Chapter 6: Ratios, Proportions, & Probability
6.1: Ratios & Rates

Objective: Find ratios and unit rates.

Vocabulary

A ratio uses division to compare two quantities.

Two ratios are called equivalent ratios when they have the same value.

A ratio of two quantities measure in different units is called a rate.

A rate that has a denominator of 1 when expressed in fraction form is called a unit rate.

Writing Ratios

You can write the ratio of two quantities, a and b, where b is not equal to 0, in three ways.

a to b $\quad a:b \quad \frac{a}{b}$

Each ratio is read "the ratio of a to b." You should write the ratio in simplest form.

Example 1: Writing Ratios

1. In a recent baseball season, the Anaheim Angels played 81 home games. Anaheim won 54 of those games and lost 27. Write each ratio in three ways.

a. The number of losses to the number of wins.

$\frac{\text{\# of losses}}{\text{\# of wins}} = \frac{27}{54} = \frac{1}{2}$ $\boxed{\frac{1}{2}, 1:2, 1 \text{ to } 2}$

b. The number of losses to the number of games.

$\frac{\text{\# of losses}}{\text{\# of games}} = \frac{27}{81} = \frac{1}{3}$ $\boxed{\frac{1}{3}, 1:3, 1 \text{ to } 3}$

c. The number of wins to the number of losses.

$\frac{\text{\# of wins}}{\text{\# of loss}} = \frac{54}{27} = \frac{2}{1}$ $\boxed{\frac{2}{1}, 2:1, 2 \text{ to } 1}$

d. The number of wins to the number of games.

$\frac{\text{\# of wins}}{\text{\# of games}} = \frac{54}{81} = \frac{2}{3}$ $\boxed{\frac{2}{3}, 2:3, 2 \text{ to } 3}$

2. The table shows the number of sailboats and motorboats in two boat stores. Use the table to write the specified ratio.

	Sailboats	Motorboats
Store A	17	38
Store B	9	22

a. Sailboats in Store B to sailboats in Store A.

$\frac{\text{B sailboats}}{\text{A sailboats}} = \frac{9}{17}$ $\boxed{\frac{9}{17}, 9:17, 9 \text{ to } 17}$

b. Motorboats in Store A to motorboats in both stores.

$\frac{\text{A motorboats}}{\text{B motorboats}} = \frac{38}{22} = \frac{19}{11}$ $\boxed{\frac{19}{11}, 19:11, 19 \text{ to } 11}$

Example 2: Finding a Unit Rate

1. On the first day of a family vacation, you and your family drive 392 miles. The amount of gasoline used is 16 gallons. What is the average miles per gallon of gasoline?

$\frac{\text{total distance}}{\text{\# of gallons}} = \frac{392 \text{ miles}}{16 \text{ gallons}} \begin{matrix} \div 16 \\ = \\ \div 16 \end{matrix} \frac{24.5 \text{ miles}}{1 \text{ gallon}}$ $\boxed{24.5 \text{ mpg}}$

Find the unit rate.

2. $\frac{220 \text{ mi}}{4 \text{ h}} \frac{\div 4}{\div 4}$ $\boxed{\frac{55 \text{ mi}}{1 \text{ h}}}$ or $\boxed{55 \text{mph}}$

3. $\frac{\$115}{5 \text{ people}} \frac{\div 5}{\div 5}$ $\boxed{\frac{\$23}{1 \text{ person}}}$

4. $\frac{100 \text{ calories}}{5 \text{ cookies}} \frac{\div 5}{\div 5}$ $\boxed{\frac{20 \text{ calories}}{1 \text{ cookie}}}$

5. Donald feeds his dog 35 dog treats in 7 days.

$\frac{35 \text{ treats}}{7 \text{ days}} \frac{\div 7}{\div 7} = \boxed{\frac{5 \text{ treats}}{1 \text{ day}}}$

Example 3: Writing Equivalent Rates

The amount of water used in a certain home is 728 gallons per week. Write this rate in gallons per day. Remember: 1 week = 7 days

$\frac{728 \text{ gallons}}{1 \text{ week}} \cdot \frac{1 \text{ week}}{7 \text{ days}} = \frac{728 \text{ gallons}}{7 \text{ days}} \frac{\div 7}{\div 7} = \boxed{\frac{104 \text{ gallons}}{1 \text{ day}}}$

Example 4: Using Equivalent Rates

1. Lightning strikes occur about 100 times per second around the world. About how many lightning strikes occur in 3 minutes? Remember: 1 min = 60 sec

$\frac{100 \text{ strikes}}{1 \text{ sec}} \cdot \frac{60 \text{ sec.}}{1 \text{ min}} = \frac{6000 \text{ strikes}}{1 \text{ min.}}$

$\frac{6000 \text{ strikes}}{1 \text{ min.}} \cdot 3 \text{ min.} = \boxed{18{,}000 \text{ times}}$

Write the equivalent rate.

