5.1 Rational Numbers

Objective: Write fractions as decimals and vice versa.

A rational number is a number that can be written as the quotient of two integers.

A terminating is a decimal that has a final digit.

A repeating decimal is a decimal that has one or more digits that repeat without end.

Example 1: Identifying Rational Numbers

Show that the number is rational by writing it as a quotient of two integers.

1. -12

$\frac{-12}{1}$ $\frac{12}{-1}$

2. $-2\frac{1}{4}$

$-2\frac{+1}{\times 4}$

$\frac{-9}{4}$ $\frac{9}{-4}$

Check It Out!

Show that the number is rational by writing it as a quotient of two integers.

1. 3

$\frac{3}{1}$

2. $4\frac{2}{3}$

$4\frac{+2}{\times 3}$

$\frac{14}{3}$

Example 2: Writing Fractions as Decimals

Write the following fractions as a decimal.

1. $\frac{5}{16}$

0.3125

$16\overline{)5.0000}$

48↓

20↓

−16

40↓

32

80

−80

0

2. $\frac{4}{9}$

$0.\overline{4}$

$9\overline{)4.000}$

36↓

40

−36

4

3. $\frac{7}{20}$

0.35

$20\overline{)7.000}$

−60↓

100

−100

0

Check It Out!

Write the following fractions as a decimal.

1. $-\frac{3}{5}$

−0.6

$5\overline{)3.000}$

−30

0

2. $2\frac{12}{25}$

$\frac{62}{25}$

2.48

$25\overline{)62.00}$

−50↓

120↓

100

200

−200

0

3. $\frac{23}{40}$

0.575

$40\overline{)23.000}$

200↓

300

−280↓

200

−200

0

Example 3: Using Decimals to Compare Fractions

Compare the following fractions.

1. $\frac{42}{38}$ and $\frac{27}{30}$

$\frac{42}{38} > \frac{27}{30}$

1.10
38 | 42.00
38↓
40
38↓
20

0.9
30 | 27.0
-270
0

Example 4: Writing Terminal Decimals as Fraction

Write the terminal decimal as a fraction.

1. 0.03 $\frac{3}{100}$

2. -9.4 $-9\frac{4}{10}$ $-9\frac{2}{5}$

Check It Out!
Write the terminal decimal as a fraction.

1. 0.8 $\frac{8}{10}$ $\frac{4}{5}$

2. -3.75 $-3\frac{75}{100}$ $-3\frac{3}{4}$

Example 5: Writing a Repeating Decimal as a Fraction

Write the repeating decimal as a fraction.

1. $0.\overline{81}$

$100x = 81.81$
$-\ x = 00.81$
$99x = 81$
$\frac{99x}{99} = \frac{81}{99}$

$x = \frac{81}{99}$ $\frac{9}{11}$

2. $0.\overline{12}$

$100x = 12.12$
$-\ x = 00.12$
$99x = 12$
$\frac{99x}{99} = \frac{12}{99}$

$x = \frac{12}{99}$ $\frac{4}{33}$

Check It Out!
Write the repeating decimal as a fraction.

1. $5.\overline{3}$

$100x = 33.33$
$-\ x = 00.33$
$99x = 33$
$\frac{99x}{99} = \frac{33}{99}$

$x = \frac{33}{99}$ $5\frac{1}{3}$

8.5
2 | 17.0
16
10

Example 6: Ordering Rational Numbers

Order the numbers from least to greatest.

1. $-\frac{17}{2}$, -1.35, 5.67, -6, $\frac{11}{3}$, $-\frac{13}{4}$

-8.5 $3.\overline{66}$ -3.25

$-\frac{17}{2}, -6, -\frac{13}{4}, -1.35, \frac{11}{3}, 5.67$

3.66
3 | 11.000
-9↓
20
18↓
20

3.25
4 | 13.00
12
10
8
20

5.2 Adding and Subtracting Like Fractions

Objective: Add and subtract like fractions.

Adding and Subtracting Like Fractions

Words To add or subtract fractions with the same denominator, write the sum or difference of the numerators over the denominator.

Numbers $\frac{4}{9}+\frac{1}{9}=\frac{5}{9}$ $\qquad \frac{9}{11}-\frac{2}{11}=\frac{7}{11}$

Algebra $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}, c\neq 0$ $\qquad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}, c\neq 0$

Example 1: Adding Like Fractions

Find the sum.

