = \frac{6}{0}$

undefined slope

Check It Out!

Find the slope of the line shown or through the given points. Tell whether the slope is positive, negative, zero, or undefined.

1. (-10, 2) (-10, -2)

$x_1 \quad y_1 \quad x_2 \quad y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-2-2}{-10-(-10)} = \frac{-4}{0}$

*cannot divide by zero

undefined slope

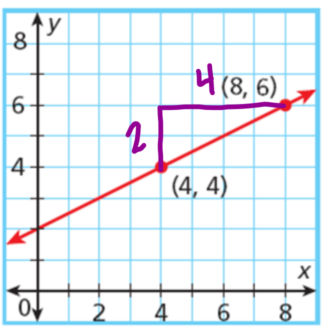
2. (1, -1), (7, -1)

$x_1 \quad y_1 \quad x_2 \quad y_2$

$m = \frac{-1-(-1)}{7-1} = \frac{0}{6} = 0$

zero slope

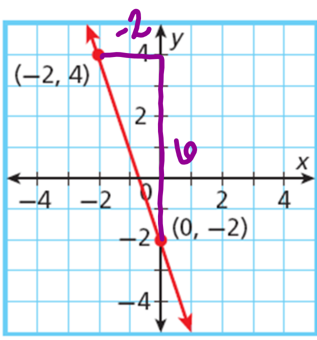
3.



$\text{slope} = \frac{2 \div 2}{4 \div 2} = \boxed{\frac{1}{2}}$

positive slope

4.



$\text{slope} = \frac{6}{-2} = \boxed{-3}$

negative slope

8.5 Slope-Intercept Form

Objective: Graph linear equations in slope-intercept form.

Slope-Intercept Form:

$y = mx + b$

Slope-Intercept Form

Words A linear equation of the form $y = mx + b$ is said to be in **slope-intercept form**. The slope is m and the y-int. is b.

Algebra $y = mx + b$ **Numbers** $y = 2x + 3$

Example 1: Identifying the Slope and y-intercept

Identify the slope and y-intercept in the equation of the line.

1. $y = 2x - 3$

$\boxed{\text{slope} = 2}$

$\boxed{\text{y-int} = -3}$

2. $4x + 3y = 9$

$-4x \quad -4x$

$\frac{3y}{3} = \frac{-4x + 9}{3}$

$\boxed{y = -\frac{4}{3}x + 3}$

$\boxed{\text{slope} = -4/3}$

$\boxed{\text{y-int} = 3}$

3. $y = -3x - 4$

$\boxed{\text{slope} = -3}$

$\boxed{\text{y-int} = -4}$

4. $x - 2y = 10$

$-x \quad -x$

$\frac{-2y}{-2} = \frac{-x + 10}{-2}$

$\boxed{y = \frac{1}{2}x - 5}$

$\boxed{\text{slope} = 1/2}$

$\boxed{\text{y-int} = -5}$

Check It Out!

Identify the slope and y-intercept in the equation of the line.

1. $y = x - 5$

$\boxed{y = 1x - 5}$

$\boxed{\text{slope} = 1}$

$\boxed{\text{y-int} = -5}$

2. $4x - 2y = -16$

$-4x \quad -4x$

$\frac{-2y}{-2} = \frac{-4x - 16}{-2}$

$\boxed{y = 2x + 8}$

$\boxed{\text{slope} = 2}$

$\boxed{\text{y-int} = 8}$

3. $y = -6x$

$\boxed{\text{slope} = -6}$

$\boxed{\text{y-int} = 0}$

4. $x - 2y = 6$

$-x \quad -x$

$\frac{-2y}{-2} = \frac{-x + 6}{-2}$

$\boxed{y = \frac{1}{2}x - 3}$

$\boxed{\text{slope} = 1/2}$

$\boxed{\text{y-int} = -3}$

Example 2: Writing an equation

Write the equation that describes the line in slope-intercept form.

1. slope = $\frac{1}{4}$; y-intercept = 4

$y = mx + b$

$\boxed{y = \frac{1}{4}x + 4}$

2. slope = -9 ; y-intercept = $-\frac{5}{4}$

$y = mx + b$

$\boxed{y = -9x - \frac{5}{4}}$

3. slope = 1 ; y-intercept = 0

$y = mx + b$

$y = 1x + 0$

$\boxed{y = x}$

4. slope = 2 ; (3, 4) is on the line

no y-int

$y = mx + b$

$4 = 2(3) + b$

$4 = 6 + b$

$-6 \quad -6$

$-2 = b$

y-int = -2

$\boxed{y = 2x - 2}$

Check It Out!

Write the equation that describes the line in slope-intercept form.

1. slope = 0 ; y-intercept = 1

$y = mx + b$

$y = 0(x) + 1$

$\boxed{y = 1}$

2. slope = 3 ; y-intercept = -1

$y = mx + b$

$\boxed{y = 3x - 1}$

3. slope = -12 ; y-intercept = $-\frac{1}{2}$

$y = mx + b$

$\boxed{y = -12x - \frac{1}{2}}$

4. slope = 8 ; (-3, 1) is on the line

no y-int

$y = mx + b$

$1 = 8(-3) + b$

$1 = 5 + b$

$-5 \quad -5$

$-4 = b$

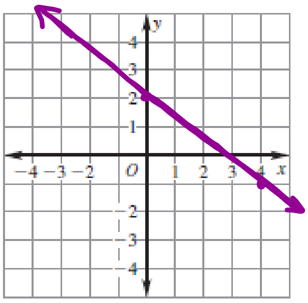
y-int = -4

$\boxed{y = 8x - 4}$

Example 3: Graphing an Equation in Slope-Intercept Form

Graph the equation.

