Chapter 1: Variables, Expressions, and Integers
1.1 Expressions and Variables

Objective: Evaluate and write variable expressions.

A numeric expression consists of numbers and operations.

A variable is a letter used to represent one or more numbers.

A variable expression consists of numbers, variables, & operations.

To evaluate a variable expression, substitute a number for each variable and calculate the resulting numeric expression.

A verbal model describes a problem using words as labels and using math symbols to relate the words.

Example 1: Evaluating a Variable Expression

Evaluate the expression for $x = 8$.

1. $3 * x$
   $3 \cdot 8$
   24
2. $x + 12$
   $8 + 12$
   20
3. $10 - x$
   $10 - 8$
   2
4. $\frac{24}{x}$
   $\frac{24}{8}$
   3

Example 2: Evaluating Expressions with Two Variables

Evaluate the expression when $x = 10$ and $y = 5$.

1. $x + y$
   $10 + 5$
   15
2. $x - y$
   $10 - 5$
   5
3. xy
   $10 \cdot 5$
   50
4. $\frac{x}{y}$
   $\frac{10}{5}$
   2

Evaluate the expression when $a = 8$ and $b = 2$.

1. $a - b$
   $8 - 2$
   6
2. $a + b$
   $8 + 2$
   10
3. $a * b$
   $8 \cdot 2$
   16
4. $\frac{a}{b}$
   $\frac{8}{2}$
   4

Common Words and Phrases that Indicate Operations			
Addition	**Subtraction**	**Multiplication**	**Division**
added to increase by more than product total	subtracted by minus difference less than	multiply times product of	divide quotient

Example 3: Writing a Variable Expression

1. You have a 350-page manuscript that needs to be edited very quickly. You are going to divide the number of pages among several editors. You want to give the same number of pages to each editor. Use a verbal model to write a variable expression for the number of pages given to each editor if you know the number of editors.

Number of pages for each editor = Total number of pages ÷ Number of editors

$350 \div e$

or

$\frac{350}{e}$

2. Grove City, Pennsylvania received 5 fewer inches of rain this year than last year. Use a verbal model to write a variable expression for the number of inches of rain Grove City received this year if you know the number of inches of rain Grove City received last year.

Number of inches this year = Number of inches last year − 5 inches

$n - 5$

Write a variable expression to represent the phrase.

3. The difference of 17 and a number $17 - n$
4. The quotient of a number and 5 $\frac{n}{5}$
5. 10 more than a number $n + 10$

1.2 Powers and Exponents

Objective: Use powers to describe repeated multiplication.

A power is the result of a repeated multiplication of the same factor.
A base is the number or variable that is used as a factor in repeated multiplication.
An exponent is the number that represents the number of times the base is used as a factor.
A formula describes a relationship between quantities.

Example 1: Using Exponents

Write the product using an exponent.

1. $7*7*7*7*7 = 7^5$ — The base 7 is used as a factor 5 times.
2. $(0.4)(0.4) = 0.4^2$ — The base 0.4 is used as a factor 2 times.
3. $a*a*a*a = a^4$ — The base a is used as a factor 4 times.
4. $r*r*r*r*r*r*r = r^7$ — The base r is used as a factor 7 times.
5. $5*5*5 = 5^3$ — The base 5 is used as a factor 3 times.
6. $(0.2)(0.2)(0.2)(0.2) = 0.2^4$ — The base 0.2 is used as a factor 4 times.
7. $x*x*x*x*x*x = x^6$ — The base x is used as a factor 6 times.

Example 2: Evaluating Powers with Variables

Evaluate the expression when $x = 0.4$

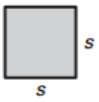
1. x^2: 0.4^2, $(0.4)(0.4)$, 0.16
2. x^3: 0.4^3, $(0.4)(0.4)(0.4)$, 0.064
3. x^4: 0.4^4, $(0.4)(0.4)(0.4)(0.4)$, 0.0256
4. x^5: 0.4^5, $(0.4)(0.4)(0.4)(0.4)(0.4)$, 0.01024

Evaluate the expression when $n = 2$.