1. $-\frac{4}{9}+\frac{3}{9}$

$\frac{-4+3}{9}=\frac{-1}{9}$

2. $\frac{2}{7}+\frac{4}{7}$

$\frac{2+4}{7}=\frac{6}{7}$

Check It Out!
Find the sum.

1. $\frac{2}{11}+\left(-\frac{3}{11}\right)$

$\frac{2+-3}{11}=\frac{-1}{11}$

2. $\frac{3}{8}+\frac{8}{8}$

$\frac{3+8}{8}=\frac{11}{8}$ or $1\frac{3}{8}$

Example 2: Subtracting Like Fractions

Find the difference.

1. $-\frac{4}{9}-\frac{3}{9}$

$\frac{-4+-3}{9}=\frac{-7}{9}$

2. $\frac{2}{7}-\frac{4}{7}$

$\frac{2+-4}{7}=\frac{-2}{7}$

Check It Out!
Find the difference.

1. $\frac{2}{11}-\left(-\frac{3}{11}\right)$

$\frac{2+3}{11}=\frac{5}{11}$

2. $\frac{3}{8}-\frac{8}{8}$

$\frac{3+-8}{8}=\frac{-5}{8}$

Example 3: Adding and Subtracting Mixed Numbers

Find the sum or difference.

1. $4\frac{3}{7} + 3\frac{6}{7}$

$\frac{31}{7} + \frac{27}{7}$

$\frac{31+27}{7} = \boxed{\frac{58}{7}}$ or $\boxed{8\frac{2}{7}}$

2. $11\frac{3}{10} - 8\frac{9}{10}$

$\frac{113}{10} - \frac{89}{10}$

$\frac{113-89}{10} = \boxed{\frac{24}{10}}$ or $\boxed{2\frac{2}{5}}$

Check It Out!

Find the sum or difference.

1. $3\frac{2}{11} + 5\frac{4}{11}$

$\frac{35}{11} + \frac{59}{11}$

$\frac{35+59}{11} = \boxed{\frac{94}{11}}$ or $\boxed{8\frac{6}{11}}$

2. $-4\frac{5}{13} - 3\frac{6}{13}$

$-\frac{57}{13} + -\frac{45}{13}$

$\frac{-57+-45}{13} = \boxed{\frac{-102}{13}}$ or $\boxed{-7\frac{11}{13}}$

Example 4: Simplifying Variable Expressions

Simplify.

1. $\frac{4a}{21} + \frac{10a}{21}$

$\frac{4a+10a}{21}$

$\frac{14a}{21} = \boxed{\frac{2a}{3}}$

2. $-\frac{9}{5b} + \left(+\frac{4}{5b}\right)$

$\frac{-9+4}{5b}$

$\frac{-5}{5b} = \boxed{\frac{-1}{b}}$

Check It Out!

Simplify.

1. $\frac{2a}{25} + \frac{8a}{25}$

$\frac{2a+8a}{25}$

$\frac{10a}{25} = \boxed{\frac{2a}{5}}$

2. $-\frac{17}{3c} + \left(+\frac{5}{3c}\right)$

$\frac{-17+5}{3c}$

$\frac{-12}{3c} = \boxed{\frac{-4}{c}}$

5.3 Adding and Subtracting Unlike Fractions

Objective: Add or subtract unlike fractions.

Example 1: Adding and Subtracting Fractions

Find the sum or difference.

1. $\frac{7}{15} + \frac{1}{5}$ .3

$\frac{7}{15} + \frac{3}{15}$

$\frac{7+3}{15} = \frac{10}{15} \div 5$

$\boxed{\frac{2}{3}}$

2. .4 $\frac{2}{3} - \frac{3}{4}$ .3

$\frac{8}{12} - \frac{9}{12}$

$\frac{8-9}{12} = \boxed{\frac{-1}{12}}$

Check It Out!

Find the sum or difference.

1. .3 $\frac{3}{7} + \frac{5}{21}$

$\frac{9}{21} + \frac{5}{21}$

$\frac{9+5}{21} = \frac{14}{21} \div 7$ $\boxed{\frac{2}{3}}$

2. .5 $\frac{1}{4} - \frac{3}{10}$ .2

$\frac{5}{20} - \frac{6}{20}$

$\frac{5-6}{20} = \boxed{\frac{-1}{20}}$

Example 2: Adding Mixed Numbers

Find the sum.

