7.1 Percents and Fractions

Objective: Use a fraction to find the percent of a number.

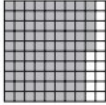A __PERCENT__ is a ratio whose denominator is 100. The symbol for percent is %.

Writing Percents

Words In the area model shown, 85 of the 100 squares are shaded. You can say that 85 percent of the squares are shaded.

Numbers $\frac{85}{100} = 85\%$

Algebra $\frac{p}{100} = p\%$

Example 1: Writing Percents as Fractions, Fractions as Percents

Write the percents as fractions in simplest form.

1. 31%

$$\frac{31}{100}$$

2. 60%

$$\frac{60}{100} = \frac{6}{10}$$

$$\frac{3}{5}$$

3. 73%

$$\frac{73}{100}$$

4. 40%

$$\frac{40}{100} = \frac{4}{10}$$

$$\frac{2}{5}$$

Write the fractions as percents.

1. $\frac{3}{10}\left(\frac{10}{10}\right)$

$$\frac{30}{100}$$

$$30\%$$

2. $\frac{4}{5}\left(\frac{20}{20}\right)$

$$\frac{80}{100}$$

$$80\%$$

3. $\frac{4}{25}\left(\frac{4}{4}\right)$

$$\frac{16}{25}$$

$$16\%$$

4. $\frac{9}{10}\left(\frac{10}{10}\right)$

$$\frac{90}{100}$$

$$90\%$$

Example 2: Writing Probability as a Percent

A computer randomly generates an integer from 1 to 10. Find the probability of the given event. Write your answer as a percent.

1. P(8) $\frac{1}{10}\left(\frac{10}{10}\right)$

$$\frac{10}{100} = 10\%$$

2. P(2) $\frac{1}{10}\left(\frac{10}{10}\right)$

$$\frac{10}{100} = 10\%$$

3. P(prime number) 2,3,5,7 $\frac{4}{10}\left(\frac{10}{10}\right)$

$$\frac{40}{100} = 40\%$$

A standard number cube is rolled. Find the probability of the given event. (1-6)

1. P(even numer)
2,4,6

$$\frac{3}{6} = \frac{1}{2}\left(\frac{50}{50}\right)$$

$$\frac{50}{100} = \boxed{50\%}$$

2. P(greater than 3)
4,5,6

$$\frac{3}{6} = \frac{1}{2}\left(\frac{50}{50}\right)$$

$$\frac{50}{100} = \boxed{50\%}$$

3. P(7)
0

$$\frac{0}{6} = 0$$

$$\boxed{0\%}$$

Example 3: Finding a Percent of a Number

In a survey of 85 people, 20% of them said they usually eat crackers with soup. How many people in the survey said they usually eat crackers with soup?

$$20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{1}{5} \cdot 85 = 17$$

17 PEOPLE EAT CRACKERS WITH SOUP.

Find the percent of the number.

1. 30% of 80

$$\frac{30}{100} = \frac{3}{10}$$

$$\frac{3}{10} \cdot 80 = 3 \cdot 8 = \boxed{24}$$

2. 60% of 105

$$\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

$$\frac{3}{5} \cdot 105 = 3 \cdot 21 = \boxed{63}$$

7.2 Percents and Proportions

Objective: Use proportions to solve percent problems.

Solving Percent Problems

You can represent "a is p percent of b" using the proportion

$$\frac{a}{b} = \frac{p}{100}$$

where a is a part of the base b and p%, or $\frac{p}{100}$, is the percent.