2. $\frac{\$20}{1 \text{ h}} \frac{\times 42}{\times 42} = \frac{? \text{ dollars}}{42 \text{ h}}$ $\boxed{\frac{\$840}{42 \text{h}}}$

3. $\frac{475 \text{ m}}{5 \text{ sec}} \frac{\div 5}{\div 5} = \frac{? \text{ m}}{1 \text{ sec}}$ $\boxed{\frac{95 \text{ m}}{1 \text{ sec.}}}$

Example 5: Ordering Ratios

Write the following ratios in order from least to greatest.

3:4, 5 to 8, $\frac{5}{9}$, 2: 6, 3 to 7

2:6, 3 to 7, $\frac{5}{9}$, 5 to 8, 3:4

6.2: Writing & Solving Proportions

Objective: Write and solve proportions.

Proportions

Words A **proportion** is an equation that states that two ratios are equivalent.

Numbers $\frac{2}{3} = \frac{8}{12}$

Algebra $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$

Example 1: Solving a Proportions Using Equivalent Ratios

Use equivalent ratios to solve the proportions.

1. $\frac{3}{5} = \frac{x}{20}$ ×4 ×4 $\boxed{x=12}$

2. $\frac{2}{9} = \frac{x}{27}$ ×3 ×3 $\boxed{x=6}$

3. $\frac{5}{6} = \frac{x}{36}$ ×6 ×6 $\boxed{x=30}$

Example 2: Solving a Proportions Using Algebra

Solve the proportions. Check your answer.

1. $\frac{x}{15} = \frac{2}{5}$ ×3 ×3 $\boxed{x=6}$

2. $\frac{3}{7} = \frac{x}{28}$ ×4 ×4 $\boxed{x=12}$

3. $\frac{8}{11} = \frac{x}{55}$ ×5 ×5 $\boxed{x=40}$

4. $\frac{x}{8} = \frac{49}{56}$

   $\frac{49}{56} \frac{\div 7}{\div 7} = \frac{x}{8}$

   $\boxed{x=7}$

5. $\frac{x}{6} = \frac{14}{3}$ ×3 ×3 $\boxed{x=42}$

6. $\frac{x}{33} = \frac{10}{3}$ ×11 ×11 $\boxed{x=110}$

Example 3: Writing and Solving a Proportion

The sap of maple trees is used to make maple syrup. It takes 40 gallons of sap to make 1 gallon of maple syrup.

1. Write and solve a proportion to find the number of gallons of maple syrup that can be made from 1520 gallons of sap.

$\frac{\text{gallons of syrup}}{\text{gallons of sap}}$ $\frac{1}{40} \overset{\times 38}{\underset{\times 38}{=}} \frac{x}{1520}$ $x = \boxed{\text{38 gallons of syrup}}$

2. Write and solve a proportion to find the number of gallons of maple syrup that can be make drom 1360 gallons of sap.

$\frac{1}{40} \overset{\times 34}{\underset{\times 34}{=}} \frac{x}{1360}$ $x = \boxed{\text{34 gallons of syrup}}$

3. Write and solve a proportion to find how many gallons of sap are needed to make 25 gallons of maple syrup.

$\frac{1}{40} \overset{\times 25}{\underset{\times 25}{=}} \frac{25}{x}$ $\boxed{x = \text{1000 gallons of sap}}$

Example 4: Writing and Solving an Exchange Rate Problem

The current exchange rate between the United states and Jamaica is 1 US dollar to 124 Jamaican dollars.

a. If you have 15 US dollars to exchange, how many Jamaican dollars will you get in exchange?