1. $-5\frac{1}{6} + \left(-\frac{3}{10}\right)$ .5 $\frac{-31}{6} + \frac{-3}{10}$ .3

$\frac{-155}{30} + \frac{-9}{30}$

$\frac{-155+-9}{30} = \frac{-164}{30}$

$\boxed{-5\frac{7}{15}}$

2. $4\frac{5}{6} + 2\frac{4}{9}$ .3 $\frac{29}{6} + \frac{22}{9}$ .2

$\frac{87}{18} + \frac{44}{18}$

$\frac{87+44}{18} = \frac{131}{18}$

$\boxed{7\frac{5}{18}}$

Check It Out!

Find the sum.

1. $-4\frac{2}{5} + \left(-2\frac{6}{11}\right)$ .11 $\frac{-22}{5} + \frac{-28}{11}$ .5

$\frac{-242}{55} + \frac{-140}{55}$

$\frac{-242+-140}{55} = \frac{-382}{55}$

$\boxed{-6\frac{52}{55}}$

2. $3\frac{5}{9} + 2\frac{1}{6}$ .2 $\frac{32}{9} + \frac{13}{6}$ .3

$\frac{64}{18} + \frac{39}{18}$

$\frac{64+39}{18} = \frac{103}{18}$

$\boxed{5\frac{13}{18}}$

Example 3: Subtracting Mixed Numbers

1. You are hiking a $12\frac{1}{5}$ mile trail. You have already hiked 6 $\frac{1}{2}$ miles. How many more miles do you have to hike before reaching the end of the trail?

$12\frac{1}{5} - 6\frac{1}{2}$ 2. $\frac{61}{5} - \frac{13}{2}$ .5

$\frac{122}{10} - \frac{65}{10}$

$\frac{122-65}{10} = \frac{57}{10}$

$5\frac{7}{10}$ left to hike

Check It Out!
Find the difference.

1. $-2\frac{1}{3} - 3\frac{3}{7}$ 7. $\frac{-7}{3} - \frac{24}{7}$ .3

$\frac{-49}{21} + \frac{-72}{21}$

$\frac{-49+-72}{21} = \frac{-121}{21}$

$-5\frac{16}{21}$

2. $6\frac{7}{18} - 8\frac{21}{54}$ 3. $\frac{115}{18} - \frac{453}{54}$

$\frac{345}{54} - \frac{453}{54}$

$\frac{345-453}{54} = \frac{-108}{54}$ -2

Example 4: Simplifying an Expression

Simplify the expression.

1. 2. $\frac{a}{4} - \frac{a}{8}$

$\frac{2a}{8} - \frac{a}{8}$

$\frac{2a-a}{8} = \frac{a}{8}$

2. 8. $\frac{b}{3} - \frac{b}{8}$ .3

$\frac{8b}{24} - \frac{3b}{24}$

$\frac{8b-3b}{24} = \frac{5b}{24}$

Check It Out!
Simplify the expression.

1. 7. $\frac{c}{5} - \frac{c}{7}$ .5

$\frac{7c}{35} - \frac{5c}{35}$

$\frac{7c-5c}{35} = \frac{2c}{35}$

2. 3. $\frac{a}{2} - \frac{a}{6}$

$\frac{3a}{6} - \frac{a}{6}$

$\frac{3a-a}{6} = \frac{2a}{6} \div 2$ $\frac{a}{3}$

5.4 Multiplying Fractions

Objective: Multiply fractions and mixed numbers.

Multiplying Fractions

Words The product of two or more fractions is equal to the product of the numerators over the product of the denominators.

Numbers $\frac{3}{5} \cdot \frac{4}{7} = \frac{3 \cdot 4}{5 \cdot 7} = \frac{12}{35}$

Algebra $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, where $b \neq 0$ and $d \neq 0$

Example 1: Multiplying Fractions

Find the product.

1. $\frac{5}{12} \cdot \left(\frac{-3}{20}\right) = \frac{-1}{16}$

2. $\frac{7}{16} \cdot \frac{5}{14} = \frac{5}{32}$

Check It Out!
Find the product.

1. $\frac{2}{15} \cdot \left(\frac{-5}{18}\right) = \frac{-1}{27}$

2. $\frac{7}{10} \cdot \left(\frac{-4}{21}\right) = \frac{-2}{15}$

Example 2: Multiplying a Mixed Number and an Integer

The showerhead in your house uses $2\frac{1}{2}$ gallons of water per minutes. If you take a 7-minute shower, how many gallons of water do you use?