Example 1: Finding a Percent

1. What percent of 35 is 14?

$(100)\dfrac{14}{35} = \dfrac{P}{100}(100)$

$\dfrac{1400}{35} = P$ $\boxed{P = 40\%}$

2. What percent of 56 is 14?

$(100)\dfrac{14}{56} = \dfrac{P}{100}(100)$

$\dfrac{1400}{56} = P$ $\boxed{P = 25\%}$

3. What percent of 80 is 30?

$\dfrac{30}{80} = \dfrac{P}{100}$ $(100)\dfrac{3}{8} = \dfrac{P}{100}(100)$

$\dfrac{300}{8} = P$ $\boxed{P = 37.5\%}$

4. What percent of 30 is 27?

$(100)\dfrac{27}{30} = \dfrac{P}{100}(100)$

$\dfrac{270}{3} = P$ $\boxed{P = 90\%}$

5. What percent of 72 is 54?

$\dfrac{54}{72} = \dfrac{P}{100}$

$(100).75 = \dfrac{P}{100}(100)$ $\boxed{P = 75\%}$

6. What percent of 125 is 98?

$\dfrac{98}{125} = \dfrac{P}{100}$

$(100).784 = \dfrac{P}{100}(100)$ $\boxed{P = 78.4\%}$

7. What percent of 80 is 40?

$\dfrac{40}{80} = \dfrac{P}{100}$

$(100).5 = \dfrac{P}{100}(100)$ $\boxed{P = 50\%}$

8. What percent of 35 is 7?

$\dfrac{7}{35} = \dfrac{P}{100}$

$(100).2 = \dfrac{P}{100}(100)$ $\boxed{P = 20\%}$

Example 2: Finding a Part of a Base

1. What number is 15% of 300?

$(300)\dfrac{X}{300} = \dfrac{15}{100}(300)$

$X = 15 \cdot 3$

$\boxed{X = 45}$

2. What number is 62% of 200?

$(200)\dfrac{X}{200} = \dfrac{62}{100}(200)$

$X = 62 \cdot 2$

$\boxed{X = 124}$

3. What number is 24% of 200?

$$(200)\frac{x}{200} = \frac{24}{100}(200)$$

$$x = 24 \cdot 2$$

$$\boxed{x = 48}$$

4. What number is 18% of 50?

$$(50)\frac{x}{50} = \frac{18}{100}(50)$$

$$x = 18 \cdot \frac{1}{2}$$

$$\boxed{x = 9}$$

Example 3: Finding a Base

1. You receive 189 votes, or 45% of the votes in the student council election. How many students voted?

$$x \cdot \frac{189}{x} = \frac{45}{100} \cdot x$$

$$(100)189 = \frac{45x}{100}(100)$$

$$\frac{18900}{45} = \frac{45x}{45}$$

$$\boxed{x = 420 \text{ VOTED}}$$

2. 117 is 78% of what number?

$$x\frac{117}{x} = \frac{78}{100}x$$

$$(100)117 = \frac{78x}{100}(100)$$

$$11700 = 78x$$

$$\boxed{x = 150}$$

3. In a heptathlon, an athlete earns points in seven track-and-field events. Suppose an athlete earns 836 points in the 100 meter hurdles. This score makes up 16% of the total score. What is the total score?

$$x\frac{836}{x} = \frac{16}{100}x$$

$$(100)836 = \frac{16x}{100}(100)$$

$$\frac{83,600}{16} = \frac{16x}{16}$$

$$\boxed{x = 5225}$$

4. 105 is 84% of what number?

$$x\frac{105}{x} = \frac{84}{100}x$$

$$(100)105 = \frac{84x}{100}(100)$$

$$\frac{10,500}{84} = \frac{84x}{84}$$

$$\boxed{x = 125}$$

7.3 Percents and Decimals

Objective: Use decimals to solve percent problems.

Percents and Decimals

- To write a decimal as a percent, move the decimal point two places to the RIGHT and WRITE A % SIGN
- To write a percent as a decimal, move the decimal point two places to the LEFT and REMOVE THE % SIGN

Example 1: Writing Decimals as Percents

Write the number as a percent.

1. 0.17

   17%

2. 2

   200%

3. 3.2

   320%

4. 0.54

   54%

5. 4

   400%

6. 0.4

   40%

7. 1.75

   175%

8. 0.03

   3%

Example 2: Writing Percents as Decimals

Write the percent as a decimal.

1. 63%

   0.63

2. 0.7%

   0.007

3. 129%

   1.29

4. 3%

   0.03

5. 41%

   0.41

6. 147%

   1.47

7. 9%

   0.09

8. 12.5%

   0.125

Example 3: Writing Fractions as Percents

Write the fraction as a percent.

1. $\dfrac{4}{9}$

0.444...