$\frac{\text{US dollars}}{\text{Jamaican dollars}}$ $\frac{1}{124} \overset{\times 15}{\underset{\times 15}{=}} \frac{15}{x}$ $x = \boxed{\text{1860 Jamaican dollars}}$

b. If you have 372 Jamaican dollars to exchange, how many US dollars would you get in exchange?

$\frac{1}{124} \overset{\times 3}{\underset{\times 3}{=}} \frac{x}{372}$ $x = \boxed{\text{3 US dollars}}$

6.3: Solving Proportions Using Cross Products

Objective: Solve proportions using cross products.

Vocabulary

A cross products of two ratios is the product of the numerator of one ratio and the denominator of the other ratio.

Example 1: Determining if Ratios Form a Proportion

Tell whether the ratio forms a proportion.

1. $\frac{4}{26}, \frac{8}{42}$

$\frac{4}{26} \stackrel{?}{=} \frac{8}{42}$

$\frac{2}{13} \stackrel{?}{=} \frac{4}{21}$

$\boxed{\text{NO}}$

2. $\frac{12}{21}, \frac{20}{35}$

$\frac{12}{21} \stackrel{?}{=} \frac{20}{35}$

$\frac{4}{7} \stackrel{?}{=} \frac{4}{7}$

$\boxed{\text{Yes}}$

3. $\frac{3}{14}, \frac{9}{42}$

$\frac{3}{14} \stackrel{?}{=} \frac{9}{42}$

$\frac{3}{14} \stackrel{?}{=} \frac{3}{14}$

$\boxed{\text{Yes}}$

4. $\frac{19}{24}, \frac{75}{96}$

$\frac{19}{24} \stackrel{?}{=} \frac{75}{96}$

$\frac{19}{24} \stackrel{?}{=} \frac{25}{32}$

$\boxed{\text{NO}}$

Example 2: Writing and Solving a Proportion

1. You earn \$68 mowing 4 lawns. How much would you earn if you mowed 7 lawns?

$\frac{\$}{\text{lawn}}$

$\frac{68}{4} = \frac{x}{7}$

$(7)17 = \frac{x}{7}(7)$

$x = \boxed{\$119}$

2. Jasmine bought 32 kiwi fruit for \$16. How many kiwi can Lisa buy if she has \$4?

$\frac{\text{Kiwi}}{\$}$

$\frac{32}{16} = \frac{x}{4}$

$(4)\ 2 = \frac{x}{4}(4)$

$8 = x$

$\boxed{\text{8 Kiwi}}$

3. Bob works for 5 hours and makes \$40. How much will he have to work to earn \$120?

$\frac{\text{hours}}{\$}$

$\frac{5}{40} \overset{\times 3}{=} \frac{x}{120}$ (×3)

$x = \boxed{\text{15 hours}}$

Solve the proportion.