$2\frac{1}{2} \cdot 7$

$\frac{5}{2} \cdot \frac{7}{1} = \frac{35}{2}$

You use $17\frac{1}{2}$ gallons of water.

Check It Out!
Find the product.

1. $5\frac{2}{9} \cdot 6$

$\frac{47}{9} \cdot \frac{6}{1} = \frac{94}{3}$

$31\frac{1}{3}$

2. $2\frac{3}{4} \cdot (-12)$

$\frac{11}{4} \cdot \frac{-12}{1} = \frac{-33}{1}$

-33

Example 3: Multiplying Mixed Numbers

Find the product.

1. $-3\frac{1}{5} \cdot 4\frac{1}{6}$

$-\frac{16}{5} \cdot \frac{25}{16} = -\frac{40}{3}$

$-13\frac{1}{3}$

2. $-2\frac{3}{4} \cdot 5\frac{1}{3}$

$-\frac{11}{4} \cdot \frac{16}{3} = -\frac{44}{3}$

$-14\frac{2}{3}$

Check It Out!

Find the product.

1. $-2\frac{3}{4} \cdot 3\frac{1}{5}$

$-\frac{11}{4} \cdot \frac{16}{5} = -\frac{44}{5}$

$-8\frac{4}{5}$

2. $4\frac{7}{8} \cdot 5\frac{2}{3}$

$\frac{39}{8} \cdot \frac{17}{3} = \frac{221}{8}$

Example 4: Simplifying Expressions

Find the product.

1. $\frac{m}{4} \cdot \left(\frac{-10}{7}\right)^{-5}$

$-5m$ / 14

2. $\frac{n^4}{12} \cdot \frac{9n^2}{10}$

$3n^6$ / 40

Check It Out!

Find the product.

1. $\frac{2x}{5} \cdot \frac{3x^2}{8}$

$3x^2$ / 20

2. $-\frac{4y^3}{15} \cdot \frac{5y^6}{16}$

$-y^9$ / 12

5.5 Dividing Fractions

Objective: Divide fractions and mixed numbers.

Two nonzero numbers whose product is 1 are reciprocals.

Using Reciprocals to Divide

Words To divide by any nonzero number, multiply by its reciprocal.

Numbers $\frac{2}{9} \div \frac{3}{7} = \frac{2}{9} \cdot \boxed{\frac{7}{3}} = \boxed{\frac{14}{27}}$

Algebra $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \boxed{\frac{d}{c}} = \boxed{\frac{ad}{bc}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$

Example 1: Dividing a Fraction by a Fraction

Find the quotient.

1. $-\frac{3}{7} \div \frac{6}{11}$ $\quad \frac{-3}{7} \cdot \frac{11}{6_2} = \frac{-11}{14}$ $\quad \boxed{\frac{-11}{14}}$

2. $\frac{8}{21} \div \frac{9}{14}$ $\quad \frac{8}{21_3} \cdot \frac{14^2}{9} = \frac{16}{27}$ $\quad \boxed{\frac{16}{27}}$

Check It Out!

Find the quotient.

1. $-\frac{5}{12} \div \frac{5}{24}$ $\quad \frac{-5}{12_1} \cdot \frac{24^2}{5_1} = \frac{-2}{1}$ $\quad \boxed{-2}$

2. $-\frac{2}{5} \div \frac{4}{7}$ $\quad \frac{-2}{5} \cdot \frac{7}{4_2} = \frac{-7}{10}$ $\quad \boxed{\frac{-7}{10}}$

Example 2: Dividing a Mixed Number by a Mixed Number

Find the quotient.

1. $2\frac{1}{2} \div -3\frac{6}{11}$ $\quad \frac{5}{2} \div \frac{-39}{11}$ $\quad \frac{5}{2} \cdot \frac{11}{-39} = \frac{55}{-78}$ $\quad \boxed{\frac{55}{-78}}$

2. $4\frac{3}{5} \div 1\frac{7}{10}$ $\quad \frac{23}{5} \div \frac{17}{10}$ $\quad \frac{23}{5_1} \cdot \frac{10^2}{17} = \frac{46}{17}$ $\quad \boxed{2\frac{12}{17}}$

Check It Out!

Find the quotient.