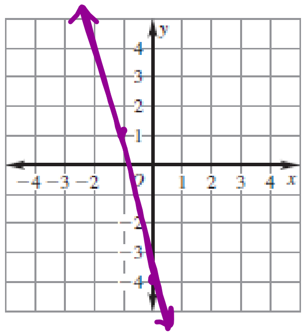
1. $y = -\frac{3}{4}x + 2$



slope $= -\frac{3}{4}$

y-int $= (0,2)$

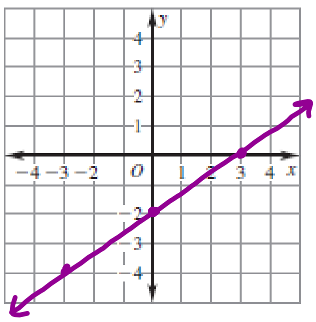
2. $y = -5x - 4$



slope $= \frac{-5}{1}$

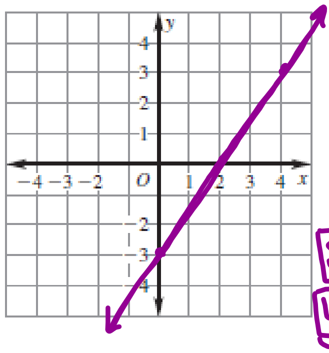
y-int $= (0,-4)$

3. $2x - 3y = 6$



$2x - 3y = 6$
$-2x \qquad -2x$
$\frac{-3y}{-3} = \frac{-2x+6}{-3}$
$\boxed{y = \frac{2}{3}x - 2}$
$\boxed{\text{slope} = 2/3}$
$\boxed{\text{y-int} = (0,-2)}$

4. $3x - 2y = 6$

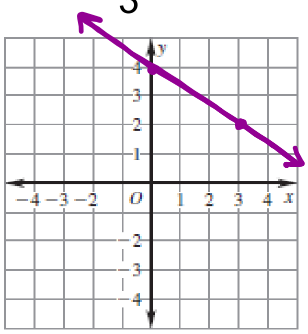


$3x - 2y = 6$
$-3x \qquad -3x$
$\frac{-2y}{-2} = \frac{-3x+6}{-2}$
$\boxed{y = \frac{3}{2}x - 3}$
$\boxed{\text{slope} = 3/2}$
$\boxed{\text{y-int} = (0,-3)}$

Check It Out!

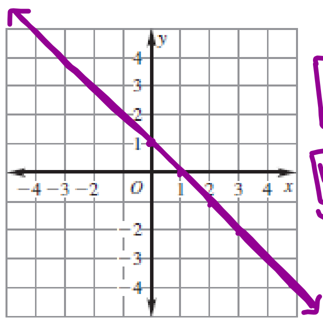
Graph the equation.

1. $y = -\frac{2}{3}x + 4$



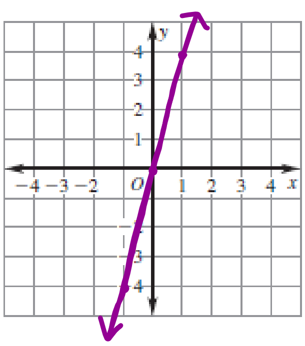
$\boxed{\text{slope} = -\frac{2}{3}}$
$\boxed{\text{y-int} = (0,4)}$

2. $y = -x + 1$



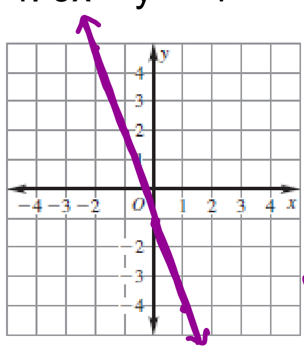
$\boxed{\text{slope} = -1/1}$
$\boxed{\text{y-int} = (0,1)}$

3. $y = 4x$



$\boxed{\text{slope} = 4/1}$
$\boxed{\text{y-int} = (0,0)}$

4. $3x + y = -1$



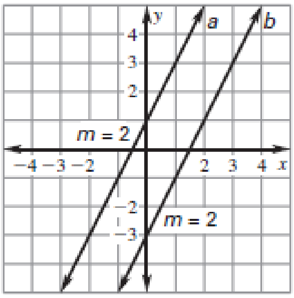
$3x + y = -1$
$-3x \qquad -3x$
$\boxed{y = -3x - 1}$
$\boxed{\text{slope} = -3/1}$
$\boxed{\text{y-int} = (0,-1)}$

If m is any nonzero number, then the negative reciprocal of m is $-\frac{1}{m}$. Note that the product of a number and its negative reciprocal is -1:

$$m\left(-\frac{1}{m}\right) = -1$$

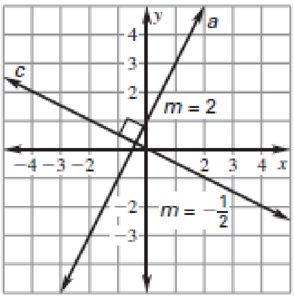
Slopes of Parallel and Perpendicular Lines

Two nonvertical parallel lines have $\boxed{\text{the same slope}}$. For example, the parallel lines a and b below $\boxed{\text{both have a slope 2}}$.