1. $(42)\dfrac{14}{42}=\dfrac{x}{6}(42)$

$(6)14=\dfrac{42x}{6}(6)$

$\dfrac{84}{42}=\dfrac{42x}{42}$

$\boxed{x=2}$

2. $(a)\dfrac{4}{9}=\dfrac{16}{a}(a)$

$(9)\dfrac{4a}{9}=16(9)$

$4a=144$

$\boxed{a=36}$

3. $(b)\dfrac{3}{b}=\dfrac{36}{60}(b)$

$(60)3=\dfrac{36b}{60}(60)$

$\dfrac{180}{36}=\dfrac{36b}{36}$

$\boxed{b=5}$

4. $(32)\dfrac{n}{32}=\dfrac{15}{60}(32)$

$n=\dfrac{480}{60}$

$\boxed{n=8}$

5. $(x)\dfrac{9}{45}=\dfrac{6}{x}(x)$

$(45)\dfrac{9x}{45}=6(45)$

$\dfrac{9x}{9}=\dfrac{270}{9}$

$\boxed{x=30}$

6. $(14)\dfrac{81}{63}=\dfrac{f}{14}(14)$

$\dfrac{1134}{63}=f$

$\boxed{18=f}$

7. $\dfrac{7}{3}=\dfrac{x}{6}$

$3x=7\cdot6$

$\dfrac{3x}{3}=\dfrac{42}{3}$

$\boxed{x=13}$

2. $\dfrac{2}{5}=\dfrac{16}{x}$

$\dfrac{2x}{2}=\dfrac{80}{2}$

$\boxed{x=40}$

3. $\dfrac{5}{x}=\dfrac{15}{21}$

$\dfrac{15x}{15}=\dfrac{105}{15}$

$\boxed{x=7}$

4. $\dfrac{x}{13}=\dfrac{15}{3}$

$\dfrac{3x}{3}=\dfrac{195}{3}$

$\boxed{x=65}$

5. $\dfrac{11}{9}=\dfrac{33}{x}$

$\dfrac{11x}{11}=\dfrac{297}{11}$

$\boxed{x=27}$

6. $\dfrac{17}{2}=\dfrac{x}{14}$

$\dfrac{2x}{2}=\dfrac{238}{2}$

$\boxed{x=119}$

Cross Products Property

Words The cross products of a proportion are equal.

Numbers Given that $\frac{2}{5}=\frac{6}{15}$, you know that $\boxed{2\cdot15}=\boxed{5\cdot6}$.

Algebra If $\frac{a}{b}=\frac{c}{d}$, where $b\neq0$ and $d\neq0$, then $\boxed{ad}=\boxed{bc}$.

6.4: Similar and Congruent Figures

Objective: Identify similar and congruent figures.

Vocabulary

Two figures are Similiar figures if they have the same shape but not necessarily the same size. ~

Corresponding parts of figures are the sides or angles that have the same relative position.

Two figures are congruent if they have the same shape and size. ≅

When naming similar figures, list the letters of the corresponding vertices in the same order. For the diagram at the right, it is not correct to say $\triangle CBA \sim \triangle EFD$, because $\angle C$ and $\angle E$ are not corresponding angles.

Properties of Similar Figures

$\triangle ABC \sim \triangle DEF$

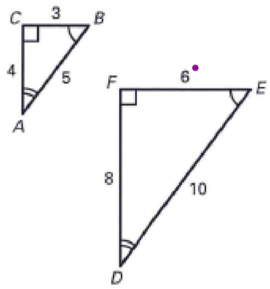
The symbol ~ indicates that two figures are similar.

1. Corresponding angles of similar figures are congruent.

   $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

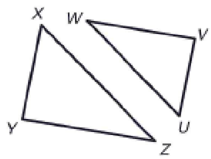
2. The ratios of the lengths of corresponding sides of similar figures are equal.

   $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$



Example 1: Identifying Corresponding Parts of Similar Figures

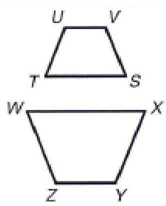
1. Given $\Delta XYZ \sim \Delta UVW$, name the corresponding angles and the corresponding sides.



Corresponding angles: $\angle X \cong \angle U$, $\angle Y \cong \angle V$, $\angle Z \cong \angle W$

Corresponding sides: $\overline{XY}$ & $\overline{UV}$, $\overline{YZ}$ & $\overline{VW}$, $\overline{ZX}$ & $\overline{WU}$

2. Given $STUV \sim WXYZ$, name the corresponding angles and the corresponding sides.



Corresponding angles: $\angle S \cong \angle W$, $\angle T \cong \angle X$, $\angle U \cong \angle Y$, $\angle V \cong \angle Z$

Corresponding sides: $\overline{ST}$ & $\overline{WX}$, $\overline{TU}$ & $\overline{XY}$, $\overline{UV}$ & $\overline{YZ}$, $\overline{VS}$ & $\overline{ZW}$

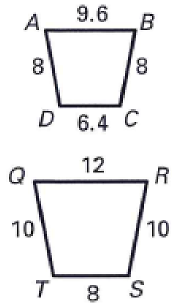
3. Given $\Delta ABC \sim \Delta XYZ$, name the corresponding angles and the corresponding sides.

Corresponding angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$

Corresponding sides: $\overline{AB}$ & $\overline{XY}$, $\overline{BC}$ & $\overline{YZ}$, $\overline{CA}$ & $\overline{ZX}$

Example 2: Finding the Ratio of Corresponding Side Lengths

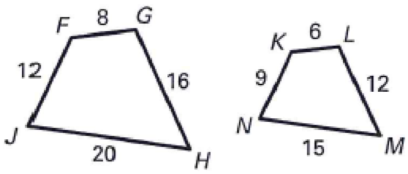
1. Given ABCD ∼ QRST, find the ratio of the lengths of the corresponding sides of ABCD to QRST.