1. n^2: 2^2, $2 \cdot 2$, 4
2. n^3: 2^3, $2 \cdot 2 \cdot 2$, 8
3. n^4: 2^4, $2 \cdot 2 \cdot 2 \cdot 2$, 16
4. n^5: 2^5, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, 32

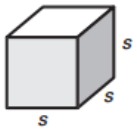
Area is measured in square units, such as square feet (ft^2) or square centimeters (cm^2). Volume is measured in cubic units, such as cubic inches ($in.^3$) or cubic meters (m^3).

Area and Volume Formulas



Area A of a square

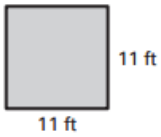
$A = s^2$



Volume V of a cube

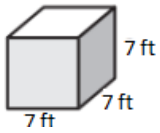
$V = s^3$

1. You are planning to put wall-to-wall carpeting in your room. To do this, you need to find the area of the square-shaped floor.



$A = s^2$
$A = 11^2$
$A = 11 \cdot 11$
$A = 121\ ft.^2$

2. You are filling a swimming pool that is shaped like a cube with water and asked how much water the pool can hold. To do this, you need to find the volume of cube.



$A = s^3$
$A = 7^3$
$A = 7 \cdot 7 \cdot 7$
$A = 343\ ft.^3$

Find the area of a square with the given side lengths.

3. 2 meters

$A = s^2$
$A = 2^2$
$A = 2 \cdot 2$
$A = 4\ m^2$

4. 3 feet

$A = s^2$
$A = 3^2$
$A = 3 \cdot 3$
$A = 9\ ft.^2$

5. 6 inches

$A = s^2$
$A = 6^2$
$A = 6 \cdot 6$
$A = 36\ in.^2$

Find the volume of a cube with the given side lengths.

6. 2 meters

$V = s^3$
$V = 2^3$
$V = 2 \cdot 2 \cdot 2$
$V = 8\ m^3$

7. 3 feet

$V = s^3$
$V = 3^3$
$V = 3 \cdot 3 \cdot 3$
$V = 27\ ft.^3$

8. 6 inches

$V = s^3$
$V = 6^3$
$V = 6 \cdot 6 \cdot 6$
$V = 216\ in.^3$

9. 1 unit

$V = s^3$
$V = 1^3$
$V = 1 \cdot 1 \cdot 1$
$V = 1\ unit^3$

1.3 Order of Operations

Objective: Use order of operations to evaluate expressions.

The order of operations are the rules established to evaluate an expression involving one or more operations.

Grouping symbols indicate which operations should be performed first. Parenthesis (), brackets [], and fraction bars are common grouping symbols.

Order of Operations

1. Evaluate expressions inside grouping symbols.
2. Evaluate powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Example 1: Use Order of Operations

Evaluate the expression.

1. $4 * 20 + 8 * 5 + 4$

$80 + 8 \cdot 5 + 4$
$80 + 40 + 4$
$120 + 4$
$\boxed{124}$

2. $20 - 6 * 3$

$20 - 18$
$\boxed{2}$

3. $56 \div 8 - 4$

$7 - 4$
$\boxed{3}$

4. $12 * 3 - 18 \div 6$

$36 - 18 \div 6$
$36 - 3$
$\boxed{33}$

5. $20 + 2 * 5 + 3 * 8$

$20 + 10 + 3 \cdot 8$
$20 + 10 + 24$
$30 + 24$
$\boxed{54}$

6. $3 * 10 - 2 + 4$

$30 - 2 + 4$
$28 + 4$
$\boxed{32}$

Example 2: Use Grouping Symbols

Evaluate the expression.