$a \parallel b$

Two nonvertical perpendicular lines, such as lines a and c below, have slopes that are $\boxed{\text{opposite reciprocals of each other}}$.



$a \perp c$

Example 4: Finding Slopes of Parallel or Perpendicular Lines

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

1. $y = -\frac{3}{4}x + 2$

slope $= -\frac{3}{4}$

parallel: $\boxed{\frac{-3}{4}}$

perpendicular: $\boxed{\frac{4}{3}}$

2. $2x - 3y = 6$

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x+6}{-3}$

$y = \frac{2}{3}x - 2$

slope $= \frac{2}{3}$

parallel: $\boxed{\frac{2}{3}}$

perpendicular: $\boxed{\frac{-3}{2}}$

Check It Out!

For the line with the given equation, find the slope of a parallel line and the slope of a perpendicular line.

1. $y = -5x - 4$

slope $= \frac{-5}{1}$

parallel: $\boxed{\frac{-5}{1}}$

perpendicular: $\boxed{\frac{1}{5}}$

2. $10x + 5y = 15$

$-10x \quad -10x$

$\frac{5y}{5} = \frac{-10x+15}{5}$

$y = -2x + 3$

slope $= \frac{-2}{1}$

parallel: $\boxed{\frac{-2}{1}}$

perpendicular: $\boxed{\frac{1}{2}}$

8.5.5 Point-Slope Form

Objective: Graph linear equations in point-slope form.

Point-Slope Form:

$y-y_1=m(x-x_1)$

Point-Slope Form of a Linear Equation

The line with slope m that contains the point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

Example 1: Identifying the Slope and Point

Identify the slope and point in equation of the line.

1. y - 2 = -7(x + 1)

$y-y_1=m(x-x_1)$

$\boxed{\text{slope}=-7}$ $\boxed{\text{point}=(-1,2)}$

2. y + 3 = -(x + 1)

$y-y_1=m(x-x_1)$

$\boxed{\text{slope}=-1}$ $\boxed{\text{point}=(-1,-3)}$

Check It Out!

Identify the slope and point in equation of the line.

1. y - 10 = $\frac{1}{4}$(x - 6)

$y-y_1=m(x-x_1)$

$\boxed{\text{slope}=\frac{1}{4}}$ $\boxed{\text{point}=(6,10)}$

2. y + 5 = 2x

$y-y_1=m(x-x_1)$

$\boxed{\text{slope}=2}$ $\boxed{\text{point}=(0,-5)}$

Example 2: Writing an equation

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. Slope = $\frac{1}{6}$; (5, 1)

$y-y_1=m(x-x_1)$

$\boxed{y-1=\frac{1}{6}(x-5)}$

2. Slope = -4 ; (0, 3)

$y-y_1=m(x-x_1)$

$y-3=-4(x-0)$

$\boxed{y-3=-4x}$

Check It Out!

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. Slope = 1 ; (-1, -4)

$y-y_1=m(x-x_1)$

$y-(-4)=1(x-(-1))$

$\boxed{y+4=1(x+1)}$

2. Slope = 0 ; (3, -4)

$y-y_1=m(x-x_1)$

$y-(-4)=0(x-3)$

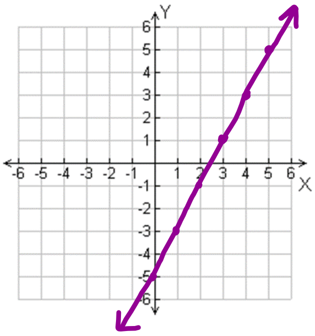
$y+4=0$

$\boxed{y=-4}$

Example 3: Graphing an Equation in Point-Slope Form

Graph the line described by the equation.