$\overline{AD}$ & $\overline{QT}$

$$\frac{AD}{QT} = \frac{8}{10} = \boxed{\frac{4}{5}}$$

2. Given FGHJ ∼ KLMN, find the ratio of the lengths of the corresponding sides of FGHJ to KLMN.

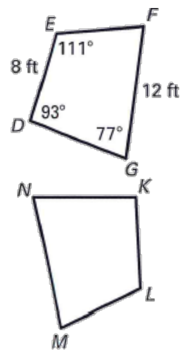


$\overline{FG}$ & $\overline{KL}$

$$\frac{FG}{KL} = \frac{8}{6} = \boxed{\frac{4}{3}}$$

Example 2: Finding the Ratio of Corresponding Side Lengths

1. Given DEFG ≅ KLMN, find the indicated measure.



a. Find $m\angle L$ $\angle L \cong \angle E$, $m\angle L = m\angle E = \boxed{111°}$

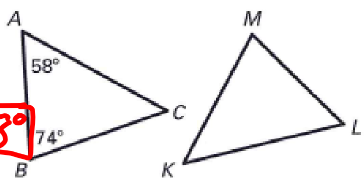
b. Find $m\angle N$ $\angle N \cong \angle G$, $m\angle N = m\angle G = \boxed{77°}$

c. Find $m\angle K$ $\angle K \cong \angle D$, $m\angle K = m\angle D = \boxed{93°}$

d. KL $\overline{KL} \cong \overline{DE}$, $KL = DE = \boxed{8 \text{ ft.}}$

e. MN $\overline{MN} \cong \overline{FG}$, $MN = FG = \boxed{12 \text{ ft.}}$

2. Given △ABC ≅ △LMK.



a. Find $m\angle L$ $\angle L \cong \angle A$, $m\angle L = m\angle A = \boxed{58°}$

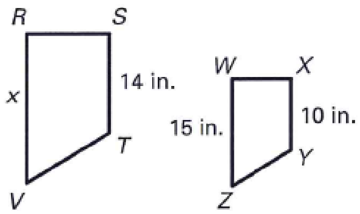
b. Find $m\angle M$ $\angle M \cong \angle B$, $m\angle M = m\angle B = \boxed{74°}$

6.5: Similarity and Measurement

Objective: Find unknown side lengths of similar figures.

Example 1: Finding Unknown Side Lengths of Similar Figures

1. Given RSTV $\sim$ WXYZ, find VR.



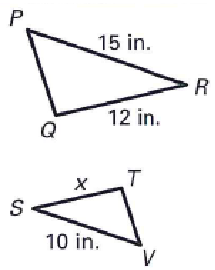
$\frac{XY}{ST} = \frac{WZ}{RV}$

$\frac{10}{14} = \frac{15}{x}$

$\frac{10x}{10} = \frac{210}{10}$

$\boxed{x = 21 \text{ in.}}$

2. Given Δ PQR $\sim$ Δ VTS. Find TS.



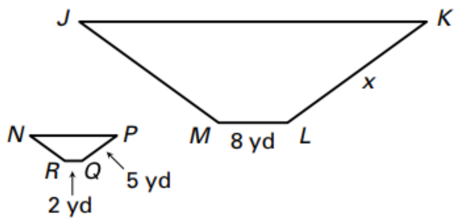
$\frac{PR}{VS} = \frac{QR}{TS}$

$\frac{15}{10} = \frac{12}{x}$

$\frac{15x}{15} = \frac{120}{15}$

$\boxed{x = 8 \text{ in.}}$

3. Given JKLM $\sim$ NPQR, find KL.



$\frac{LM}{QR} = \frac{KL}{PQ}$

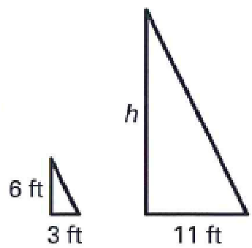
$\frac{8}{2} = \frac{x}{5}$

$\frac{2x}{2} = \frac{40}{2}$

$\boxed{x = 20 \text{ yd.}}$

Example 2: Use Indirect Measurement

1. At a certain time of day, a person who is 6 feet tall casts a 3-foot shadow. At the same time, a tree casts an 11-foot shadow. The triangles formed are similar. Find the height of the tree.