1. $5(14 - 3)$

$5(11)$
$\boxed{55}$

2. $\dfrac{27 - 3}{4 + 2}$

$\dfrac{24}{4+2}$
$\dfrac{24}{6}$
$\boxed{4}$

3. $4 * [35 - (11 + 9)]$

$4[35 - 20]$
$4[15]$
$\boxed{60}$

4. $5(3) + 3(2)$

$15 + 3(2)$

$15 + 6$

$\boxed{21}$

5. $\dfrac{36 - 12}{2 + 6}$

$\dfrac{24}{2+6}$

$\dfrac{24}{8}$

$\boxed{3}$

6. $24 \div [(18 - 16) * 3]$

$24 \div [2 \cdot 3]$

$24 \div [6]$

$\boxed{4}$

Example 3: Evaluate Variable Expressions

Evaluate the expression when $x = 3$ and $y = 6$.

1. $3(x + y)$

$3(3+6)$

$3(9)$

$\boxed{27}$

2. $5(y - x)^2$

$5(6-3)^2$

$5(3)^2$

$5 \cdot 9$

$\boxed{45}$

3. $x + y \div 2$

$3+6 \div 2$

$3+3$

$\boxed{6}$

Evaluate the expression when $x = 4$ and $y = 5$.

4. $y(19 - x^2)$

$5(19-4^2)$

$5(19-16)$

$5(3)$

$\boxed{15}$

5. $\dfrac{6y}{x + 1}$

$\dfrac{6 \cdot 5}{4+1}$

$\dfrac{30}{4+1}$

$\dfrac{30}{5}$

$\boxed{6}$

6. $(4x - 3y)^3$

$(4 \cdot 4 - 3 \cdot 5)^3$

$(16 - 3 \cdot 5)^3$

$(16 - 15)^3$

1^3

$\boxed{1}$

1.3.5 Using Order of Operations

Objective: Use a calculator to evaluate expressions using the order of operations.

Alex Rodriguez played for the Texas Rangers during the 2002 baseball season. Use the following information to calculate his batting average for that season.

To find a baseball player's batting average, you divide the number of hits he made by the number of times he was a bat and round the quotient to the nearest thousand. The table gves Alex Rodriguez's 2002 batting statistics.

2002 season	Hits	At bats
Before All-Star Game	100	328
After All-Star Game	87	296

$(100+87) \div (328+296) = 0.300$

Note: grouping symbols are important, without them you get a different answer

Derek Jeter played for the New York Yankees during the 2002 season. Use the information in the table to calculate his batting average for the entire 2002 season.

2002 season	Hits	At bats
Before All-Star Game	109	349
After All-Star Game	82	295

$(109+82) \div (349+295) = 0.297$

Use a calculator to evaluate the expression.

1. $50 + 21 \div 3$ — 57

2. $15 * (24 + 8)$ — 480

3. $(8 + 10) \div 2$ — 9

4. $(5 + 2)^2 - 3^2$ — 40

5. $(24 - 16) \div 2$ — 4

6. $(12 - 7)^2 - 1$ — 24

7. $38 \div (2 + 17)$ — 2

8. $(8 + 3)^2 + 2$ — 123

1.4 Comparing and Ordering Integers

Objective: Compare and order integers.

Integers are the numbers..., -3, -2, -1, 0, 1, 2, 3,...

Negative integers are the integers that are less than zero.

Positive integers are the integers that are greater than zero.

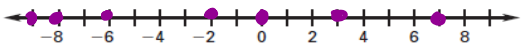
The absolute value of a number is its distance from 0 on a number line. The absolute value of a number *a* is written as $|a|$.

Example 1: Graphing and Ordering Integers

Use a number line to order these integers from least to greatest.

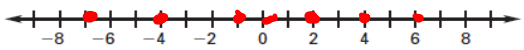
1. 0, -6, -2, -8, 7, -9, 3

-9, -8, -6, -2, 0, 3, 7



2. 2, -7, 6, 4, 0 , -4, -1

-7, -4, -1, 0, 2, 4, 6



Example 2: Using the Quotient of Powers Property

State the absolute value of the number.