1. $y - 1 = 2(x - 3)$

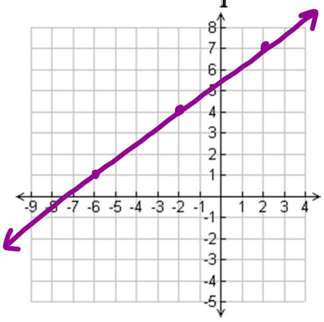


$y-y_1=m(x-x_1)$

slope $=2$

point $=(3,1)$

2. $y - 4 = \frac{3}{4}(x + 2)$

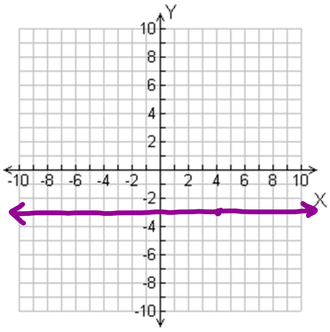


$y-y_1=m(x-x_1)$

slope $=\frac{3}{4}$

point $=(-2,4)$

3. $y + 3 = 0(x - 4)$

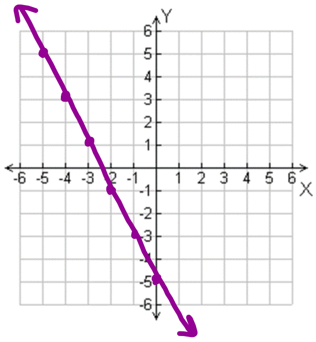


$y-y_1=m(x-x_1)$

slope $=0$

point $=(4,-3)$

4. $y - 3 = -2(x + 4)$



$y-y_1=m(x-x_1)$

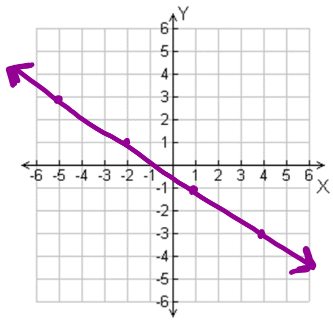
slope $=-2$

point $=(-4,3)$

Check It Out!

Graph the line described by the equation.

1. $y - 1 = -\frac{2}{3}(x + 2)$

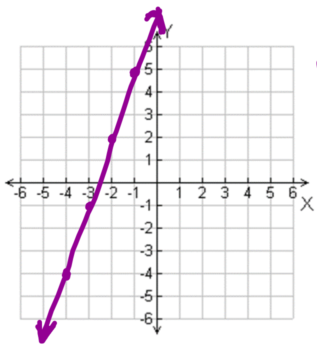


$y-y_1=m(x-x_1)$

slope $=-\frac{2}{3}$

point $=(-2,1)$

2. $y + 4 = 3(x + 4)$

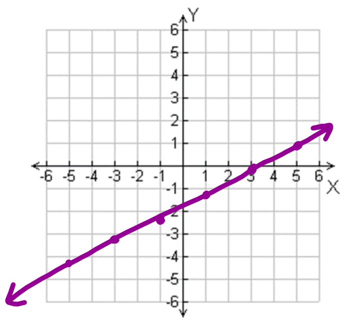


$y-y_1=m(x-x_1)$

slope $=3$

point $=(-4,-4)$

3. $y = \frac{1}{2}(x - 3)$

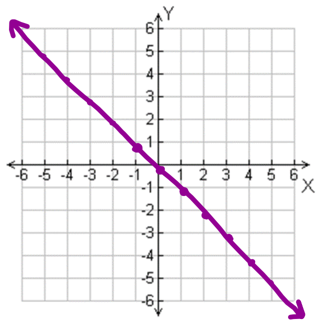


$y-y_1=m(x-x_1)$

slope $=\frac{1}{2}$

point $=(3,0)$

4. $y - 1 = -(x + 1)$



$y-y_1=m(x-x_1)$

slope $=-1$

point $=(-1,1)$

8.6 Writing Linear Equations

Objective: Write linear equations.

The best-fitting line is the line that lies as close as possible to the points in a data set.

Example 1: Writing an Equation

Write an equation of the line with the given information in the given form.

1. slope of -2 ; y-intercept of -5
   slope-intercept form
   $y=mx+b$
   $y=-2x+(-5)$
   $\boxed{y=-2x-5}$

2. slope of $\frac{1}{2}$; through (-3, -3)
   point-slope form
   $y-y_1=m(x-x_1)$
   $y-(-3)=\frac{1}{2}(x-(-3))$
   $\boxed{y+3=\frac{1}{2}(x+3)}$

3. slope of 1 ; through (-1, 2)
   point-slope form
   $y-y_1=m(x-x_1)$
   $y-2=1(x-(-1))$
   $\boxed{y-2=x+1}$

4. slope of 4 ; y-intercept of -3
   slope-intercept form
   $y=mx+b$
   $y=4x+(-3)$
   $\boxed{y=4x-3}$

Check It Out!

Write an equation of the line with the given information in the given form.

1. slope of 1 ; y-intercept of -2
   slope-intercept form
   $y=mx+b$
   $y=1x+(-2)$
   $\boxed{y=x-2}$

2. slope of 0 ; through (2, 4)
   point-slope form
   $y-y_1=m(x-x_1)$
   $y-4=0(x-2)$
   $y-4=0$
   $\boxed{y=4}$

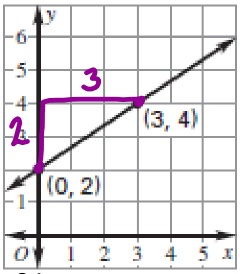
3. slope of -10 ; through (0, 7)
   point-slope form
   $y-y_1=m(x-x_1)$
   $y-7=-10(x-0)$
   $\boxed{y-7=-10x}$

4. slope of -6 ; y-intercept of 3
   slope-intercept form
   $y=mx+b$
   $\boxed{y=-6x+3}$

Example 2: Writing an Equation of a Graph

Write an equation of the line shown.