$\boxed{44.\overline{4}\%}$

2. $\dfrac{7}{4}$

1.75

$\boxed{175\%}$

3. $\dfrac{5}{8}$

0.625

$\boxed{62.5\%}$

4. $\dfrac{1}{2}$

0.5

$\boxed{50\%}$

5. $\dfrac{8}{9}$

0.888...

$\boxed{88.\overline{8}\%}$

6. $\dfrac{11}{5}$

2.2

$\boxed{220\%}$

7. $\dfrac{3}{4}$

0.75

$\boxed{75\%}$

8. $\dfrac{13}{6}$

2.166...

$\boxed{216.\overline{6}\%}$

.

Example 3: Writing Fractions as Percents

In a survey of 1300 adults, 18% said the day they dread the most is Monday. How many adults chose Monday?

$18\% \cdot 1300$

$0.18\,(1300)$

$\boxed{234 \quad \text{ADULTS}}$

Find the percent of the number.

1. 25% of 76

$25\% \cdot 76$

$0.25\,(76)$

$\boxed{19}$

2. 110% of 65

$110\% \cdot 65$

$1.1\,(6.5)$

$\boxed{71.5}$

3. 0.7% of 500

$0.7\% \cdot 500$

$0.007\,(500)$

$\boxed{3.5}$

1. 20% of 85

$0.2\,(85)$

$\boxed{17}$

2. 3.8% of 45

$0.038\,(45)$

$\boxed{1.71}$

3. 125% of 64

$1.25\,(64)$

$\boxed{80}$

7.4 The Percent Equation

The Percent Equation

You can represent "a is p percent of b" using the equation

$a = p\% \cdot b$

where a is a part of the base b and p% is the percent.

Example 1: Finding a Part of a Base

In a newspaper's survey, 1100 adults were asked to name their favorite condiment. The most frequent response was ketchup, which was given by 47% of adults. How many adults chose ketchup?

$47\%(1100)$

$0.47(1100)$ 517 ADULTS

Use the percent equation to answer the question.

1. What number is 15% of 60?

$X = 0.15(60)$

$X = 9$

2. What number is 76% of 105?

$X = 0.76(105)$

79.8

3. What number is 16% of 75?

$X = 0.16(75) = 12$

4. What number is 89% of 110?

$X = 0.89(110) = 97.9$

Example 2: Find a Commission

1. A sales person earns 5.5% commission on every car sold. The sales person sells a car for $41,200. What is the commission?

$0.055(41,200)$

$\$2266$

What would the commission be if the sales person sold a car for $45,000?

$0.055(45,000)$

$\$2475$

2. A car salesman earns 6.5% commission on every car sold. The salesman sells a car for $21,800. What is the commission?

$0.065(21,800)$

$\$1417$

Find the commission if the car is sold for $23,000.

$0.065(23,000)$ $\$1495$

Example 3: Find a Percent

1. What percent of 24 is 84?

$$\frac{x \cdot 24}{24} = \frac{84}{24}$$

$$x = 3.5 \quad \boxed{350\%}$$

2. What percent of 15 is 21?

$$\frac{x \cdot 15}{15} = \frac{21}{15}$$

$$x = 1.4 \quad \boxed{140\%}$$

3. What percent of 72 is 45?

$$\frac{x \cdot 72}{72} = \frac{45}{72}$$

$$x = 0.625 \quad \boxed{62.5\%}$$

4. What percent of 25 is 60?

$$\frac{x \cdot 25}{25} = \frac{60}{25}$$

$$x = 2.4 \quad \boxed{240\%}$$

5. What percent of 48 is 45?

$$\frac{x \cdot 48}{48} = \frac{45}{48}$$

$$x = 0.9375 \quad \boxed{93.75\%}$$

6. 2.7 is 7.5% of what number?

$$\frac{2.7}{0.075} = \frac{0.075x}{0.075}$$

$$x = \boxed{36}$$

7. 33 is 30% of what number?

$$33 = 0.3x \quad \boxed{x = 110}$$

8. 90 is 37.5% of what number?

$$90 = 0.375x \quad \boxed{x = 240}$$

Example 4: Finding a Base

1. Your friend paid $48 for a ticket to a professional football game. This amount was 64% of the total amount your friend spent at the game. How much money did your friend spend?

$$\frac{48}{0.64} = \frac{0.64x}{0.64}$$

$$75 = x \quad \boxed{\$75}$$

2. Your friend paid $9 for a movie ticket. This amount was 72% of the total amount your friend spent at the theater. How much money did your friend spend?

$$\frac{9}{0.72} = \frac{0.72x}{0.72}$$

$$12.5 = x \quad \boxed{\$12.50}$$

7.5 Percent of Change

Objective: Find a percent of change in a quantity.