1. 7 — 7
2. -5 — 5
3. -6 — 6
4. 2 — 2
5. |8| — 8
6. |3| — 3
7. |-10| — 10
8. |-4| — 4

Example 3: Finding Opposites

State the opposite of the number.

1. 3 — -3
2. -8 — 8
3. 9 — -9
4. -12 — 12
5. 8 — -8
6. -3 — 3
7. -2 — 2
8. 1 — -1

1.5 Adding Integers

Objective: Add integers.

The opposite of a number is also call its additive inverse.

The additive inverse property states that the sum of a number and its opposite is 0.

Example 1: Adding Integers

Find the sum.

1. 7 + (-10) -3
2. -6 + 5 -1
3. 9 + (-6) 3
4. -9 + (-3) -12

5. 5 + 3 8
6. -4 + (-2) -6
7. 6 +(-4) 2
8. -12 + 3 -9

Adding Integers

Words	Numbers
1. **Same Sign** Add the absolute values and use the common sign.	8 + 12 = 20 −6 + (−4) = -10
2. **Different Signs** Subtract the lesser absolute value from the greater absolute value and use the sign of the number with the greater absolute value.	5 + (−8) = -3 −11 + 13 = 2
3. **Opposites** The sum of a number and its opposite is 0.	7 + (−7) = 0

Example 2: Adding Two Integers

Find the sum

1. -35 + (-18) -53
2. 27 + (-13) 14
3. -19 + 36 17
4. -20 + (-5) -25

5. 25 + (-10)

15

6. -15 + (-8)

-23

7. 21 + 6

27

8. -20 + 5

-15

Example 3: Adding More Than Two Integers

Find the sum.

1. -7 + (-41) + 32

-48 + 32

-16

2. -19 + 36 + (-5)

-24 + 36

12

3. -29 + (-31) + 47

-60 + 47

-13

4. 8 + (-20) + 42

-12 + 42

30

5. -10 + 25 + (-5)

15 + -5

10

3. -41 + (-14) + 17

-55 + 17

-38

1.6 Subtracting Integers

Objective: Subtract integers.

Subtracting Integers

Words To subtract an integer, add its opposite.

Numbers $3 - 7 = 3 + (-7) = -4$

Algebra $a - b = a + (-b)$

Example 1: Subtracting Integers

Find the difference.

1. 5 - 9
   5 + -9
   -4
2. -4 - (-10)
   -4 + 10
   6
3. 3 - 8
   3 + -8
   -5
4. -2 - 9
   -2 + 9
   7

Example 2. Evaluate Variable Expressions

Evaluate the expression when $x = -8$.

1. $x - (-22)$
   -8 - -22
   -8 + 22
   14
2. $9 - x$
   9 - -8
   9 + 8
   17
3. $x - 8$
   -8 - 8
   -8 + -8
   -16

Evaluate the expression when $y = -12$.

4. $y - 8$
   -12 - 8
   -12 + -8
   -20
5. $19 - y$
   19 - -12
   19 + 12
   31
6. $-7 - y$
   -7 - -12
   -7 + 12
   5

Example 3: Evaluate Change

1. Write a verbal model to find the change in temperature given the start temperature and the end temperature. Use the model to find the change in temperature from -5°F to 12°F.

Change in temp. = ending temp. − starting temp.

12 − -5

12 + 5

17°F

Find the change in temperature.

2. -3°F to -8°F

-8 − -3

-8 + 3

-5°F

3. -15°C to -2°C

-2 − -15

-2 + 15

13°C

4. -10°C to -18°C

-18 − -10

-18 + 10

-8°C

1.6.5 Mean, Median, Mode, & Range

Objective: Identify the mean, median, mode, and range of a data set.

The mean is the sum of the values divided by the number of values.

The median is the middle value when the value are written in numerical order. If a data set has an even number of values, the median is the middle of the two middle values.

The mode is the value that occurs most often. A data set can have no mode, one mode, or more than one modes.