1.



slope $= \frac{2}{3}$ y-int $= 2$

$y = mx + b$

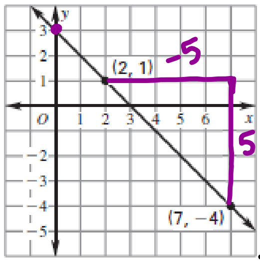
$\boxed{y = \frac{2}{3}x + 2}$

Slope $= 2/3$ pt $= (3,4)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 4 = 2/3(x - 3)}$

2.



slope $= \frac{5}{-5} = -1$

y-int $= 3$

$y = mx + b$

$\boxed{y = -x + 3}$

slope $= -1$ pt $= (2,1)$

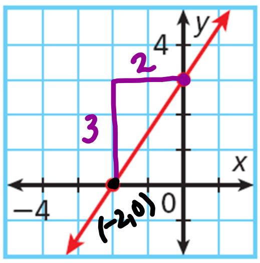
$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = -(x - 2)}$

Check It Out!

Write an equation of the line shown.

1.



slope $= \frac{3}{2}$

y-int $= 3$

$y = mx + b$

$\boxed{y = \frac{3}{2}x + 3}$

slope $= \frac{3}{2}$

pt $= (-2, 0)$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{2}(x - (-2))$

$\boxed{y = \frac{3}{2}(x + 2)}$

Example 3: Writing an Equation in Slope-Intercept Form

Write the equation in slope-intercept form. Then identify the slope and y-intercept.

* solve for y

1. y - 3 = -2(x + 4)

$y - 3 = -2x - 8$

$+3 \qquad +3$

$\boxed{y = -2x - 5}$

$\boxed{\text{Slope}: -2}$

$\boxed{\text{y-int}: -5}$

2. 6x + 2y =10

$-6x \qquad -6x$

$\frac{2y}{2} = \frac{-6x + 10}{2}$

$\boxed{y = -3x + 5}$

$\boxed{\text{slope}: -3}$

$\boxed{\text{y-int}: 5}$

Check It Out!

Write the equation in slope-intercept form. Then identify the slope and y-intercept.

1. y - 10 = 2(x - 1)

$y - 10 = 2x - 2$

$+10 \qquad +10$

$\boxed{y = 2x + 8}$

$\boxed{\text{Slope}: 2}$

$\boxed{\text{y-int}: 8}$

2. 2y - 4x = 16

$+4x \qquad +4x$

$\frac{2y}{2} = \frac{4x + 16}{2}$

$\boxed{y = 2x + 8}$

$\boxed{\text{Slope}: 2}$

$\boxed{\text{y-int}: 8}$

8.7 Function Notation

Objective: Use Function Notation.

An equation written in function notation uses f(x) to represent the output of the function *f* for an input of *x*.

Example 1: Working with Function Notation

Let f(x) = 2x - 5. Find the missing value with the given information.

1. x = -3 ; Find f(x)

$f(-3) = 2(-3) - 5$

$f(-3) = -11$

$\boxed{f(x) = -11}$

2. f(x) = 13 ; Find x

$13 = 2x - 5$

$+5 \qquad +5$

$\frac{18}{2} = \frac{2x}{2}$

$\boxed{x = 9}$

Check It Out!

Let f(x) = -x + 7. Find the missing value with the given information.

1. x = 4 ; Find f(x)

$f(4) = -4 + 7$

$f(4) = 3$

$\boxed{f(x) = 3}$

2. f(x) = 9 ; Find x

$9 = -x + 7$

$-7 \qquad -7$

$\frac{2}{-1} = \frac{-x}{-1}$

$\boxed{x = -2}$

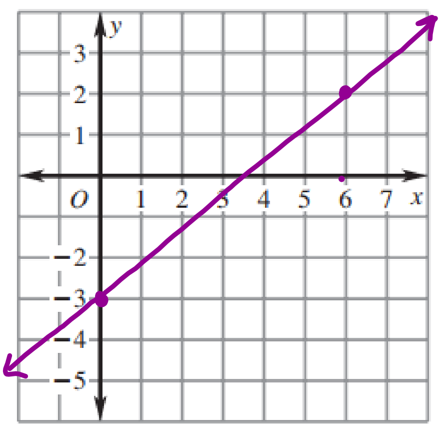
Example 2: Graphing a function

Graph the function.

1. $f(x) = \frac{5}{6}x - 3$

$y = \frac{5}{6}x - 3$

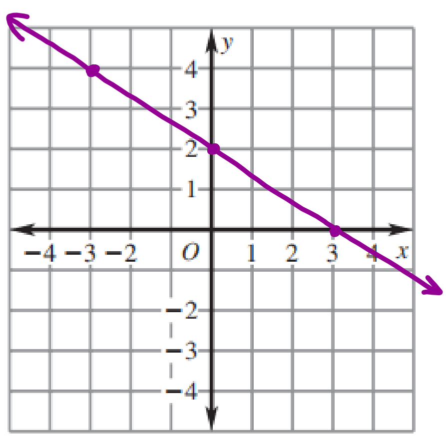
slope: $\frac{5}{6}$ y-int: -3



2. $g(x) = -\frac{2}{3}x + 2$

$y = \frac{-2}{3}x + 2$

slope: $\frac{-2}{3}$ y-int: 2



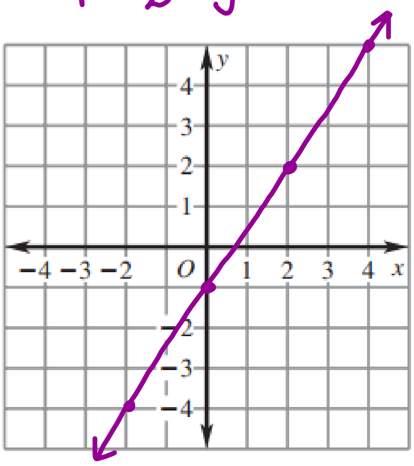
Check It Out!