1. $-3\frac{1}{4} \div 5\frac{1}{2}$ $\quad \frac{-13}{4} \div \frac{11}{2}$ $\quad \frac{-13}{2_4} \cdot \frac{2^1}{11} = \frac{-13}{22}$ $\quad \boxed{\frac{-13}{22}}$

2. $\frac{3}{8} \div 9\frac{1}{6}$ $\quad \frac{3}{8} \div \frac{55}{6}$ $\quad \frac{3}{8_4} \cdot \frac{6^3}{55} = \frac{9}{220}$ $\quad \boxed{\frac{9}{220}}$

Example 3: Dividing a Mixed Number by a Mixed Number

You have two dogs that eat about $1\frac{1}{5}$ pounds of dog food per day. How many whole days will a 5-pound bag of dog good last?

Number of Days = Number of pounds in bag ÷ Number of pounds eaten per day

$= 5 \div 1\frac{1}{5}$

$= \frac{5}{1} \div \frac{6}{5}$

$= \frac{5}{1} \cdot \frac{5}{6}$

$= \frac{25}{6}$

$= \boxed{4\frac{1}{6}}$

They will need to buy a 5 pound bag for 4 days.

Check It Out!

You want to join strips of wood that are 15 inches long and $1\frac{5}{8}$ inches wide to make a cutting board that is at least 12 inches wide. How many strips are needed?

Number of strips = Cutting Board Width ÷ Strip Width

$= 12 \div 1\frac{5}{8}$

$= \frac{12}{1} \div \frac{13}{8}$

$= \frac{12}{1} \cdot \frac{8}{13}$

$= \frac{96}{13} = \boxed{7\frac{5}{13}}$

They will need 8 strips to make sure that the cutting board is at least 12 inches wide.

5.6.5 Reasonableness of Answers

Objective: Assess the Reasonableness of answers by using mental math and estimation.

Example 1: Estimating with Decimals and Fractions

Evaluate the expression. Use mental math and estimation to assess the reasonableness of your answer.

1. 5.3 - 2 + 8.9

3.3+8.9

12.2

2. 4 1/2 - 25 - 7.25

45-25-7.25

-20.5-7.25

-27.75

3. 8.7 + 5 1/4 + 2.4

8.7+5.25+2.4

13.95+2.4

16.35

Check It Out!

Evaluate the expression. Use mental math and estimation to assess the reasonableness of your answer.

1. 1.03 + 2 + 5.4 + 7.9

3.03+5.4+7.9

8.43+7.9

16.33

2. 15.3 + 18.1 + 9

33.1+9

42.1

3. 6.2 + 3 3/4 - 5.9

6.2+3.75-5.9

9.95-5.9

4.05

Example 2: Assessing Reasonableness

1. A skit has a 1.9 minute song, a 2 1/4 minute dialoge, and a 3.2 minute song. The teacher calculates that the skit is 7 minutes and 20 seconds long. Is this reasonable? Explain.

$1.9+2\frac{1}{4}+3.2$

$=1.9+3.2+2.25$

$\approx 5+2.25$

≈ 7.25 minutes

Since the skit is about 7.25 minutes and 7 minutes and 20 seconds is close to 7.25 minutes, 7 minutes and 20 seconds is a reasonable answer.

2. You buy a sweater for $30.99, a pair of shoes for $46.79, a book bag for $15.20, and a DVD for $8.00. While in the checkout line you guess that your total bill before tax will be about $80. Is $80 a reasonable answer? Explain.

$30.99+46.79+15.20+8$

$31+62+8$

$\$101$

Since the total is approximately $\$101$, $\$80$ is not a reasonable answer.

3. Your teacher assigns you 130 pages to read in a book over the weekend. Friday night you read 28 1/2 pages, Saturday morning you read 42 3/4 pages, and Saturday night you read 18 pages. You tell your pages that you only have about 40 pages left to read. Is this a reasonable estimate? Explain.

$28\frac{1}{2}+42\frac{3}{4}+18$

$28.5+42.75+18$

≈ 89 pages

$130-89=41$ pages

Yes, it is a reasonable answer to say that you have about 40 pages left to read.

4. You have $25 to spend in a gift shop. You choose four items with prices of $4, $7.79, $16.20, and $2.09. Is it reasonable to assume you can buy all of these items? Explain.

$4+7.79+16.20+2.09$

$4+24+2.09$

$\approx \$30$

NO, it is NOT reasonable to assume that you can buy all of these items.