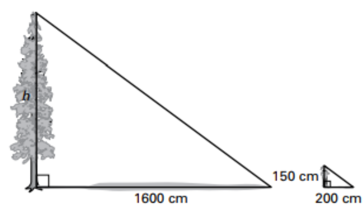


$\frac{h}{6} = \frac{11}{3}$

$\frac{3h}{3} = \frac{66}{3}$

$\boxed{h = 22 \text{ ft.}}$

2. A girl who is 150 centimeters tall is standing next to a tree. The tree and the girl are perpendicular to the ground. The sun's rays strike the tree and the girl at the same angle, forming two similar triangles. The length of the girl's shadow is 200 centimeters, and the length of the tree's shadow is 1600 centimeters. How tall is the tree?



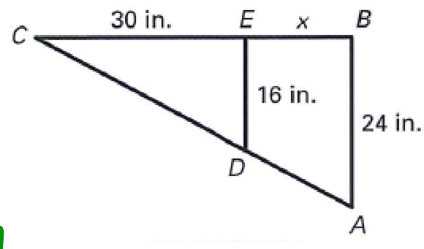
$\frac{h}{150} = \frac{1600}{200}$

$\frac{200h}{200} = \frac{240{,}000}{200}$

$\boxed{h = 1200 \text{ cm}}$

Example 3: Using Algebra and Similar Triangles

1. Given $\Delta ABC \sim \Delta DEC$, find BE.



$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{24}{16} = \frac{x+30}{30}$$

$$16(x+30) = 720$$

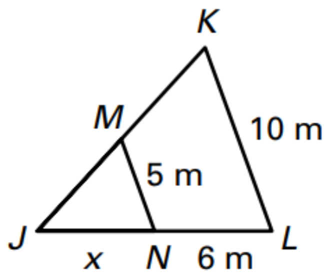
$$16x + 480 = 720$$

$$-480 \quad -480$$

$$\frac{16x}{16} = \frac{240}{16}$$

$$\boxed{x = 15 \text{ in.}}$$

2. Given $\Delta JKL \sim \Delta JMN$, find JN.



$$\frac{KL}{MN} = \frac{JL}{JN}$$

$$\frac{10}{5} = \frac{x+6}{x}$$

$$5(x+6) = 10x$$

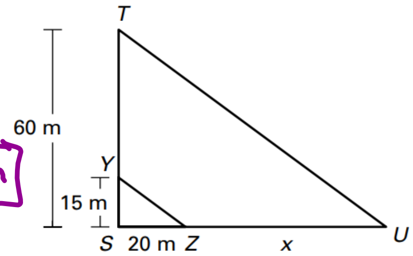
$$5x + 30 = 10x$$

$$-5x \quad -5x$$

$$\frac{30}{5} = \frac{5x}{5}$$

$$\boxed{x = 6 \text{ m}}$$

3. Given $\Delta STU \sim \Delta SYZ$, find ZU.



$$\frac{ST}{SY} = \frac{SU}{SZ}$$

$$\frac{60}{15} = \frac{x+20}{20}$$

$$1200 = 15(x+20)$$

$$1200 = 15x + 300$$

$$-300 \quad -300$$

$$\frac{900}{15} = \frac{15x}{15}$$

$$\boxed{x = 60 \text{ m}}$$

6.6: Scale Drawings

Objective: Use proportions with scale drawings.

Vocabulary

A <u>scale drawing</u> is a two-dimensional drawing that is similar to the object it represents.

A <u>scale model</u> is a three-dimensional model that is similar to the object it represents.

The <u>scale</u> of a scale drawing or scale model gives the relationship between the drawing or model's dimensions and the actual dimensions.

Example 1: Using a Scale Drawing

1. On a map, the distance between two cities is 3 inches. What is the actual distance (in miles) between the two cities if the map's scale is 1 in.:125 mi?

$$\frac{1\text{ in.}}{125\text{ mi.}} = \frac{3\text{ in.}}{x}$$

$$x = \boxed{375\text{ miles}}$$

2. On a map, the distance between two cities is 4 inches. What is the actual distance (in miles) between the two cities if the map's scale is 1 in.: 80 mi?

$$\frac{1\text{ in.}}{80\text{ mi.}} = \frac{4\text{ in.}}{x} \qquad x = \boxed{320\text{ miles}}$$