A _PERCENT OF CHANGE_ indicates how much a quantity increases or decreases with respect to the original amount.

The _PERCENT OF INCREASE_ in a quantity when the new amount of the quantity is greater than the original amount.

The _PERCENT OF DECREASE_ in a quantity when the new amount of the quantity is less than the original amount.

Percent of Change

The percent of change is the ratio of the amount of increase or decrease to the original amount.

$$\text{Percent of change, } p\% = \frac{\text{Amount of increase or decrease}}{\text{Original amount}}$$

Example 1: Finding a Percent of Increase

A school had 720 students enrolled last year. This year, 745 students are enrolled. By about what percent did they number of students change from last year to this year?

$$\frac{745-720}{720} = \frac{25}{720} \approx 0.035$$

$$\boxed{3.5\% \quad \text{INCREASE}}$$

Example 2: Finding a Percent of Decrease

Determine whether the change is increase or decrease. Then find the percent of increase or decrease.

1. Original: 576 $\frac{576-216}{576}$
   New: 216
   $\frac{360}{576} = 0.625$

   __62.5%__

   __DECREASE__

2. Original: 512 $\frac{512-320}{512}$
   New: 320
   $\frac{192}{512} = 0.375$

   __37.5%__

   __DECREASE__

3. Original: 20 $\frac{20-15}{20}$
   New: 15
   $\frac{5}{20} = 0.25$

   __25%__

   __DECREASE__

4. Original: 35 $\frac{75-35}{35}$
   New: 75
   $\frac{40}{35} = 1.143$

   __114.3%__

   __INCREASE__

5. Original: 102 $\frac{102-51}{102}$
   New: 51
   $\frac{51}{102} = 0.5$

   __50%__

   __DECREASE__

6. Original: 25 $\frac{31-25}{25}$
   New: 31
   $\frac{6}{25} = 0.24$

   __24%__

   __INCREASE__

Example 3: Using a Percent of Increase

1. A professional baseball team announces that the average ticket price to one of their games will be 8% more than last year. If the average of a ticket was $12 last year, how much will the average ticket cost this year?

$$\frac{\text{TICKET COST}}{\text{THIS YEAR}} = \frac{\text{TICKET COST}}{\text{LAST YEAR}} + \frac{\text{AMOUNT OF}}{\text{INCREASE}}$$

$$X = 12 + (0.08)12$$
$$X = 12 + 0.96$$
$$X = 12.96$$

$$\boxed{\$12.96}$$

2. There were about 198,000 spectators at an action sports event in 1995. The number of spectators increased by about 12% from 1995 to 2002. About how many spectators were there in 2002?

$$X = 198,000 + (0.12)198,000$$
$$X = 198,000 + 23,760$$
$$X = \boxed{221,760 \quad \text{SPECTATORS}}$$

Example 4: Finding a New Amount

In 1990, the average price per pound of light chunk tuna was $2.11. By 2001, the average price per pound had decreased by 7.1%. What was the average price per pound in 2001?

$$\text{PRICE IN 2001} = \text{PRICE IN 1990}(100\% - p\%)$$
$$= 2.11(1 - 0.071)$$
$$= 2.11(0.929) \qquad \boxed{\$1.96}$$
Find the new amount. $= 1.96019$

1. Increase 54 by 25%
   $$54(1+.25)$$
   $$54(1.25) = \boxed{67.5}$$
3. Decrease 85 by 28%
   $$85(1-0.28)$$
   $$85(0.72) = \boxed{61.2}$$
5. Increase 25 by 24%
   $$25(1+0.24)$$
   $$25(1.24) = \boxed{31}$$
7. Decrease 35 by 60%
   $$35(1-0.6)$$
   $$35(0.4) = \boxed{14}$$

2. Decrease 78 by 40%
   $$78(1-0.4)$$
   $$78(0.6) = \boxed{46.8}$$
4. Increase 45 by 20%
   $$45(1+0.2)$$
   $$45(1.2) = \boxed{54}$$
6. Increase 120 by 75%
   $$120(1+0.75)$$
   $$120(1.75) = \boxed{210}$$
8. Decrease 72 by 65%
   $$72(1-0.65)$$
   $$72(0.3) = \boxed{25.2}$$

7.6 Percent Applications

Objective: Find markups, discounts, sales tax, and tips.