5.6 Using Multiplicative Inverses to Solve Equations

Objective: Use multiplicative inverses to solve equations.

The multiplicative inverses of a nonzero number is the number's reciprocal.

Multiplicative Inverse Property

Words The product of a number and its multiplicative inverse is 1.

Numbers $\frac{3}{5} \cdot \frac{5}{3} = 1$

Algebra $\frac{a}{b} \cdot \frac{b}{a} = 1$, where $a \neq 0, b \neq 0$

Example 1: Solving a One-Step Equation

Solve the equation. Show your work and check your answer.

1. $\frac{3}{5}x = 15$ $\left(\frac{5}{3}\right) \cdot \frac{3}{5}x = \frac{15}{1} \cdot \left(\frac{5}{3}\right)$ $x = \frac{25}{1}$ $x = 25$

2. $\frac{6}{11}x = 18$ $\left(\frac{11}{6}\right) \cdot \frac{6}{11}x = \frac{18}{1} \cdot \left(\frac{11}{6}\right)$ $x = \frac{33}{1}$ $x = 33$

Check It Out!

Solve the equation. Show your work and check your answer.

1. $-\frac{7}{13}x = 28$ $\left(\frac{13}{-7}\right) \cdot \frac{-7}{13}x = \frac{28}{1} \cdot \left(\frac{13}{-7}\right)$ $x = \frac{52}{-1}$ $x = -52$

2. $\frac{4}{7}x = -12$ $\left(\frac{7}{4}\right) \cdot \frac{4}{7}x = \frac{-12}{1} \cdot \left(\frac{7}{4}\right)$ $x = \frac{-21}{1}$ $x = -21$

Example 2: Solving a Two-Step Equation

Solve the equation. Show your work and check your answer.

1. $-\frac{7}{13}x + \frac{3}{4} = \frac{1}{2}$

$\frac{2}{4} - \frac{3}{4}$

$-\frac{3}{4} \quad -\frac{3}{4}$

$\left(\frac{13}{-7}\right) \cdot \frac{-7}{13}x = \frac{-1}{4} \cdot \left(\frac{13}{-7}\right)$

$x = \frac{-13}{-28}$ $x = \frac{13}{28}$

Check It Out!

Solve the equation. Show your work and check your answer.

1. $-\frac{11}{15}x + \frac{4}{5} = \frac{1}{3}$

$\frac{1}{3} - \frac{4}{5}$

$\frac{5}{15} - \frac{12}{15}$

$-\frac{4}{5} \quad -\frac{4}{5}$

$\left(\frac{15}{-11}\right) \cdot \frac{-11}{15}x = \frac{-7}{15} \cdot \left(\frac{15}{-11}\right)$

$x = \frac{-7}{-11}$ $x = \frac{7}{11}$

Example 3: Writing and Solving a Two-Step Equation

The height of a certain Norway Spruce is 10 feet. If the tree's height grows $2\frac{1}{2}$ feet per year, find how long it will take the tree to reach a height of 25 height.

current height + growth rate · number of years = new height

$10 + 2\frac{1}{2}x = 25$

$10 + \frac{5}{2}x = 25$

$-10 \qquad -10$

$\left(\frac{2}{5}\right) \cdot \frac{5}{2}x = \frac{15}{1} \cdot \left(\frac{2}{5}\right)$

$x = \frac{6}{1}$

$\boxed{x = 6}$

The tree will be 25 feet after 6 years.

Check It Out!

Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is 10 inches long and is growing at a rate of about $\frac{1}{8}$ inch per decade. How long will it take for the stalactite to reach a length of 1 foot?

current length + growth rate · number of decades = new length

$10 + \frac{1}{8}x = 12$

$-10 \qquad -10$

$\left(\frac{8}{1}\right) \cdot \frac{1}{8}x = \frac{2}{1} \cdot \left(\frac{8}{1}\right)$

$x = \frac{16}{1}$

$\boxed{x = 16}$

The stalactites will be 1 foot long after 16 decades or 160 years.

5.7 Equations and Inequalities with Rational Numbers

Objective: Use the LCD to solve equations and inequalities.

Another way to solve an equation with fractions is to clear fractions by multiplying each side by the LCD of the fractions. The resulting equation is equivalent to the original equation.

Example 1: Solving an Equation by Clearing Fractions

Solve each equation by eliminating the fractions.