The range is the difference of the greatest value and the least value.

Measures of Central Tendency and Dispersion

Measures of Central Tendency: Mean, median, mode

Measure of Dispersion: range

Example 1: Finding the Mean

Kelsey's test scores are listed below. Find the mean of the data.

82, 85, 84, 88, 92, 94, 86, 79, 94, 100

$\frac{82+85+84+88+92+94+86+79+94+100}{10} = \frac{884}{10} = 88.4$

Example 2: Find the Median and Mode

Find the median and mode of the given data set.

82, 85, 84, 88, 92, 94, 86, 79, 94, 100

79, 82, 84, 85, 86, 88, 92, 94, 94, 100

median: $\frac{86+88}{2} = \frac{174}{2} = 87$

mode: 94

Example 3: Find the Range

Find the range of the given data set.

82, 85, 84, 88, 92, 94, 86, 79, 94, 100

$100 - 79 = 21$

Find the mean, median, mode, and range of the given data sets.

1. Daily low temperatures in degrees Fahrenheit:
   -7, -5, 0, 2, 0, -6, -5

   -7, -6, -5, -5, 0, 0, 2

   mean: $\frac{-7+-6+-5+-5+0+0+2}{7} = \frac{-21}{7} = \boxed{-3}$

   median: $\boxed{-5}$

   mode: $\boxed{0}$

   range: $2--7 = 2+7 = \boxed{9}$

2. Number of students in different classes:
   22, 21, 26, 25, 24, 18, 23, 19

   18, 19, 21, 22, 23, 24, 25, 26

   mean: $\frac{18+19+21+22+23+24+25+26}{8} = \frac{178}{8} = \boxed{22.25}$

   median: $\frac{22+23}{2} = \frac{45}{2} = \boxed{22.5}$

   mode: $\boxed{\text{none}}$

   range: $26-18 = \boxed{8}$

3. Golf scores relative to par:
   -2, -3, 0, 1, 3, 4, -1, 2

   -3, -2, -1, 0, 1, 2, 3, 4

   mean: $\frac{-3+-2+-1+0+1+2+3+4}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$

   median: $\frac{0+1}{2} = \boxed{\frac{1}{2}}$

   mode: $\boxed{\text{none}}$

   range: $4--3 = 4+3 = \boxed{7}$

4. Height in feet of trees in a park:
   51, 65, 75, 43, 58, 85, 75, 60

   43, 51, 58, 60, 65, 75, 75, 85

   mean: $\frac{43+51+58+60+65+75+75+85}{8} = \frac{512}{8} = \boxed{64}$

   median: $\frac{60+65}{2} = \frac{125}{2} = \boxed{62.5}$

   mode: $\boxed{75}$

   range: $85-43 = \boxed{42}$

1.7 Multiplying & Dividing Integers

Objective: Multiply and divide integers.

Multiplying Integers

Words	Numbers
The product of two integers with same sign is positive.	$2(4) = 8$ $\quad -2(-4) = 8$
The product of two integers with different signs is negative.	$2(-4) = -8$ $\quad -2(4) = -8$
The product of any integer and 0 is 0.	$2(0) = 0$ $\quad -2(0) = 0$

Example 1: Multiplying Integers

Find the product.

1. -5(-8) 40
2. -8(7) -56
3. -51(0) 0
4. 7(-12) -84
5. -9(-5) 45
6. -250(0) 0
7. -4(11) -44
8. -2(-3) 6

Dividing Integers

Words	Numbers
The quotient of two integers with same sign is positive.	$8 \div 4 = 2$ $-8 \div (-4) = 2$
The quotient of two integers with different signs is negative.	$-8 \div 4 = -2$ $8 \div (-4) = -2$
The quotient of 0 and any nonzero integer is 0.	$0 \div 4 = 0$ $0 \div (-4) = 0$

Example 1: Dividing Integers

Find the quotient.