An increase from the wholesale price of an item to the retail price is a
__MARKUP__.

A decrease from the original price of an item to the sale price is a
__DISCOUNT__.

Example 1: Finding a Retail Price

1. A store buys decorative pillows from a manufacturer for $2 each. The store marks up the price by 400%. What is the retail price?

RETAIL PRICE = WHOLESALE PRICE (100% + MARKUP %)

$$= 2(1+4)$$
$$= 2(5)$$
$$= 10$$

$\boxed{\$10}$

Use the given information to find the new price.

2. Wholesale Price: $7

   Markup Percent: 150%

   $7(1+1.5)$
   $7(2.5) = 17.5$ $\boxed{\$17.50}$

3. Wholesale price: $13

   Markup Percent: 110%

   $13(1+1.1)$
   $13(2.1) = 27.3$ $\boxed{\$27.30}$

4. Wholesale Price: $34

   Markup Percent: 125%

   $34(1+1.25)$
   $34(2.25) = 76.5$ $\boxed{\$76.50}$

5. Wholesale price: $125

   Markup Percent: 50%

   $125(1+0.5)$
   $125(1.5) = 187.5$

   $\boxed{\$187.50}$

Example 2: Finding a Sale Price

1. You buy a backpack that is on sale for 25% off the original price of $20. What is the sale price?

SALE PRICE = ORIGINAL PRICE (100% - DISCOUNT %)

$$= 20(1-0.25)$$
$$= 20(0.75)$$
$$= 15$$

$\boxed{\$15}$

Use the given information to find the new price.

2. Original Price: $15

   Discount Percent: 40%

   $15(1-0.4)$
   $15(0.6) = 9$ $\boxed{\$9}$

3. Original Price: $42

   Discount Percent: 25%

   $42(1-0.25)$
   $42(0.75) = 31.5$ $\boxed{\$31.50}$

Example 3: Using Sales Tax and Tips

1. You order pizza to be delivered. The bill is $18. You give the delivery person a 15% tip. The sales tax is 5%. What is the total cost of the pizza?

$$\text{TOTAL} = \text{FOOD BILL} + \text{SALES TAX} + \text{TIP}$$
$$= 18 + (0.05)18 + (0.15)18$$
$$= 18 + 0.9 + 2.7$$
$$= 21.6$$

$\boxed{\$ 21.60}$

2. The bill for your restaurant meal is $22. You leave a 15% tip. The sales tax is 6%. What is the total cost of your meal?

$$22 + (0.06)22 + (0.15)25$$
$$22 + 1.32 + 3.3$$
$$26.62$$

$\boxed{\$26.62}$

Use the given information to find the total cost.

3. Original Price: $25

Sales Tax: 5%

Tip: 15%

$$25 + (0.05)25 + (0.15)25$$
$$25 + 1.25 + 3.75$$
$$30$$

$\boxed{\$30}$

4. Original price: $11

Sales Tax: 6%

Tip: 15%

$$11 + (.06)11 + (.15)11$$
$$11 + 0.66 + 1.65$$
$$13.31$$

$\boxed{\$13.31}$

Example 4: Finding an Original Amount

1. A store marks up the wholesale price of a blender by 125%. The retail price is $30. What is the wholesale price?

$$\text{RETAIL PRICE} = \text{WHOLESALE PRICE}(100\% + \text{MARKUP }\%)$$
$$30 = x(1 + 1.25)$$
$$30 = x(2.25)$$
$$13.33 \approx x$$

$\boxed{\$13.33}$

2. A furniture store discounts the price of a desk lamp by 25%. The sale price is $150. What is the original price?

$$150 = x(1 - 0.25)$$
$$150 = x(0.75)$$
$$200 = x$$

$\boxed{\$200}$

Use the given information to find the original price.