1. $\left(\frac{1}{4}x + \frac{3}{10} = \frac{2}{5}\right) \cdot 20$

$20\left(\frac{1}{4}x\right) + 20\left(\frac{3}{10}\right) = 20\left(\frac{2}{5}\right)$

$5x + 6 = 8$

$-6 \quad -6$

$\frac{5x}{5} = \frac{2}{5}$

$\boxed{x = \frac{2}{5}}$

2. $\left(\frac{1}{3}x + \frac{5}{6} = \frac{7}{9}\right) \cdot 18$

$18\left(\frac{1}{3}x\right) + 18\left(\frac{5}{6}\right) = 18\left(\frac{7}{9}\right)$

$6x + 15 = 14$

$-15 \quad -15$

$\frac{6x}{6} = \frac{-1}{6}$

$\boxed{x = \frac{-1}{6}}$

Check It Out!

Solve each equation by eliminating the fractions.

3. $\left(\frac{3}{10} - \frac{7}{15}x = \frac{2}{3}\right) \cdot 30$

$30\left(\frac{3}{10}\right) - 30\left(\frac{7}{15}x\right) = 30\left(\frac{2}{3}\right)$

$9 - 14x = 20$

$-14x = 11$

$\boxed{x = \frac{-11}{14}}$

4. $\left(-\frac{2}{9} = \frac{3}{4} - \frac{1}{6}x\right) \cdot 36$

$36\left(\frac{-2}{9}\right) = 36\left(\frac{3}{4}\right) - 36\left(\frac{1}{6}x\right)$

$-8 = 27 - 6x$

$-35 = -6x$

$\boxed{x = \frac{35}{6}}$

Example 2: Solving an Inequatily by Clearing Fractions

Solve each inequality by eliminating the fractions.

1. $\left(\frac{3}{7}x + \frac{1}{4} < \frac{1}{2}\right) \cdot 28$

$28\left(\frac{3}{7}x\right) + 28\left(\frac{1}{4}\right) < 28\left(\frac{1}{2}\right)$

$12x + 7 < 14$

$12x < 7$

$\boxed{x < \frac{7}{12}}$

2. $\left(\frac{5}{6}x - \frac{1}{5} < -\frac{8}{15}\right) \cdot 30$

$30\left(\frac{5}{6}x\right) - 30\left(\frac{1}{5}\right) < 30\left(\frac{-8}{15}\right)$

$25x - 6 < -16$

$25x < -10$

$x < \frac{-10}{25} \div 5$

$\boxed{x < \frac{-2}{5}}$

Check It Out!
Solve each inequality by eliminating the fractions.

3. $\left(\frac{8}{15}x - \frac{17}{30} > \frac{7}{10}\right) \cdot 30$

$30\cdot\left(\frac{8}{15}x\right) - 30\left(\frac{17}{30}\right) > 30\left(\frac{7}{10}\right)$

$16x - 17 > 21$

$16x > 38$

$x > \frac{38}{16}$

$\boxed{x > \frac{19}{8}}$

4. $\left(\frac{4}{5} > \frac{2}{3} - \frac{2}{5}x\right) \cdot 15$

$15\left(\frac{4}{5}\right) > 15\left(\frac{2}{3}\right) - 15\left(\frac{2}{5}x\right)$

$12 > 10 - 6x$

$2 > -6x$

$\frac{2}{-6} > x$

$\boxed{x > \frac{-1}{3}}$

Example 3: Solving an Equation by Clearing Decimals
Solve each inequality by first clearing the decimals.

1. $(0.5m - 4.9 = 2.6) \cdot 10$

$10(0.5m) - 10(4.9) = 10(2.6)$

$5m - 49 = 26$

$5m = 75$

$\boxed{m = 15}$

2. $(0.2x + 5.7 = 9.3) \cdot 10$

$10(0.2x) + 10(5.7) = 10(9.3)$

$2x + 57 = 93$

$2x = 36$

$\boxed{x = 18}$

Check It Out!
Solve each inequality by first clearing the decimals.

3. $(0.55 - 0.2k = 0.15) \cdot 100$

$100(0.55) - 100(0.2k) = 100(0.15)$

$55 - 20k = 15$

$-20k = -40$

$k = \frac{-40}{-20}$

$\boxed{k = 2}$

4. $(0.5 - 0.4p = 1.3) \cdot 10$

$10(0.5) - 10(0.4p) = 10(1.3)$

$5 - 4p = 13$

$-4p = 8$

$\boxed{p = -2}$