Graph the function.

1. $h(x) = \frac{3}{2}x - 1$

$y = \frac{3}{2}x - 1$

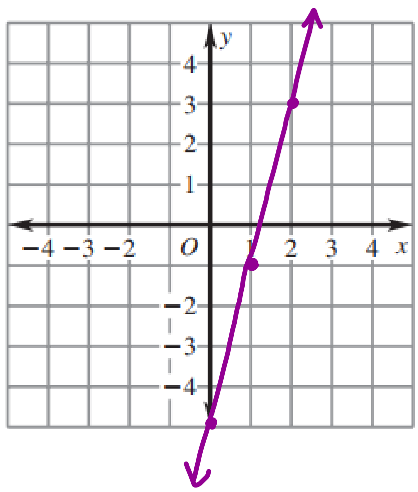
slope: $\frac{3}{2}$ y-int: -1



2. $f(x) = 4x - 5$

$y = 4x - 5$

slope: 4 y-int: -5



Example 3: Using Function Notation in real life

You ride your bike at a speed of 12 miles per hour. Use function notation to write an equation giving the distance traveled as a function of time. How long will it take you to travel 30 miles ?

Distance traveled = speed of bike × time

$\boxed{d(t) = 12t}$

$d(t) = 30$

$\frac{30}{12} = \frac{12t}{12}$

$2.5 = t$

$\boxed{\text{It will take you 2.5 hours to travel 30 miles.}}$

8.8 Systems of Linear Equations

Objective: Graph and solve systems of linear equations.

A ___system___ of linear equations consists of two or more linear equations with the same variables.

A ___solution___ of a linear system in two variables is an ordered pair that is a solution of each equation in the system.

Example 1: Solutions of systems of equations

Determine whether the given ordered pair is a solution to the systems of equations.

1. $(0, 3); \begin{cases} y = 3x + 3 & \#1 \\ y = -3x + 3 & \#2 \end{cases}$

Equation 1:

$3 = 3(0) + 3$

$3 = 3$ ✓

Equation 2:

$3 = -3(0) + 3$

$3 = 3$ ✓

$\boxed{(0,3) \text{ is a solution}}$

2. $(3, -2); \begin{cases} y = -4x + 10 & \#1 \\ y = \frac{1}{3}x + 3 & \#2 \end{cases}$

Equation 1:

$-2 = -4(3) + 10$

$-2 = -2$ ✓

Equation 2:

$-2 = \frac{1}{3}(3) + 3$

$-2 \neq 4$

$\boxed{(3,-2) \text{ is } \underline{\text{NOT}} \text{ a solution}}$

Check It Out!

Determine whether the given ordered pair is a solution to the systems of equations.

1. $(2, 4); \begin{cases} y = 4x - 4 & \#1 \\ y = \frac{1}{2}x + \frac{3}{2} & \#2 \end{cases}$

Equation 1:

$4 = 4(2) - 4$

$4 = 4$ ✓

Equation 2:

$4 = \frac{1}{2}(2) + \frac{3}{2}$

$4 \neq 2.5$

$\boxed{(2,4) \text{ is NOT a solution}}$

2. $(-2, 1); \begin{cases} y = 2x + 5 & \#1 \\ y = -x - 1 & \#2 \end{cases}$

Equation 1:

$1 = 2(-2) + 5$

$1 = 1$ ✓

Equation 2:

$1 = -(-2) - 1$

$1 = 1$ ✓

$\boxed{(-2,1) \text{ is a solution}}$

Example 2: Solving a System of Linear Equations

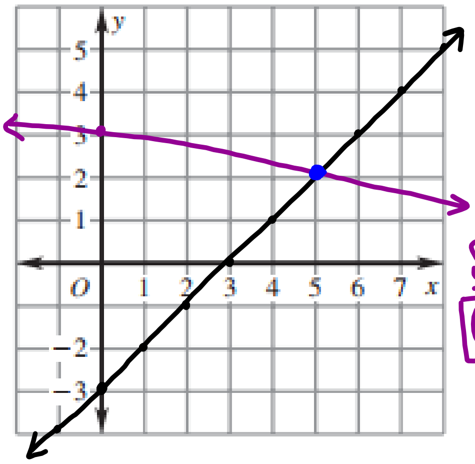
Solve the linear equations by graphing.