3. The scale of a blueprint is 1 in. : 3 ft. The actual length of the living room is 18 feet. Find the length of the living room on the blueprint

$$\frac{1\text{ in.}}{3\text{ ft.}} = \frac{x}{18\text{ ft.}} \qquad \frac{3x}{3} = \frac{18}{3}$$

$$x = \boxed{6\text{ in.}}$$

Example 2: Finding the Scale of a Drawing

1. In a scale drawing, a wall is 2 inches long. The actual wall is 12 feet long. Find the scale of the drawing.

$$\frac{2\text{ in.}}{12\text{ ft.}} \begin{matrix}\div 2\\ = \\ \div 2\end{matrix} \frac{1\text{ in.}}{6\text{ ft.}} \qquad \boxed{1\text{ in.} : 6\text{ ft.}}$$

2. A gardener is making a scale drawing of a garden. The length of the actual garden is 9 yards. The length of the garden in the drawing is 3 inches. Find the drawing's scale.

$$\frac{3\text{ in.}}{9\text{ yds.}} \begin{matrix}\div 3\\ = \\ \div 3\end{matrix} \frac{1\text{ in.}}{3\text{ yd.}} \qquad \boxed{1\text{ in.} : 3\text{ yd.}}$$

3. The distance between two cities on a map is 12 millimeters. The actual distance between the cities is 60 miles. Find the map's scale.

$$\frac{12\text{ mm}}{60\text{ mi}} \begin{matrix}\div 12\\ = \\ \div 12\end{matrix} \frac{1\text{ mm}}{5\text{ mi}} \qquad \boxed{1\text{ mm} : 5\text{ mi.}}$$

4. The length of a drawing of a fireplace is 10 inches. The actual length is 4 feet. Find the drawing's scale.

$$\frac{10\text{ in.}}{4\text{ ft.}} \begin{matrix}\div 10\\ = \\ \div 10\end{matrix} \frac{1\text{ in}}{.4\text{ ft}} \qquad \boxed{1\text{ in.} : 0.4\text{ ft.}}$$

Example 3: Finding a Dimension of a Scale Drawing

1. A model of the Sears Tower in Chicago has a scale of 1:103. The height of the Sears Tower's observation deck is about 412 meters. Find the height of the observation deck of the model.

$$\frac{1}{103} = \frac{x}{412} \qquad \frac{103x}{103} = \frac{412}{103}$$

$$x = \boxed{4\text{ meters}}$$

2. A model of a boat has a scale of 1 : 15. The model's length is 2 feet. Find the actual length of the boat.

$$\frac{1}{15} = \frac{2}{x} \qquad x = \boxed{30\text{ ft.}}$$

3. A model of a ladybug has a scale of 1 in. : 0.3 cm. The ladybug's actual length is 1.2 centimeters. Find the model's length.

$$\frac{1}{0.3} = \frac{x}{1.2} \qquad \frac{0.3\cancel{x}}{0.3} = \frac{1.2}{0.3}$$

$$\boxed{x = 4\text{ in.}}$$

4. A model of a building has a scale of 1 : 40. The building is 100 feet tall. Find the height of the model.

$$\frac{1}{40} = \frac{x}{100} \qquad \frac{40x}{40} = \frac{100}{40}$$

$$\boxed{x = 2.5\text{ft.}}$$

6.7: Probability and Odds

Objective: Use proportons with scale drawings.

Vocabulary

The possible results of an experiment are outcomes.

An event is an outcome or a collection of outcomes.

The outcomes for a specified event are called favorable outcomes.

The probability that an event occurs is a measure of the likelihood that the even will occur.

A theoretical probability is based on knowing all of the equally likely outcomes of an experiement.

A probability that is based on repeated trials of an experiment is called an experimental probability. Each trial in which the event occurs is a success.

The ratio of the number of favorable outcomes to the number of unfavorable outcomes is called the odds in favor of an event.

The ratio of the number of unfavorable outcomes to the number of favorable outcomes is called the odds against an event.

Probability of an Event

The probability of an event when all outcomes are equally likely is:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

Example 1: Finding a Probability

Find the probability of the given event.

1. Suppose you roll a standard number cube.

a. Rolling a 4.

$P(4) = \boxed{\frac{1}{6}}$

b. Rolling a number less than 5.

1, 2, 3, 4

$P(\# < 5) = \frac{4}{6} = \boxed{\frac{2}{3}}$

c. Multiple of 3.