3. Retail Price: $50

Markup Percent: 90%

$$50 = x(1 + .9)$$
$$50 = x(1.9)$$
$$26.32 = x$$

$\boxed{\$26.32}$

4. Sale Price: $210

Discount Percent: 30%

$$210 = x(1 - 0.3)$$
$$210 = x(.7)$$
$$300 = x$$

$\boxed{\$300}$

7.7 Simple and Compound Interest

Objective: Calculate interest earned and account balances.

___INTEREST___ is the amount earned or paid for the use of money.

The amount of money deposited or borrowed is the ___PRINCIPAL___.

Interest that is earned or paid only on the principal is called
___SIMPLE INTEREST___.

The percent of the principal earned or paid per year is the
___ANNUAL INTEREST RATE___.

The ___BALANCE___ A of an account that earns simple annual interest is
the sum of the principal P and the interest Prt. In symbols this means

$$A = P + Prt$$

OR

$$A = P(1 + rt)$$

Simple Interest Formula

Simple interest I is given by the formula

$$I = Prt$$

where P is the principal, r is the annual interest rate (written as a decimal), and t is the time in years.

Example 1: Finding Simple Interest

1. A $2000 bond earns 3% simple interest per year on its purchase price. Find the interest earned after 5 years and the balance of the account.

$$I = Prt$$
$$I = 2000(0.03)5$$
$$I = 300$$
$$TOTAL = 2000 + 300 = 2300$$

INTEREST = $300
TOTAL = $2300

2. If a $1500 bond earns 4% simple interest per year on its purchase price, how much will it earn in interest after 2 years? What is the total balance of the account?

$$I = Prt$$
$$I = 1500(0.04)2$$
$$I = 120$$
$$T = 1500 + 120 = 1620$$

INTEREST = $120
TOTAL = $1620

For an account that earns simple annual interest, find the interest and balance of the account.

3. $P=\$500$, $r=7\%$, $t=4$ years

$I = 500(0.07)4$
$I = 140$

$\boxed{I = \$140 \qquad T = \$640}$

4. $P=\$2500$, $r=3\%$, $t=9$ months

$I = 2500(0.03)\left(\frac{9}{12}\right)$
$I = 2500(0.03)(0.75) = 56.25$

$\boxed{I = \$56.25 \qquad T = \$2556.25}$

Example 2: Finding an Interest Rate

1. You deposit $900 into an account that earns simple annual interest. After 8 months, the balance is $913.20. Find the annual interest rate.

$A = P(1+rt)$
$913.20 = 900\left(1 + r\left(\frac{8}{12}\right)\right)$
$913.20 = 900 + 600r$

$13.2 = 600r$
$0.022 = r$

$\boxed{\text{RATE IS } 2.2\%}$

2. Suppose you save $1400 of your pay and deposit it into an account that earns 2% interest. The balance reaches $1421. Find the amount of time needed to reach this balance.

$1421 = 1400(1 + 0.02t)$
$1421 = 1400 + 28t$

$21 = 28t$
$0.75 = t$

$0.75(12) = 9$

$\boxed{9 \text{ MONTHS}}$

Find the unknown quantity for an account that earns simple annual interest.

3. $P=\$1000$, $r=2.5\%$, $t=2$ years

$A = 1000(1+(0.025)2)$
$A = 1000(1 + 0.05)$
$A = 1000(1.05)$
$A = 1050 \qquad \boxed{\$1,050}$

4. $A=\$1424.50$, $r=3.5\%$, $t=6$ months

$1424.50 = P\left(1 + (0.035)\frac{1}{2}\right)$
$1424.50 = P(1.0175)$
$1400 = P$

$\boxed{\$1400}$

5. $A=\$563$, $P=\$500$, $t=7$ years

$563 = 500(1 + r\,7)$
$563 = 500 + 3500r$
$63 = 3500r$
$0.018 = r$

$\boxed{1.8\%}$

6. $A=\$1670$, $P=\$1600$, $r=3.5\%$

$1670 = 1600(1 + 0.035t)$
$1670 = 1600 + 56t$
$70 = 56t$
$1.25 = t$

$\boxed{1.25 \text{ YEARS}}$