1. $y = x - 3$ #1

   $y = -\frac{1}{5}x + 3$ #2

Equation 1:
slope : 1
y-int : -3

Equation 2:
slope : $-1/5$
y-int : 3



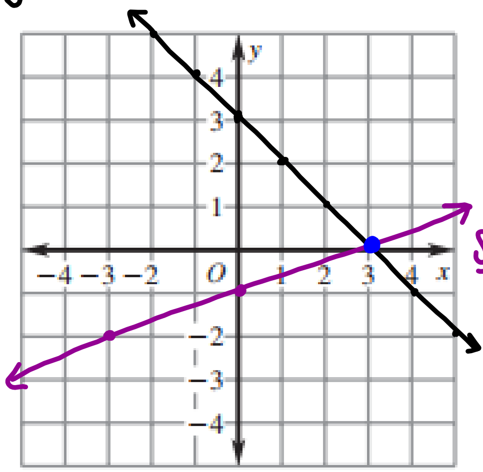
Solution: $\boxed{(5,2)}$

2. $y = -x + 3$ #1

   $y = \frac{1}{3}x - 1$ #2

Equation 1:
slope : -1
y-int : 3

Equation 2:
slope : $1/3$
y-int : -1



Solution: $\boxed{(3,0)}$

Check It Out!

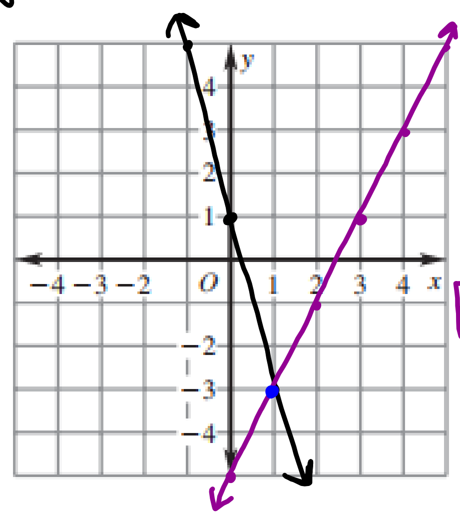
Solve the linear equations by graphing.

1. $y = -4x + 1$ #1

   $y = 2x - 5$ #2

Equation 1:
slope : -4
y-int : 1

Equation 2:
slope : 2
y-int : -5



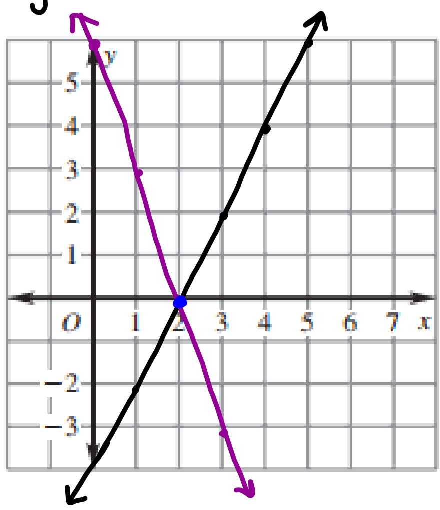
Solution: $\boxed{(1,-3)}$

2. $y = 2x - 4$ #1

   $y = -3x + 6$ #2

Equation 1:
slope : 2
y-int : -4

Equation 2:
slope : -3
y-int : 6



Solution: $\boxed{(2,0)}$

8.9 Graphs of Linear Inequalities

Objective: Graph inequalities in two variables.

A Linear inequality in two variables is the result of replacing the equal sign in a linear equation with $<, >, \leq, or\ \geq$.
The solution of a linear inequality is an ordered pair (x, y) that makes the inequality true when the values of x and y are substituted into the inequality.
The graph of a linear inequality in two variables is the set of points in a coordinate plane that represent the inequality's solutions.

Example 1: Checking Solutions of a Linear Inequality

Tell whether the ordered pair is a solution of 3x - y > 2.

1. (3, 0)

$3x-y>2$

$3(3)-0>2$

$9>2$ ✓

$\boxed{(3,0) \text{ is a solution}}$

2. (-1, 5)

$3x-y>2$

$3(-1)-5>2$

$-8 \not> 2$

$\boxed{(-1,5) \text{ is NOT a solution}}$

Check It Out!

Tell whether the ordered pair is a solution of -x + 2y > 4.

1. (1, 6)

$-x+2y>4$

$-1+2(6)>4$

$11>4$ ✓

$\boxed{(1,6) \text{ is a solution}}$

2. (-7, -2)

$-x+2y>4$

$-(-7)+2(-2)>4$

$3 \not> 4$

$\boxed{(-7,-2) \text{ is NOT a solution}}$

3. (2, 3)

$-x+2y>4$

$-2+2(3)>4$

$4 \not> 4$

$\boxed{(2,3) \text{ is NOT a solution}}$

4. (0, 5)

$-x+2y>4$

$-0+2(5)>4$

$10>4$

$\boxed{(0,5) \text{ is a solution}}$

Graphing Linear Inequalities

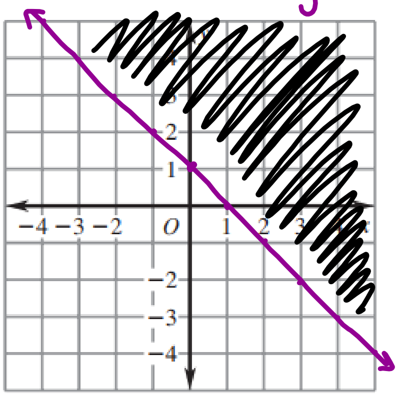
1. Find the equation of the boundary line by replacing the inequality symbol with =. Graph this equation. Use a dashed line for < or >. Use a solid line for ≤ or ≥.
2. Test a point in one of the half-planes to determine whether it is a solution of the inequality.
3. If the test point is a solution, shade the half-plane that contains the point. If not, shade the other half-plane.