3, 6

$P(\text{multiple of } 3) = \frac{2}{6} = \boxed{\frac{1}{3}}$

d. Rolling an odd number.

1, 3, 5

$P(\text{odd}) = \frac{3}{6} = \boxed{\frac{1}{2}}$

2. Suppose you flip a coin.

a. flipping a heads

$P(\text{Heads}) = \boxed{\frac{1}{2}}$

b. flipping a tails

$P(\text{tails}) = \boxed{\frac{1}{2}}$

3. A jar contains 15 red marbles, 16 blue marbles, 5 yellow marbles, and 10 green marbles. You randomly choose one marble from the jar. What is the probability that you choose a green marble?

$P(\text{Green}) = \frac{10}{46} = \boxed{\frac{5}{23}}$

4. A drawer contains 12 white socks, 4 red socks, 6 green socks, and 10 blue socks. You randomly choose one sock from the drawer. Find the probability that you choose a red sock.

$P(\text{Red}) = \frac{4}{32} = \boxed{\frac{1}{8}}$

5. Suppose you draw a card from a standard deck of cards.

a. drawing a heart

$P(\text{Heart}) = \frac{13}{52} = \boxed{\frac{1}{4}}$

b. drawing a 3

$P(3) = \frac{4}{52} = \boxed{\frac{1}{13}}$

c. drawing a black card

$P(\text{Black}) = \frac{26}{52} = \boxed{\frac{1}{2}}$

d. drawing a face card → K, Q, J

$P(\text{face card}) = \frac{12}{52} = \boxed{\frac{3}{13}}$

Experimental Probability

The experimental probability of an event is:

$$P(\text{event}) = \frac{\text{Number of successes}}{\text{Number of trials}}$$

Example 2: Finding Experimental Probability

1. You plant 32 seeds of a certain flower and 18 of them sprout. Find the experimental probability that the next flower seed planted will sprout.

$P(\text{sprout}) = \frac{18}{32} = \boxed{\frac{9}{16}}$

2. You surveyed 65 randomly chosen students. Of the students you surveyed, 25 went camping during summer break. Find the experimental probability that the next randomly chosen student went camping during summer break.

$P(\text{camping}) = \frac{25}{65} = \boxed{\frac{5}{13}}$

3. You are playing darts. Out of 45 attempts, you hit the bull's eye 18 times. Find the experimental probability that you hit the bull's eye on your next attempt.

$P(\text{Bull's Eye}) = \frac{18}{45} = \boxed{\frac{2}{5}}$

Example 3: Finding the Odds

1. Suppose you randomly choose a number between 1 and 16.

   a. What are the odds in favor of choosing a prime number? 2,3,5,7,11,13

   $\frac{\text{\# favorable outcomes}}{\text{\# unfavorable outcomes}} = \frac{6}{10} = \boxed{\frac{3}{5}}$

   b. What are the odds against choosing a prime number?

   $\frac{\text{\# unfavorable outcomes}}{\text{\# favorable outcomes}} = \frac{10}{6} = \boxed{\frac{5}{3}}$

   c. What are the odds in favor of choosing a factor of 15?

   1, 3, 5, 15

   $\frac{4}{12} = \boxed{\frac{1}{3}}$

   d. What are the odds against choosing a factor of 15?

   $\frac{12}{4} = \boxed{\frac{3}{1}}$

2. You randomly choose a letter from a bag containing one of each letter of the alphabet.

   a. What are the odds in favor of choosing a vowel? A, E, I, O, U, Y

   $\frac{6}{20} = \boxed{\frac{3}{10}}$

   b. What are the against choosing a vowel?

   $\frac{20}{6} = \boxed{\frac{10}{3}}$

3. You spin a spinner that is divided into 20 equal parts. Six parts are orange, 8 parts are white, 3 parts are black, 2 parts are red, and 1 part is blue.

   a. Find the odds in favor of spinning white.

   $\frac{8}{12} = \boxed{\frac{2}{3}}$

   b. Find the odds against spinning white.

   $\frac{12}{8} = \boxed{\frac{3}{2}}$

   c. Find the odds in favor of spinning black.

   $\boxed{\frac{3}{17}}$

   c. Find the odds against spinning black.

   $\boxed{\frac{17}{3}}$