1. -63 ÷ (-9) 7
2. 24 ÷ (-4) -6
3. 32 ÷ (-8) -4
4. 0 ÷ (-43) 0
5. 32 ÷ (-4) -8
6. 0 ÷ (-2) 0
7. -38 ÷ (-19) 2
8. -2 ÷ (-1) 2

1.8 The Coordinate Plane

Objective: Identify and plot points on the coordinate plane.

A Coordinate plane is formed by the intersection of a horizontal number line and a vertical number line.

The horizontal axis in a coordinate plane is the x-axis.

The vertical axis in a coordinate plane is the y-axis.

The origin is the point (0, 0) in a coordinate plane at which the horizontal axis intersects the vertical axis.

A quadrant is one of four parts into which the axes divide the coordinate plane.

Each point in the coordinate plane corresponds to a pair of real numbers that gives the location of the point on the plane is called an ordered pair.

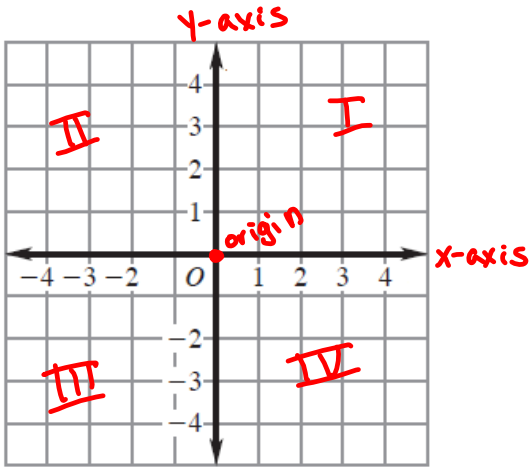
The first number in an ordered pair is the x-coordinate.

The second number in an ordered pair is the y-coordinate.

A scatter plot uses a coordinate plane to display paired data.

Identify the following features of a coordinate plane:

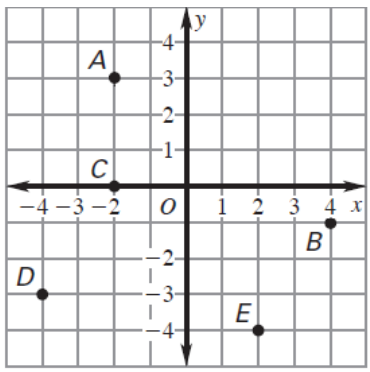
- x-axis
- y-axis
- origin
- the four quadrants



Example 1: Naming Points on a Coordinate Plane

Give the coordinate of the point.

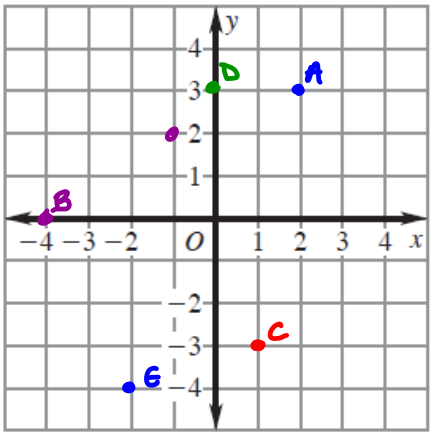
1. A (-2,3)
2. B (4,-1)
3. C (-2,0)
4. D (-4,-3)
5. E (2,-4)



Example 2: Plotting Points on a Coordinate Plane

Plot the point in the coordinate plane. Describe the location of the point.

1. A (2, 3) Quadrant I
2. B (-4, 0) x-axis
3. C (1, -3) Quadrant IV
4. D (0, 3) y-axis
5. E (-2, -4) Quadrant III
6. F (-1, 2) Quadrant II



Example 3: Making a Scatter Plot

The number of hours you spent studying for 5 different math tests and the score you got on each test is given in the table. Make a scatter plot of the data and describe the relationship you see.

Hours studying	1	3	4	4	5
Test score	55	78	86	89	98

The points increase from left to right, so as the number of hours studying increases the test score increases.