Example 2: Graphing a Linear Inequality

Graph the inequalities in the coordinate plane.

1. $y \geq -x + 1$

$y = -x + 1$

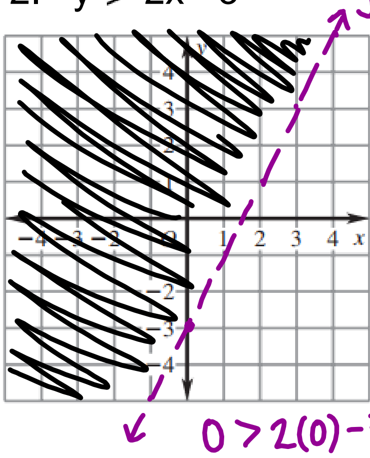


slope: -1
y-int: 1

$0 \geq -0 + 1$
$0 \ngeq 1$

2. $y > 2x - 3$

$y = 2x - 3$



slope: 2
y-int: -3

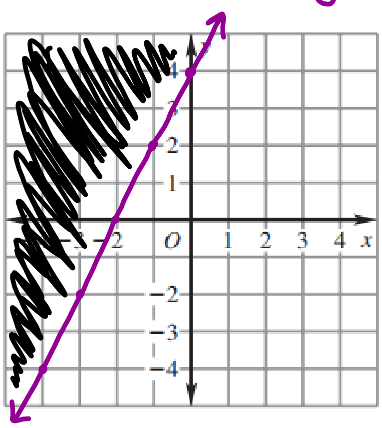
$0 > 2(0) - 3$
$0 > -3$ ✓

Check It Out!

Graph the inequalities in the coordinate plane.

1. $y \geq 2x + 4$

$y = 2x + 4$

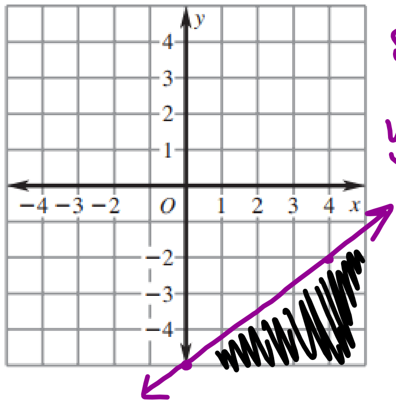


slope: 2
y-int: 4

$0 \geq 2(0) + 4$
$0 \ngeq 4$

2. $y \leq \frac{3}{4}x - 5$

$y = \frac{3}{4}x - 5$



slope: $\frac{3}{4}$
y-int: -5

$0 \leq \frac{3}{4}(0) - 5$
$0 \nleq -5$

x = # Vertical Line

y = # Horizonal Line

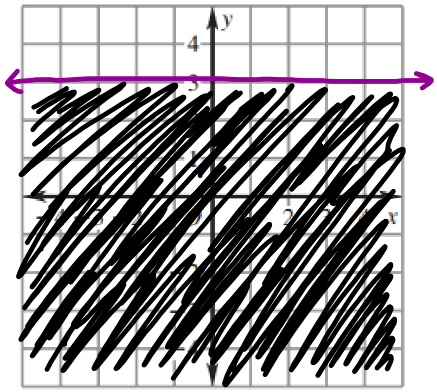
Example 3: Graphing Inequalities with One Variable

Graph the inequalities in the coordinate plane.

1. $x > -2$ $\quad x=-2$

$0 > -2$ ✓

2. $y \leq 3$ $\quad y=3$

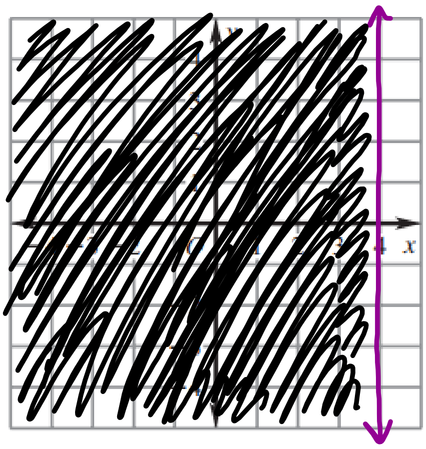


$0 \leq 3$ ✓

Check It Out!

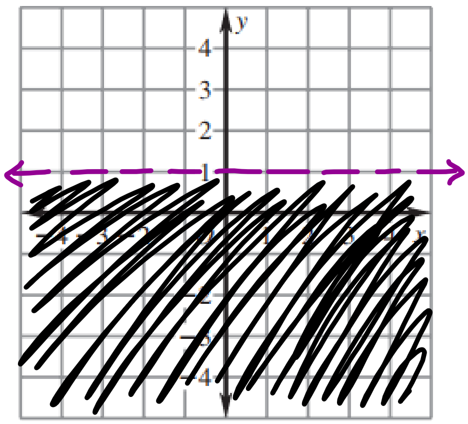
Graph the inequalities in the coordinate plane.

1. $x \leq 4$ $\quad x=4$



$0 \leq 4$ ✓

2. $y < 1$ $\quad y=1$



$0 < 1$ ✓